Suppress the accumulated effect of channel noise on ILC systems over wireless channels

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Abstract: In this paper, we address the problem of suppressing the accumulated effect of channel noises on the convergence of ILC system. First, the relation between iterative difference values of input, controller-to-actuator (CA) noise and sensor-to-controller (SC) noise is derived using super-vector formulation. After that, a method is proposed to suppress the accumulated effect of channel noise on the convergence of system based on this relation. Specifically, the learning gain is used as a filter, and adjusted adaptively in iteration domain through minimizing the trace of the iterative difference value of input covariance matrix. With the adjusted learning gain, the contribution of the iterative difference values of CA and SC noise on that of input are all filtered from the perspective of covariance matrix norm. Finally, some numerical experiments are given to illustrate the effectiveness of the proposed method.

Key Words: iterative learning control (ILC), channel noise, convergence

1 INTRODUCTION

As a control algorithm, iterative learning control (ILC) aims to improve the desired output of the system that operate repeatedly over a finite time cycle, and has a well-established research history [1, 2, 3]. The main feature of this algorithm is to adjust the control signal for the present cycle using the control signal and the output errors in the previous cycle. Due to its potential merits and ease of implementation, ILC has attracted considerable attention in many areas and applications.

Recently, motivated by network control systems, the problem of convergence with respect to ILC systems over wireless channels has been considered by researchers [4, 5]. In this system, the wireless channel introduced can separate iterative learning controller from the system platform. Obviously, the system not only has advantages of classic ILC systems, but also satisfies the need for rapid deployment, flexible installation and node mobility in many industrial applications.

However, the introduction of the wireless channel brings greater challenge to guarantee the convergence of ILC system over wireless channels. Besides the influence of time delay and data dropouts, another important issue is wireless channel-induced noise, which occurs during control and measurement signals are exchanged among sensors linked by the wireless network. The channel noise may affect the convergence of the ILC system if the system is designed without taking them into account.

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On the one hand, studies on ILC systems considering communication constraints appeared in the last few years. For example, In [6], authors proposed an ILC method for a class of sampled-data non-linear systems with constant time delay, and studied the effect of data dropouts on the convergence of the system. A compensation method was proposed in a further study when an ILC system is implemented in presence of random time delay and data dropouts in measurement signals [7]. Authors in [8] discussed the stability of ILC systems with data dropouts using an asynchronous system method, and gave a stability condition in the form of linear matrix inequalities to guarantee convergence of the system. For a class of nonlinear systems with data dropouts in input and measurement signals, authors pointed out that ILC systems converges even though some packet are lost if a given condition is met [9]. Kalman filtering approach was used in [10] to design a learning gain, which can guarantee the convergence performance of the ILC system in presence of measurement signals dropouts. Authors in [11] proposed to use the control data at the same time with the dropped ones in last iteration to compensate the effect of control data dropouts on the convergence of ILC systems over networks at actuator side.

On the other hand, studies on ILC systems subject to state disturbance and/or measurement noise can be found in many literatures (see, e.g., [12, 13, 14] and references therein). In particular, optimal and suboptimal ILC algorithms shown in [15, 16] provide a worthy consultation to our study. Based on two-dimensional Roessor Model, the state error and input error were augmented, and then a relation between the augmented error vector and noise vector was obtained. Through minimizing the trace of covariance matrix of the augmented error vector, forgetting ma-

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trix and/or learning gain matrix were selected adaptively, so the effect of noise on the system was suppressed. In [17], the author proposed a PID (proportional plus integral and derivative) type ILC update law for control discrete-time single input single-output (SISO) linear time-invariant (LTI) systems. An error dynamic relation of closed-loop system was derived, and the P, I, D learning gains were designed optimally through minimizing the upper bound of the eigenvalue of the system matrix in the derived relation. The convergence of the system with the optimally designed P, I, D learning gains was also proved.

We would like to highlight that the channel noise not only differ from time delay or data dropouts introduced by network congestion, but also unlike state disturbance or measurement noise. Channel noise is an additive interference, while time delay and data dropouts belong to multiplicative disturbance. Furthermore, it can be easily seen that channel noise is introduced externally, while state disturbance or measurement noise is introduced internally, which reveals the model of ILC system over wireless channels in presence of channel noise is different from that of ILC system with state disturbance and/or measurement noise. As a result, the problem of ILC system over wireless channels in presence of channel noise needs to be addressed.

These observations motivate us to pursue our studies along this track. Specially, we have analyzed the convergence of the ILC system over wireless channels in presence of channel noise using super-vector formulation [18]. The analysis revealed that the effect of channel noise on the system contains non-accumulated and accumulated effect. And what's more, we proved the accumulated part domains the effect of channel noise on the convergence of system in certain circumstances. In [19], we proposed a method to improve the convergence of the ILC system over wireless channels in presence of channel noise, but the method requires knowledge of system state, which cannot be obtained easily in practice.

Keeping to this track of our research activities, in this paper, we assume there are no time delay and data dropouts so that we can continue concentrate our efforts to address the problem of suppressing the accumulated effect of channel noise on ILC system over wireless channels. Compared with the above existing works, this paper makes the following contributions:

- Using super-vector formulation, the relation between iterative difference value of input, controller-to-actuator (CA) noise and sensor-to-controller (SC) noise in iteration domain is derived, which reveals the contribution of the iterative difference values of CA and SC noise to that of input are all restricted by learning gain;

- Based on this relation, a method is proposed to suppress the accumulated effect of channel noise on the ILC system over wireless channels. To be specific, the learning gain is used as a filter, and adjusted adaptively in iteration domain through minimizing the trace of the iterative difference value of input covariance matrix. Convergence of the system with this method is proved theoretically, and some numerical examples are given to illustrate the effectiveness of the proposed method.

The remainder of this paper is organized as follows. In next section, the problem of ILC systems taking the channel noise into account is formulated. After that, the idea behind the method design is given, and the method to suppress the accumulated effect of channel noise on ILC systems over wireless channels is proposed in Section III. In Section IV, the convergence of the system with the proposed method is proven from the perspective of first order difference of input error covariance matrix norm and the limit of output error covariance matrix norm is derived, which shows the accumulated effect of channel noise is suppressed. Then, numerical experiments are given to illustrate the effectiveness of the proposed method in Section V. Finally, some conclusions wrap up this paper in Section VI.

Notations: throughout the paper, the superscripts "-1" and "T" represent the inverse and transpose operations, respectively; "$\xi[\cdot]$" represents expectation operator; "$||\cdot||$" represents norm operator.

2 PROBLEM FORMULATION

Consider the deterministic, discrete-time, linear, time-invariant system described as following equation

$$\begin{cases} x_k(t+1) = Ax_k(t) + Bu_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{1}$$

where $x_k(t) \in R^n$, $u_k(t) \in R^m$ and $y_k(t) \in R^l$ are state, control input and output variables, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{l\times n}$ are system matrices, respectively. The subscript $k = 0, 1, \cdots$ is used to indicate iteration number, $t \in [0, 1, \cdots, T]$ is used to denote time.

If CB is full rank, the desired trajectory $y_d(t)$ which the system needs to track in the time interval is realizable with a unique control input $u_d(t)$. That is to say the following equation are satisfied

$$\begin{cases} x_d(t+1) = Ax_d(t) + Bu_d(t) \\ y_d(t) = Cx_d(t) \end{cases} \tag{2}$$

where $x_d(t)$ is the desired state. In order to track the $y_d(t)$ accurately, the controller adopts ILC algorithm is an effective method. The first order P-style ILC algorithm can be given in the following form

$$u_{k+1}(t) = u_k(t) + \Gamma(t)e_k(t+1) \tag{3}$$

where $\Gamma(t) \in R^{r\times l}$ is learning gain, $e_k(t) = y_d(t) - y_k(t)$ is output error, .

For ILC systems, there are two convergence concepts: asymptotic stability and monotonic convergence [2]. As to the latter, the condition that the learning gain satisfied to guarantee monotonic convergence is given after transforming in next section.

The ILC system over wireless networks is illustrated in Figure 1. In this system, measurement and control signals are transmitted from the sensor to the controller and

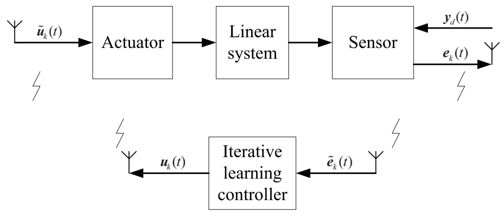

Figure 1: Block ILC system over wireless channels

from the controller to the actuator over wireless channels, respectively. However, the introduction of wireless channels brings some new challenges in contrast to dedicated connections. The most important one is channel noise, including SC noise and CA noise. Taking effect of the SC noise and the CA noise into account, the system can be given as following

$$\begin{cases} x_k(t+1) = Ax_k(t) + B\tilde{u}_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{4}$$

$$u_{k+1}(t) = u_k(t) + \Gamma(t)\tilde{e}_k(t+1) \tag{5}$$

where $\tilde{u}_k(t) = u_k(t) + m_k(t)$ is the actuator received control signal, $\tilde{e}_k(t+1) = e_k(t+1) + n_k(t+1)$ is the controller received measurement signal, $m_k(t)$ is the CA noise, and $n_k(t)$ is the SC noise, respectively. The analysis in [18] shows that the accumulated of the SC noise and the CA noise has a significant effect on the convergence of the system. In order to suppress the accumulated effect of the SC noise and the CA noise, an adaptively adjusted learning gain method is proposed in next section.

3 SUPPRESS THE ACCUMULATED EFFECT OF CHANNEL NOISE

In this section, the relation between iterative difference value of input, CA noise and SC noise in iteration domain is derived first. After that, the method to suppress the accumulated effect of channel noise through adjusting learning gain adaptively in iteration domain is given. The following assumptions are used:

Assumption 1: CA noise and SC noise are uncorrelated white Gaussian noise, the mean of which are all 0, and the variance of which are m and n, respectively;

Assumption 2: Initial errors are zero-mean white Gaussian noise.

In [18], the two-dimensional ILC system over wireless channels is changed into a one-dimensional multi-input multi-output (MIMO) ILC system using super-vector method, which is given as following:

$$Y_k = G(U_k + M_k) \tag{6}$$

$$U_{k+1} = U_k + \Gamma(E_k + N_k) \tag{7}$$

whereY_k, E_k, U_k, M_k, N_k are output vector, output error vector, input vector, CA noise vector and SC noise vector, and described in (8)-(12), respectively. Γ is learning gain matrix, G is a lower-triangular Toeplitz matrix of the system, elements of which are the Markov parameters, which are given in (13) and (14), respectively.

$$Y_k = [y_k(1) \;\; y_k(2) \cdots \;\; y_k(T)]^T \tag{8}$$

$$E_k = [e_k(1) \;\; y_k(2) \cdots \;\; e_k(T)]^T \tag{9}$$

$$U_k = [u_k(0) \;\; u_k(1) \cdots \;\; u_k(T-1)]^T \tag{10}$$

$$M_k = [m_k(0) \;\; m_k(1) \cdots \;\; m_k(T-1)]^T \tag{11}$$

$$N_k = [n_k(1) \;\; n_k(2) \cdots \;\; n_k(T)]^T \tag{12}$$

$$\Gamma = \begin{bmatrix} \Gamma(0) & 0 & \cdots & 0 \\ 0 & \Gamma(1) & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \Gamma(T-1) \end{bmatrix} \tag{13}$$

$$G = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ g_2 & g_1 & & & 0 \\ g_3 & g_2 & g_1 & & 0 \\ \vdots & & & \ddots & \vdots \\ g_T & g_{T-1} & g_{T-2} & \cdots & g_1 \end{bmatrix}_{lT \times mT} \tag{14}$$

and $g_t = CA^{t-1}B, t = 1, 2, \cdots, T$.
From (6), we have

$$Y_{k+1} - Y_k = G(U_{k+1} + M_{k+1}) - G(U_k + M_k) \tag{15}$$

According to the definition of output error, $E_k = Y_d - Y_k$ can be easily derived, where $Y_d = [y_d(1) \;\; y_d(2) \;\; \cdots \;\; y_d(T)]^T$ is the desired output vector. The relation between output error vector and input vector can be readily derived

$$E_{k+1} - E_k = -G\left(U_{k+1} + M_{k+1}\right) + G\left(U_k + M_k\right) \tag{16}$$

Using (7) and (16), the recurrence relation between the first order difference of input vector $U_{k+1} - U_k$, the first order difference of CA noise $M_{k+1} - M_k$, and the first order difference of SC noise $N_{k+1} - N_k$ can be represented as

$$\begin{aligned} U_{k+1} - U_k &= U_k - U_{k-1} + \Gamma\left(E_k - E_{k-1}\right) \\ &\quad + \Gamma\left(N_k - N_{k-1}\right) \\ &= U_k - U_{k-1} + \Gamma\left(N_k - N_{k-1}\right) \\ &\quad - \Gamma G\left(\left(U_k + M_k\right) - \left(U_{k-1} + M_{k-1}\right)\right) \\ &= \left(I - \Gamma G\right)\left(U_k - U_{k-1}\right) \\ &\quad - \Gamma G\left(M_k - M_{k-1}\right) \\ &\quad + \Gamma\left(N_k - N_{k-1}\right) \end{aligned} \tag{17}$$

For compactness, we set $\Delta U_{k+1} = U_{k+1} - U_k$, $\Delta M_{k+1} = M_{k+1} - M_k$, $\Delta N_{k+1} = N_{k+1} - N_k$, and then (17) can be rewritten as

$$\Delta U_{k+1} = \left(I - \Gamma G\right)\Delta U_k + \Gamma G \Delta M_k + \Gamma G \Delta N_k \tag{18}$$

From (18), it can be easily seen that the SC noise and the CA noise are all ruled by the learning gain matrix. Based on this relation, a method is proposed to suppress the accumulated effect of the SC noise and the CA noise on the ILC system over wireless networks. Specifically, the learning gain is regarded as an adjustable "filter" in iteration domain, which is defined as Γ_k, and the trace of covariance matrix of ΔU_k is used as the criterion to adjust the learning gain with iteration number increasing. Through adjusting the value of the learning gain Γ_k adaptively in iteration domain, the accumulated effect of channel noise on the convergence of the system is suppressed.

In order to develop the covariance matrix of ΔU_k, the mean of ΔU_k needs to be considered first. According to **Assumption 1**, it can be easily derived that $\xi(\Delta M_k) = 0$ and $\xi(\Delta M_k) = 0$ for all k . Then, we have $\xi(\Delta U_{k+1}) = (I - \Gamma G)\xi(\Delta U_k)$. If **Assumption 2** is satisfied, i.e., $\xi(U_0) = 0$, then we have $\xi(\Delta U_{k+1}) = (I - \Gamma G)\xi(\Delta U_k) = 0$.

For compactness, we set $V_{\Delta U_k} = \xi\left(\Delta U_k \Delta U_k^T\right)$, $V_{\Delta M_k} = \xi\left(\Delta M_k \Delta M_k^T\right)$ and $V_{\Delta N_k} = \xi\left(\Delta N_k \Delta N_k^T\right)$. It can be easily seen that $V_{\Delta U_k}$, $V_{\Delta M_k}$ and $V_{\Delta N_k}$ are all positive semi-definite matrices. From (18), $V_{\Delta U_k+1}$ is developed as following

$$\begin{aligned} V_{\Delta U_{k+1}} & \\ &= (I-\Gamma_k G)\, V_{\Delta U_k} (I-\Gamma_k G)^T \\ &+ \Gamma_k G V_{\Delta M_k} (\Gamma_k G)^T + \Gamma_k V_{\Delta N_k} \Gamma_k^T \\ &= V_{\Delta U_k} - \Gamma_k G V_{\Delta U_k} - V_{\Delta U_k} G^T \Gamma_k^T \\ &+ \Gamma_k G \left(V_{\Delta U_k} + V_{\Delta M_k}\right) G^T \Gamma_k^T + \Gamma_k V_{\Delta N_k} \Gamma_k^T \end{aligned} \tag{19}$$

For compactness, we set $\tilde{V}_{\Delta U_k} = \left(G V_{\Delta U_k} G^T + G V_{\Delta M_k} G^T + V_{\Delta N_k}\right)$. Setting derivative of the trace of the $V_{\Delta U_k+1}$ w.r.t. the Γ_k to zero, and minimizing the trace, proper value of Γ_k is selected adaptively as following

$$\begin{aligned} &\partial\left(trace\left(V_{\Delta U_{k+1}}\right)\right)/\partial\left(\Gamma_k\right) \\ &= 2\Gamma_k \tilde{V}_{\Delta U_k} - 2V_{\Delta U_k} G^T \\ &\equiv 0 \end{aligned} \tag{20}$$

Solving (20), we have

$$\Gamma_k = V_{\Delta U_k} G^T \tilde{V}_{\Delta U_k}^{-1} \tag{21}$$

The proper value of Γ_k is derived as shown in (21), but its calculation is complicated. In order to facilitate the calculation of Γ_k, in the remainder of this subsection, the method to adjust the learning gain adaptively is given.

Substituting (21) into (19), the update expression of $V_{\Delta U_k+1}$ can be represented as

$$\begin{aligned} V_{\Delta U_{k+1}} &= V_{\Delta U_k} - V_{\Delta U_k} G^T \tilde{V}_{\Delta U_k}^{-1} G V_{\Delta U_k} \\ &- V_{\Delta U_k} G^T V_{\Delta U_k} G^T \tilde{V}_{\Delta U_k}^{-1} \\ &+ V_{\Delta U_k} G^T V_{\Delta U_k} G^T \tilde{V}_{\Delta U_k}^{-1} \\ &= (I-\Gamma_k G)\, V_{\Delta U_k} \end{aligned} \tag{22}$$

(21) and (22) facilitate the calculation of Γ_k.

4 CONVERGENCE

In this section, we prove that the learning gain adjusted using the method guarantees the convergence of the norm of $V_{\Delta U_k}$.

Theorem 1. *With the learning gain adjusted using the method, the norm of* $V_{\Delta U_k}$ *satisfies* $\lim_{k\to\infty} \|V_{\Delta U_k}\| = 0$.

Proof. Using a matrix inversion function $\left(A - UD^{-1}V\right)^{-1} = A^{-1} + A^{-1}U\left(D - VA^{-1}U\right)^{-1}VA^{-1}$ [20], $I - \Gamma_k G$ can be rewrote as

$$I - \Gamma_k G = \left(I + V_{\Delta U_k} G^T \tilde{V}_{\Delta U_k}^{-1} G\right)^{-1} \tag{23}$$

If CB is full rank, the system matrix G is also full rank. According to statistic characteristics of SC and CA noise, it can be easily found that $V_{\Delta U_k}$, $V_{\Delta M_k}$ and $V_{\Delta N_k}$ are all symmetric and positive-definite matrices. If $V_{\Delta U_k}$ is a symmetric and positive-definite matrix, all eigenvalues of $I - \Gamma_k G$ are greater than zero and less than one, which guarantees that

$$\|I - \Gamma_k G\| < 1, \forall k \tag{24}$$

From (22) and (24), it can be easily seen that

$$\lim_{k\to\infty} \|V_{\Delta U_k}\| = 0 \tag{25}$$

This completes the proof. □

According to (21), it can be easily seen that the adjusted learning gain converges to zero monotonically with the iteration number goes on.

5 SIMULATION RESULTS

In this section, some numerical examples are given to illustrate the effectiveness of the proposed method. Consider the system (4) with matrices given by

$$\begin{cases} x_k(t+1) = \begin{bmatrix} -0.5 & 0 & 0 \\ 1 & 1.24 & -0.87 \\ 0 & 0.87 & 0 \end{bmatrix} x_k(t) \\ + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [u_k(t) + m_k(t)] \\ y_k(t) = \begin{bmatrix} 2 & 2.6 & -2.8 \end{bmatrix} x_k(t) \end{cases} \tag{26}$$

The desired trajectory is as follows

$$y_d(t) = 10(1 + \sin(2\pi t/T - \pi/2)) \tag{27}$$

The ILC method is described in (5). Initial state and control are all zero, $T = 100$, the mean of $m_k(t)$ and $n_k(t)$ are zero, and covariance of $m_k(t)$ and $n_k(t)$ are all 0.05. In order to illustrate the effectiveness of the proposed method, a constant learning gain $\Gamma(t) = 0.5$ is used.

The effectiveness of the proposed method is illustrated from the perspective of tracking trajectories. As shown in Figure 2-4, due to the accumulated effect of the channel noise on the system is suppressed significantly with the

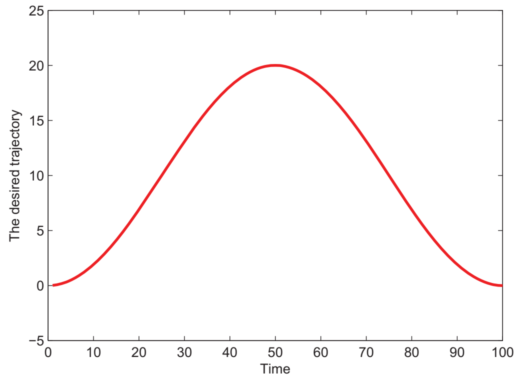

Figure 2: The desired trajectory

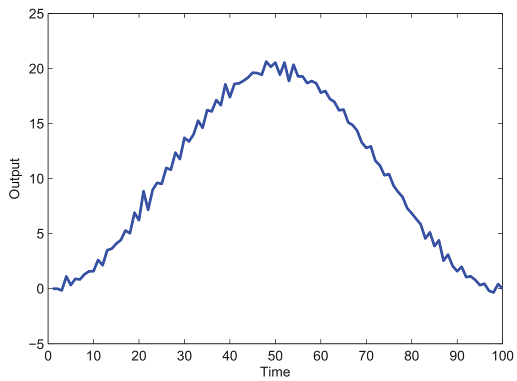

Figure 3: The output with constant learning gain

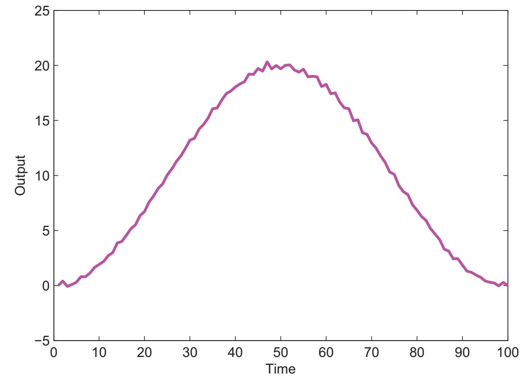

Figure 4: The output with adjusted learning gain

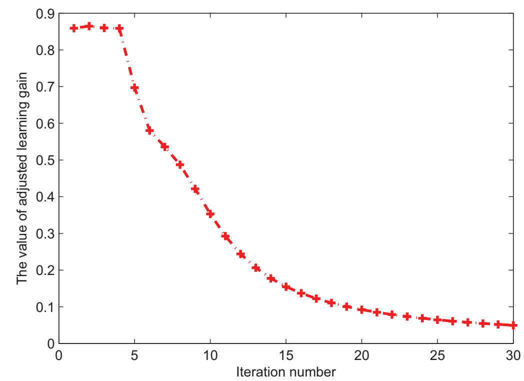

Figure 5: The value of adjusted learning gain

proposed method, the output with adjusted learning gain is more smooth than that with constant gain. Figure 5 shows that the adjusted learning gain converges to zero monotonically, which corroborate the theoretical analysis in Section IV about the variation of adjusted learning gain with the proposed method.

6 CONCLUSIONS

In this paper, iterative learning control (ILC) system over wireless channels is considered, in which control and measurement signals are all be added on white Gaussian noises. Using super-vector formulation, the relation between iterative difference value of input, CA noise and SC noise in iteration domain is derived. Based on this relation, a method is proposed to suppress the accumulated effect of channel noise on the ILC system over wireless channels. The method uses learning gain as a filter, and adjusts it adaptively in iteration domain through minimizing the trace of the iterative difference value of input covariance matrix. As a result, the convergence performance of the ILC system is improved greatly in presence of channel noises. Channel noise is one of uncertainties introduced by wireless channel. In the future study, we need to take other sub-problems into account such as multi-delay and channel fading.

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