The EMD Based on Adaptive Stochastic Resonance for Weak Signal Detection

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Abstract: Empirical mode decomposition (EMD) is a powerful tool for general signal feature extraction and signal detection, but not for weak signal under strong noise background. Because stochastic resonance (SR) can improve SNR by energy transfer, the method which combines empirical mode decomposition with stochastic resonance for weak signal detection is proposed. The simulation results show that output signal-to-noise ratio is higher and the EMD algorithm is improved at the same time.

Keywords: Empirical Mode Decomposition (EMD), Stochastic Resonance (SR), Adaptive Structural Parameters, Signal-to-Noise Ratio (SNR), Consecutive Mean Square Error (CMSE)

1 INTRODUCTION

The weak signal detection from strong noise background is widely used in the fields of communication, mechanical fault diagnosis, biological medicine and physical measurement and it has been a hot research topic in the field of signal detection. The EMD, proposed by HUANG, is a new signal processing method, which has great advantage in nonlinear and non-stationary signal processing [1]. Although the EMD shows good characteristics in the signal feature extraction, it is not suitable for feature extraction of the weak signal under strong noise background. With the increase of decomposition layers gradually, boundary error accumulation and the number of false component increases, leading to weaken the features of useful signal and have a bad effect on weak signal detection.

There is a method which uses noise to improve the signal-to-noise ratio instead of eliminating noise, named stochastic resonance proposed by BENZI in the 1980s [2]. By selecting appropriate system parameters, the stochastic resonance occurs. The weak signal under the background of strong noise achieves energy transfer between noise and signal by stochastic resonance, so as to improve the output SNR and overcome signals submerged in noise.

In this paper, taking advantage of SR in the treatment of the weak signal and the EMD in the analysis principle of signal amplitude frequency characteristics, a method which combines EMD with stochastic resonance of adaptive structural parameters is proposed. Firstly, selecting the appropriate parameters of the bistable system makes the weak signal lead to stochastic resonance, so as to increase output SNR. Then we use EMD to decompose the signal generated by stochastic resonance. As a result, we gain signal components and feature extraction of each frequency thereby completing the detection of weak signal. Thus, this method can increase SNR, also improve the effectiveness of the EMD algorithm for a weak signal at the same time.

In section II, the stochastic resonance of adaptive structural parameters is introduced. The method of combining EMD with the adaptive structural parameters stochastic resonance is described in section III. In section IV, simulation is performed and the result is analyzed. Finally, conclusion is given in section V.

2 STOCHASTIC RESONANCE OF ADAPTIVE STRUCTURAL PARAMETERS

According to the theory of stochastic resonance, when the signal and noise are added into a nonlinear system, signal and noise will resonate by the synergy from each other at the same time. Meanwhile the noise energy transfer to signal energy so that it makes the SNR increase. Through the analysis of the relationship between stochastic resonance model parameter, the input signal, the noise signal and SNR, we can match the model parameter under the maximum SNR. As to the input signal of high frequency, the system parameters should be taken normalized transformation in order to meet the conditions of stochastic resonance.

Stochastic resonance of nonlinear bistable system can be described by the Langevin equation [3]

$$\frac{dx}{dt} = ax - bx^3 + A*\sin(2\pi ft + \varphi) + n(t) \tag{1}$$

Where a, b are the structure parameters of a given nonlinear system. A is the amplitude of the input signal. f is frequency. n(t) is white noise, its strength is D, and x(t)

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is output signal.

Generally, the stochastic resonance is apply for the signal with low frequency, and the parameters of the bistable system a is set as 1. Due to the limitation of adiabatic approximation theory, this method usually can only detect low frequency signal. Generally, stochastic resonance request that the frequency of input signal should be far less than 1 [4]. Introduce the transformation as follow [5]

$$y = x\sqrt{\frac{b}{a}}, \quad \tau = at \tag{2}$$

Add into equation (1), it can be rewritten as

$$\frac{dy}{d\tau} = y - y^3 + \sqrt{\frac{b}{a^3}} * [A * \sin(2\pi \frac{f}{a}\tau + \varphi) + n(\tau)] \tag{3}$$

It is easily to see that the structural parameters of bistable system remain to 1 after transformation. The frequency of periodic input signal change to the 1 / a of the former frequency. The amplitudes of periodic signal and the noise multiply by the scale factor $\sqrt{b/a^3}$. Because the equation (1) and equation (2) are similar, it can be thought that the stochastic resonance of low frequency signals occurs in a small parameter bistable system, which transforms from high frequency signals in a big parameter bistable system. Whereas, according to matching larger bistable system parameters, the high frequency signal can be transformed to low frequency signal in order to generate stochastic resonance.

In general, the most obvious effect of stochastic resonance is to gain the maximum output SNR. According to the theory of adiabatic approximation, the output SNR can be approximately described by input of weak signal amplitude, noise intensity and the parameters of the system [6]

$$SNR = \frac{S}{N} \approx \frac{\sqrt{2}a^2 A^2}{4bD^2}\exp(-\frac{\Delta U}{D}) = \frac{\sqrt{2}a^2 A^2}{4bD^2}\exp(-\frac{a^2}{4bD}) \tag{4}$$

When $a = b = 1$, the relationship between the output SNR and different input noise intensity is shown in figure 1. From the curve on the figure, it can be seen that as the noise power getting larger, SNR increases gradually and appears a peak, resulting from noise energy transferring to signal. Continuing to increase intensity, the noise will play a leading role, and then the SNR will decrease gradually. When the output SNR gets to the peak, effect of stochastic resonance is most obvious. That is to say, there is the best relationship for matching between system, signal and noise at that time [7]. By derivation analysis, the position of the peak of SNR is the point of $D=\Delta U/2$. When the output SNR reaches the maximum, the relationships, between the system parameters a and b, signal amplitude A and noise intensity D, can be acted as a selection criterion of system parameters.

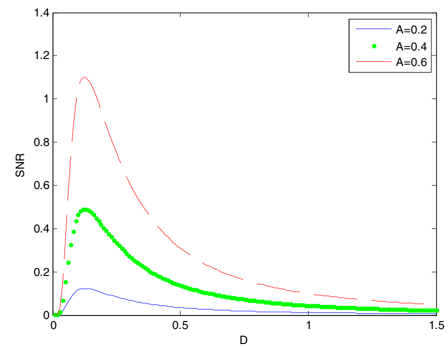

Figure 1. The curve of the output SNR change with high noise intensity D

For a linear system, the nonlinear deformation of the output signal does not exist, so the correlation between the input and output is equals to 1. But for a nonlinear system, with the nonlinear deformation when signal passes, the signal loses its information so that the correlation between the input and output will be less than 1. The correlation coefficient is larger, the degree of matching between output and input is better. Therefore, the correlation coefficient between input and output can be played a role in a criterion to measure stochastic resonance of parameter adjustment. It can be described by

$$\rho = \frac{\mathrm{cov(s, x)}}{\sqrt{\mathrm{var(s)} \cdot \mathrm{var(x)}}} = \frac{E(\mathrm{sx}) - \mathrm{E(s)\,E(x)}}{\sqrt{\mathrm{var(s)} \cdot \mathrm{var(x)}}} \tag{5}$$

Where the symbols, E and var, represent the mean and variance respectively.

3 THE METHOD OF EMD COMBINED WITH STRUCTURAL PARAMETERS ADAPTIVE STOCHASTIC RESONANCE FOR WEAK SIGNAL DETECTION

3.1 The EMD Algorithm of Combining Endpoint Extremum Continuation And Cosine Window Function

The EMD decompose nonlinear signal into several Intrinsic Mode functions (IMF) and the residual error. By spectrum analysis on each IMF Hilbert transforms, the meaningful instantaneous frequency can be obtained. Then it gives the exact expression for the frequency scal of non-stationary signal with time. The IMFs need to meet two conditions: (a) for a list of data, the number of extrema and the number of zero crossings must either equal or differ at most by one; (b) at any point, the mean value of the envelope, respectively composed of local maximum points and local minimum point, is equal to zero.

The EMD primary algorithm is as follows, where X (t) as the original signal sequence.

1) initialize h=X(t).
2) find all the minimum and maximum of *h*.
3) fit the upper and lower envelope of h and calculate

the envelope mean *m*.

4) The series *h* minus the envelope mean, *h=h-m*.
5) Repeat step (2) to (4), until *h* meets the IMF's conditions.
6) *I=I-h*. Repeat steps (1) to (5) until the final sequence r_n not be decomposed.

The results of the EMD can be expressed as

$$X(t)=\sum_{i=1}^{n} IMF_i + r_n \tag{6}$$

In the use of the EMD method, the endpoint effect becomes the main factors influencing the precision of this method [8]. Extending data sequence or increasing the extreme point at both ends to suppress the endpoint effect is a method of universally recognized [9]. But it still can't ensure that the endpoints of the continuation sequence locate in the maximum or minimum points, leading to the envelope distortion and affecting the decomposition quality.

In this paper, based on extremum of continuation, adding cosine window function into extended data to make endpoint converges. Then it makes spline function fit envelope better and makes EMD is more effective.

The both endpoints of the window function designed are set to *0*, and the function is defined as

$$w(t)=\begin{cases} 0.5-0.5\cos(\dfrac{2\pi t}{2T_1-1}), t\in T_1 \\ 1, t\in T \\ 0.5-0.5\cos[\dfrac{2\pi t}{2(T_2-T)-1}], t\in T_2 \end{cases} \tag{7}$$

Among them, *T1*, *T2* are the left and right time series separately. *T* is the original signal sequence. The shape of the special window function can be described as a rectangular window in the middle and hanning windows near the each endpoint, as shown in figure 2.

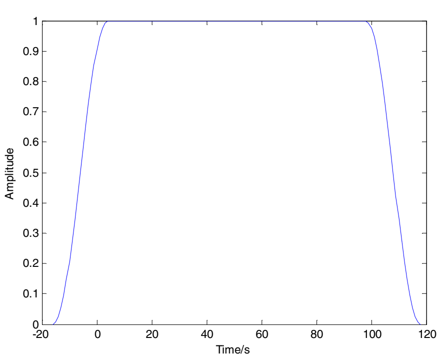

Figure 2. The Special Cosine Window Function

3.2 EMD Combined with Structural Parameters Adaptive Stochastic Resonance for Weak Signal Detection

As to EMD for high SNR, it has a good result for decomposition and gains the time-frequency characteristics of the signal by extreme continuation and cosine window function. But when the input SNR is low, namely a weak signal with strong noise acts as the input, the noise may have a great effect on decomposing. The IMFs exists serious distortion, leading decomposition to meaningless.

To reduce the noise influence on decomposition, EMD with wavelet de-noising is proposed by the literature [10]. Using wavelet filter to process input signal, lots of noise have been removed, then make EMD for the pure signal. Because the input signal is weak while the noise is strong, resulting in signal annihilation, wavelet denoising is bound to lose some useful information at the same time. In this paper, the method of EMD combined with structural parameters adaptive stochastic resonance can inhibit noise and increase SNR, also can extract the signal characteristics of each frequency component. In this method, noise energy transfer into signal by stochastic resonance, then we analyze the improved EMD of the new signal which is the result of original signal after stochastic resonance, as well each component of the time domain and frequency spectrum distribution. Algorithm steps are as follows

1) Match proper structure parameters of system by maximum output SNR, in order to make the weak signal stochastic resonance.
2) Use the input and output correlation coefficient to measure the formation of adaptive stochastic resonance whether reasonable or not.
3) Make EMD of the results of stochastic resonance, then extract and analyze the signal amplitude frequency characteristics.

In order to verify the effectiveness of the method, the mean square error (MSE) from EMD is usually used as evaluate methods. Where the MSE is defined as

$$MSE=\frac{1}{N}\sum_{i=1}^{N}[s(i)-s^*(i)]^2 \tag{8}$$

In In order to calculate easily, the continuous mean square error (CMSE) can be used [11].

$$\begin{aligned} \mathrm{CMSE}(\tilde{y}_k,\tilde{y}_{k+1}) &\triangleq \frac{1}{N}\sum[\tilde{y}_k-\tilde{y}_{k+1}]^2 \\ &\triangleq \frac{1}{N}\sum[\mathrm{IMF}_k]^2 \end{aligned} \tag{9}$$

Where, the CMSE measures the squared Euclidean distance between two consecutive reconstructions of the signal.

The SNR of EMD between each component is defined as

$$SNR=10\times\lg\left[\frac{\sum_{i=1}^{N}(s(i)-\bar{s})^2}{CMSE}\right] \tag{10}$$

Among them, s is the original signal without noise. s^* is the de-noising signal. $\bar{s}$ is the average of s.

SNR determines that the signal processing effect is good or bad. The larger the SNR, the better de-noising effect is. It can be seen that CMSE and SNR assumes the inverse correlation. From the point of view of CMSE, the CMSE value is smaller, the error produced by the EMD successive decomposition is less, the result is closer to the real signal characteristics, and de-noising effect is more ideal.

4 SIMULATION AND ANALYSIS

When the input signal amplitude A=0.2, noise intensity D = 0.5, signal frequency f = 10 Hz and sampling frequency f_s= 5 Hz, the time domain and frequency spectrum of signal with noise are shown by figure 3. From the figure, it can be seen that the signal is almost submerged in noise and the effective information cannot be told apart either from the perspective of time domain or frequency domain.

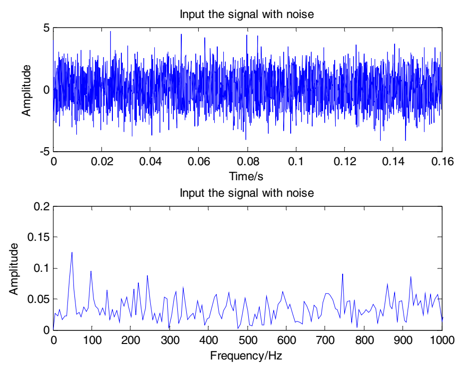

Figure 3. The input signal with noise and its spectrum

If input signal directly decompose by improved EMD, the results are shown in figure 4. According to analyzing the frequency spectrum of each component, we can see that the decomposed IMFs contained much high frequency ingredients of noise. The IMFs are distorted and the result of decomposition is not ideal. Thus, the input signal needs to improve the signal-to-noise ratio before EMD in order to reduce the noise effect on the decomposition.

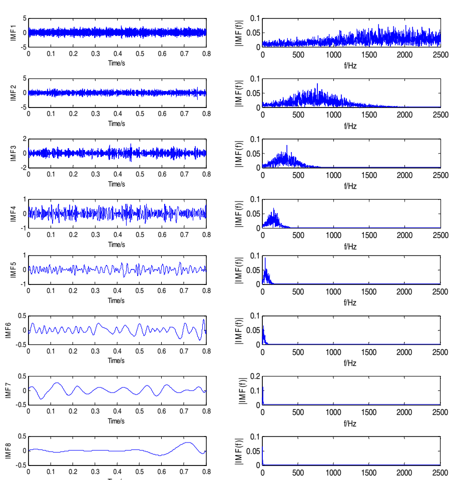

Figure 4. The result of EMD directly

Due to the stochastic resonance using noise energy transformation to improve SNR when dealing with a weak signal, stochastic resonance with adaptive structure parameters occurs, thus a higher output SNR will be got. Simulation shows that the result of input and output correlation coefficient is 0.6491. It can be thought that the output with stochastic resonance and pure signal input are matched well and keep the characteristics of the original signal. The effect of the stochastic resonance is shown in figure 5. From the figure, the output of the stochastic resonance has certain periodicity and the input spectrum peak apparently concentrate on the frequency of the original signal.

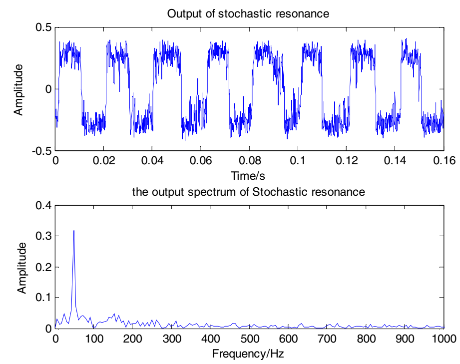

Figure 5. Stochastic resonance output and spectrum

The decomposition result of EMD is shown in figure 6. From the figure, the high frequency noise component has been greatly reduced, and each component of decomposition concentrates on low frequency spectrum of the original signal.

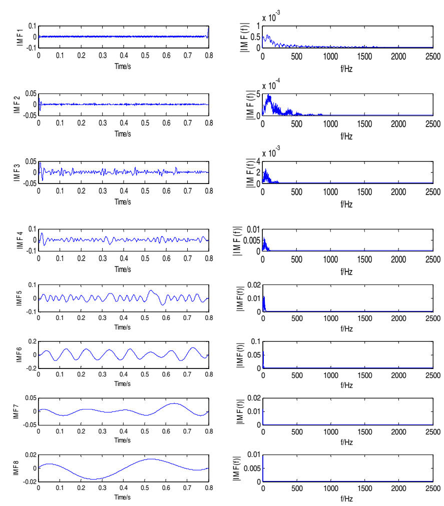

Figure 6. EMD for stochastic resonance

In order to compare advantages and disadvantages of various methods better, several performance indicators by simulation are shown in table 1 (the input SNR is 6.4728).

Table 1. Comparison with Several Methods Results

Methods	Output SNR	CMSE
Improved EMD directly	-3.8987	2.0962
Improved EMD based on wavelet transform	2.4719	0.3759
Improved EMD with adaptive SR (the method used in this paper)	22.1226	0.0450

From table 1, when the input has low SNR, whether improved EMD to signal directly or improved EMD based on wavelet transform, neither of them can obtain high output SNR, resulting in decomposition quality decreasing. The method used in this paper can obviously improve signal-to-noise ratio and greatly reduce the noise impact on the decomposition. By the point of CMSE criterion, CMSE value from the methods in this paper is significantly lower than that from other methods.

5 CONCLUSION

Stochastic resonance of adaptive structure parameters is proposed. It is suitable for adapting to stochastic resonance of weak signal at random frequency and it improves the signal-to-noise ratio of the output signal. Combining improved EMD with stochastic resonance of adaptive structural parameters, it achieves signal noise reduction processing and effectively improves the accuracy of EMD on weak signal. By analyzing the simulation and results, the feasibility and effectiveness of the method in this paper have been verified. Results show that this method can decompose the signal characteristic information more accurately, with reducing the false component of EMD and improving the quality of the decomposition.

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