H_∞ Robust Control with Improved Prescribed Performance for a Class of Strict Feedback Nonlinear Systems

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Abstract: In this paper, an H_∞ robust prescribed tracking controller is designed for a class of strict feedback nonlinear systems based on backstepping technique. The improved prescribed performance constraint guarantees the transient and steady state performance of nonlinear system. The dynamic surface control is adopted to overcome the differential explosion problem in the backstepping design procedure. The impacts of uncertainties in the system can be attenuated by H_∞ control. The stability analysis proves that the controller design procedure is simple with low complexity and robustness. Finally, the simulation results verify the effectiveness of the controller.

Key Words: H_∞ control, backstepping, prescribed performance, dynamic surface control and nonlinear system

1 Introduction

In recent years, prescribed performance control (PPC) is one of hot research topics in current control area, its main idea is to guarantee the transient and steady state behavior on the premise of ensuring the stabilization of the system. The relative remarkable PPC method was developed by G. Rovithakis $et\ al$ [1] and [2]. In [3], S. I. Han and J. M. Lee proposed an improved prescribed performance constraint (IPPC) method by using a new transformation function, which successfully avoids the singularity problem and makes the design process simpler. Backstepping is a systematic control design method for nonlinear systems, and the concept of virtual control is introduced. The real control law can be designed [4] and [5]. Unfortunately, the conventional backstepping control method has a large number of complex terms with high order system due to differentiations of virtual control functions. Hence, dynamic surface control (DSC) method overcomes the differential explosion problem in the backstepping control, and simplifies the design of the control law [6]- [8]. By considering the unknown and time-varing uncertainties, some approximation methods are adopted for the controller design to attenuate the impacts of uncertainties such as fuzzy approximation [3], neural network approximation [9] and [10], and H_∞ robust control [11]. Currently, the researches of PPC or IPPC with H_∞ robust control are few. Therefore, an H_∞ robust controller with IPPC approach is designed for a class of strict feedback nonlinear systems by using backstepping technique.The contributions of this work are including: (1) In order to improve the system performance and robustness, it is the first attempt to combine IPPC method with H_∞ robust control for nonlinear systems. (2) IPPC method can avoid the repeated differentiations of inverse of the transformation function $S^{-1}\left(\frac{e(t)}{\rho(t)}\right)$ in the recursive steps, and surface control can avoid the repeated differentiations of virtual controls in backstepping design. The combination of these two methods simplifies the controller design.

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2 Problem Formulation and Prelimaries

2.1 System Formulation

Consider a strict feedback nonlinear system below

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_1, \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta_n, \\ y = x_1. \end{cases} \tag{1}$$

where $x_i \in \Re, i = 1, \cdots, n,$ is the system state, $\bar{x}_i = [x_1, \cdots, x_i]^T \in \Re^i$, $u \in \Re$ and $y \in \Re$ represent the control input and output respectively. $f_i(\cdot)$ and $g_i(\cdot)$ are known, continuous and smooth functions. $\Delta_i, i = 1, \cdots, n,$ are the unknown bounded disturbances.

Assumption 1. The signs of $g_i(\cdot)$ are known, and there exist constants $0 < g_{\min} < g_{\max}$ such that $g_{\min} \leqslant |g_i(\cdot)| \leqslant g_{\max}, i = 1, \cdots, n$. Without losing generality, assume that $g_{\min} \leqslant g_i(\cdot) \leqslant g_{\max}$.

Assumption 2. The desired trajectory $y_r(t)$ is a known and bounded function with time, and its derivatives are also known and bounded.

2.2 Performance Function and Error Transformation

A continuous and smooth function $\rho(t) : \Re_+ \rightarrow \Re_+$ with $\lim_{t\rightarrow\infty} \rho(t) = \rho_\infty$ can be defined as follows:

$$\rho(t) = (\rho_0 - \rho_\infty)e^{-lt} + \rho_\infty \tag{2}$$

where ρ_0, ρ_∞, and l are appropriately selected positive constants. The transient and steady state performance can be guaranteed by the following prescribed constraint conditions:

$$-\delta\rho(t) < e(t) < \rho(t), \quad if \quad e(0) \geqslant 0, \tag{3}$$

or

$$-\rho(t) < e(t) < \delta\rho(t), \quad if \quad e(0) < 0, \tag{4}$$

where $e(t)$ is the output tracking error, and $0 < \delta < 1$ is the designed parameter.
According to [3], the transformed error ε can be defined as follows:

$$\varepsilon(t) = \frac{e(t)}{\zeta(t)}, \tag{5}$$

$$\zeta = a\zeta_H + (1-a)\zeta_L, \tag{6}$$

where $a = 1$, if $e(t) \geq 0$, and $a = 0$, if $e(t) < 0$. ζ_H and ζ_L are defined as follows:

$$\zeta_H = \begin{cases} \rho(t), & if \quad e(0) \geq 0 \\ \delta\rho(t), & if \quad e(0) < 0 \end{cases}, \tag{7}$$

$$\zeta_L = \begin{cases} -\delta\rho(t), & if \quad e(0) \geq 0 \\ -\rho(t), & if \quad e(0) < 0 \end{cases}, \tag{8}$$

[3] has proved that the transformed error ε satisfies the inequality

$$0 < \varepsilon(t) < 1, \forall t > 0. \tag{9}$$

Remark 1. In equation (2), the constant ρ_∞ confines the maximum allowable steady state error, l regulates the convergence speed of tracking error, and $\delta\rho_0$ is the upper bound of maximum overshoot. Therefore, the steady state error of system is able to converge into a prescribed area by selecting appropriate ρ_0, ρ_∞, and l, thus the maximum overshoot and convergence speed can be guaranteed to satisfy the requirements of prescribed performance.

3 H_∞ Robust Controller Design

A group of dynamic surface variables are defined as follows:

$$\begin{cases} S_1 = \frac{\varepsilon_1}{1-\varepsilon_1}, \\ S_i = x_i - x_{i,out}, \end{cases} \tag{10}$$

where $\varepsilon_1 = \frac{e_1}{\zeta_1}$, which is obtained from (5) and $x_{i,out}$, $i = 2, \cdots, n$, are virtual filtering control functions.

According to (10), the transformation system is provided by

$$\begin{cases} \dot{S}_1 = \Xi(f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 + \Delta_1 - \dot{y}_r - \varepsilon_1\dot{\zeta}_1), \\ \dot{S}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_i - \dot{x}_{i,out}, \\ \dot{S}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta_n - \dot{x}_{n,out}, \\ z = S_1, \end{cases} \tag{11}$$

where $\Xi = \frac{1}{(1-\varepsilon_1)^2\zeta_1}$ and z is control output. The derivatives of virtual filtering controls are obtained by passing designed virtual controls $x_{i,in}$, $i = 2, \cdots, n$ through first-order filters with time constants τ_i, $i = 2, \cdots, n$, such that

$$\tau_{i-1}\dot{x}_{i,out} + x_{i,out} = x_{i,in}, x_{i,out}(0) = x_{i,in}(0). \tag{12}$$

The approximation error of above filters are determined by

$$\sigma_i = x_{i,out} - x_{i,in}, i = 2, \cdots, n. \tag{13}$$

The H_∞ control problem is defined as follows:
Definition 1. If there exist a control law $u = \alpha(x)$, an appropriate Lyapunov function candidate V_N, and a positive constant γ so that the following three objectives (O1)-(O3) can be achieved, then the H_∞ control problem is solvable.
(O1) All signals are bounded in the closed loop system.
(O2) The output tracking error $e(t) = y(t) - y_r(t)$ satisfies the prescribed performance during both transient process and steady state with the desired trajectory $y_r(t)$.
(O3) The L_2 gain from the external disturbances and modeling errors to the output is less than or equal to γ, that is,

$$V_N - V_N(0) \leq \int_0^T (\gamma^2 \left|\left|\bar{\Delta}\right|\right|^2 - ||z||^2)dt, \tag{14}$$

for any final time $T > 0$ with $\bar{\Delta} = [\Delta_1, \cdots, \Delta_n, \sigma_2, \cdots, \sigma_n]^T$.
Remark 2. $\bar{\Delta}_i$ is the system uncertainty composed by external disturbances and approximation errors of surface control. A better robust performance can be achieved by selecting a smaller γ.
The design procedure of H_∞ controller is divided into n steps.
Step 1:
Consider the first subsystem in (11) and choose the following Lyapunov function candidate

$$V_1(S_1) = \frac{1}{2}S_1^2. \tag{15}$$

Define the following function

$$H_1 = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\Delta_1^2 + \dot{V}_1(S_1). \tag{16}$$

Substituting the derivative of $V_1(S_1)$, $z = S_1$, $x_2 = S_2 + x_{2,out}$, and $x_{2,out} = \sigma_2 + x_{2,in}$ into (16), we obtain

$$\begin{aligned} H_1 \quad \leq \quad & -\frac{1}{4}\gamma^2\Delta_1^2 - \left(\frac{\gamma}{2}\Delta_1 - \frac{S_1\Xi}{\gamma}\right)^2 + S_1\Xi g_1(\bar{x}_1)S_2 \\ & + S_1^2\frac{\Xi^2 g_1^2(\bar{x}_1)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_2^2 \\ & + S_1\Xi(\Gamma_1 S_1 + f_1(\bar{x}_1) + g_1(\bar{x}_1)x_{2,in} - \dot{y}_r - \varepsilon_1\dot{\zeta}_1), \end{aligned} \tag{17}$$

where $\Gamma_1 = \frac{\Xi}{\gamma^2} + \frac{1}{2\Xi}$ and the inequality above is obtained by using the following relation

$$S_1 \Xi g_1(\bar{x}_1)\sigma_2 \leq \frac{S_1^2 \Xi^2 g_1^2(\bar{x}_1)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_2^2. \tag{18}$$

Introduce the virtual control $x_{2,in}$ as follows:

$$\begin{aligned} x_{2,in} &= -\frac{1}{g_1(\bar{x}_1)}\left(k_1\zeta_1 S_1 + \Gamma_1 S_1 + f_1(\bar{x}_1) - \dot{y}_r - \varepsilon_1\dot{\zeta}_1\right) \\ &\quad -\frac{1}{g_1(\bar{x}_1)}\frac{S_1\Xi g_1^2(\bar{x}_1)}{2\gamma^2}, \end{aligned} \tag{19}$$

where k_1 is a positive design parameter. It follows from substituting $x_{2,in}$ into (17) that

$$\begin{aligned} H_1 &\leq -\frac{1}{4}\gamma^2\Delta_1^2 - \left(\frac{\gamma}{2}\Delta_1 - \frac{S_1\Xi}{\gamma}\right)^2 - \frac{k_1}{(1-\varepsilon_1)^2}S_1^2 \\ &\quad + S_1\Xi g_1(\bar{x}_1)S_2 + \frac{\gamma^2}{2}\sigma_2^2. \end{aligned} \tag{20}$$

Step 2:
Consider the second subsystem in (11) and choose the following Lyapunov function candidate

$$V_2(S_1, S_2) = V_1(S_1) + \frac{1}{2}S_2^2. \tag{21}$$

By differentiating both sides of (21), the following relation can be obtained

$$\dot{V}_2(S_1, S_2) = \dot{V}_1(S_1) + S_2\dot{S}_2. \tag{22}$$

Substituting (16) into the equation above gives

$$\dot{V}_2(S_1, S_2) = H_1 - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\Delta_1^2 + S_2\dot{S}_2. \tag{23}$$

Define the following function

$$H_2 = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{2}(\Delta_j^2 + \sigma_j^2) + \dot{V}_2(S_1, S_2). \tag{24}$$

Then, by substituting (23) into (24) and taking (20) into consideration, it follows that

$$\begin{aligned} H_2 &\leq \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 \\ &\quad + S_2\left(\frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3\right) \\ &\quad + S_2\left(S_1\Xi g_1(\bar{x}_1) - \dot{x}_{2,out}\right), \end{aligned} \tag{25}$$

where $\Psi_1 = -\frac{\gamma^2}{4}\Delta_1^2 - \left(\frac{\gamma}{2}\Delta_1 - \frac{\Xi}{\gamma}S_1\right)^2 - \frac{k_1}{(1-\varepsilon_1)^2}S_1^2$. Substituting $x_3 = S_3 + x_{3,out}$ and $x_{3,out} = \sigma_3 + x_{3,in}$ into (25) produces

$$\begin{aligned} H_2 &\leq \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 + S_2 g_2(\bar{x}_2)S_3 \\ &\quad + \frac{\gamma^2}{2}\sigma_3^2 + S_2\left(\frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + g_2(\bar{x}_2)x_{3,in}\right) \\ &\quad + S_2\left(S_1\Xi g_1(\bar{x}_1) + \frac{S_2 g_2^2(\bar{x}_2)}{2\gamma^2} - \dot{x}_{2,out}\right), \end{aligned} \tag{26}$$

where the inequality is obtained by using the following relation

$$S_2 g_2(\bar{x}_2)\sigma_3 \leq \frac{S_2^2 g_2^2(\bar{x}_2)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_3^2. \tag{27}$$

Define the virtual control $x_{3,in}$ as follows:

$$\begin{aligned} x_{3,in} &= -\frac{1}{g_2(\bar{x}_2)}\left(k_2 S_2 + \frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + S_1\Xi g_1(\bar{x}_1)\right) \\ &\quad -\frac{1}{g_2(\bar{x}_2)}\left(\frac{S_2 g_2^2(\bar{x}_2)}{2\gamma^2} - \dot{x}_{2,out}\right), \end{aligned} \tag{28}$$

where k_1 is a positive design parameter. It follows from substituting $x_{3,in}$ into (26) that

$$\begin{aligned} H_2 &\leq \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 \\ &\quad - k_2 S_2^2 + S_2 g_2(\bar{x}_2)S_3 + \frac{\gamma^2}{2}\sigma_3^2. \end{aligned} \tag{29}$$

Step $i, i = 3, \cdots, n-1$:
Suppose that at Step $i-1$, the function

$$H_{i-1} = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{i-1}(\Delta_j^2 + \sigma_j^2) + \dot{V}_{i-1}(S_1, \cdots, S_{i-1}). \tag{30}$$

satisfies the inequality

$$\begin{aligned} H_{i-1} &\leq \Psi_1 - \sum_{j=2}^{i-1}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] \\ &\quad - \sum_{j=2}^{i-1}k_j S_j^2 + S_{i-1}g_{i-1}(\bar{x}_{i-1})S_i + \frac{\gamma^2}{2}\sigma_i^2, \end{aligned} \tag{31}$$

where

$$V_{i-1}(S_1, \cdots, S_{i-1}) = V_{i-2}(S_1, \cdots, S_{i-2}) + \frac{1}{2}S_{i-1}^2. \tag{32}$$

Consider the ith subsystem in (11) and choose the following Lyapunov function candidate

$$V_i(S_1, \cdots, S_i) = V_{i-1}(S_1, \cdots, S_{i-1}) + \frac{1}{2}S_i^2. \tag{33}$$

By differentiating both sides of (33), the following can be obtained.

$$\dot{V}_i(S_1,\cdots,S_i)=\dot{V}_{i-1}(S_1,\cdots,S_{i-1})+S_i\dot{S}_i. \quad (34)$$

Substituting (11) and (30) into the equation above yields

$$\dot{V}_i(S_1,\cdots,S_i)=H_{i-1}-\frac{1}{2}z^2+\frac{\gamma^2}{2}\sum_{j=1}^{i-1}(\Delta_j^2+\sigma_j^2)+S_i\dot{S}_i. \quad (35)$$

Define the function

$$H_i=\frac{1}{2}z^2-\frac{\gamma^2}{2}\sum_{j=1}^{i}(\Delta_j^2+\sigma_j^2)+\dot{V}_i(S_1,\cdots,S_i). \quad (36)$$

Then, by using (35) and (31), it can be easily verified that

$$\begin{aligned} H_i \quad \leq \quad & \Psi_1-\sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2+\left(\frac{\gamma}{2}\Delta_j-\frac{S_j}{\gamma}\right)^2\right] \\ & -\sum_{j=2}^{i-1}k_jS_j^2+S_i\left(\frac{S_i}{\gamma^2}+f_i(\bar{x}_i)+g_i(\bar{x}_i)x_{i+1}\right) \\ & +S_i\left(S_{i-1}g_{i-1}(\bar{x}_{i-1})-\dot{x}_{i,out}\right). \end{aligned} \quad (37)$$

Substituting $x_{i+1} = S_{i+1} + x_{i+1,out}$ and $x_{i+1,out} = \sigma_{i+1} + x_{i+1,in}$ into (37) gives

$$\begin{aligned} H_i \leq \Psi_1 & -\sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2+\left(\frac{\gamma}{2}\Delta_j-\frac{S_j}{\gamma}\right)^2\right] \\ & -\sum_{j=2}^{i-1}k_jS_j^2+S_ig_i(\bar{x}_i)S_{i+1}+\frac{\gamma^2}{2}\sigma_{i+1}^2 \\ & +S_i\left(\frac{S_i}{\gamma^2}+f_i(\bar{x}_i)+g_i(\bar{x}_i)x_{i+1,in}+S_{i-1}g_{i-1}(\bar{x}_{i-1})\right) \\ & +S_i\left(+\frac{S_ig_i^2(\bar{x}_i)}{2\gamma^2}-\dot{x}_{i,out}\right). \end{aligned} \quad (38)$$

where the inequality is obtained by

$$S_ig_i(\bar{x}_i)\sigma_{i+1}\leq\frac{S_i^2g_i^2(\bar{x}_i)}{2\gamma^2}+\frac{\gamma^2}{2}\sigma_{i+1}^2. \quad (39)$$

Design the virtual control $x_{i+1,in}$ as follows:

$$\begin{aligned} x_{i+1,in} \quad = \quad & -\frac{1}{g_i(\bar{x}_i)}\left(k_iS_i+\frac{S_i}{\gamma^2}+f_i(\bar{x}_i)\right) \\ & -\frac{1}{g_i(\bar{x}_i)}S_{i-1}g_{i-1}(\bar{x}_{i-1}) \\ & -\frac{1}{g_i(\bar{x}_i)}\left(+\frac{S_ig_i^2(\bar{x}_i)}{2\gamma^2}-\dot{x}_{i,out}\right), \end{aligned} \quad (40)$$

where k_i is a positive design parameter. It follows from substituting $x_{i+1,in}$ into (38) that

$$\begin{aligned} H_i \quad \leq \quad & \Psi_1-\sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2+\left(\frac{\gamma}{2}\Delta_j-\frac{S_j}{\gamma}\right)^2\right] \\ & -\sum_{j=2}^{i}k_jS_j^2+S_ig_i(\bar{x}_i)S_{i+1}+\frac{\gamma^2}{2}\sigma_{i+1}^2. \end{aligned} \quad (41)$$

Step n:
Consider the nth subsystem in (11) and choose the following Lyapunov function candidate

$$V_n(S_1,\cdots,S_n)=V_{n-1}(S_1,\cdots,S_{n-1})+\frac{1}{2}S_n^2. \quad (42)$$

The following relation can be obtained by differentiating both sides of (42) and using (41) with $i = n - 1$.

$$\dot{V}_n(S_1,\cdots,S_n)=H_{n-1}-\frac{1}{2}z^2+\frac{\gamma^2}{2}\sum_{j=1}^{n-1}(\Delta_j^2+\sigma_j^2)+S_n\dot{S}_n. \quad (43)$$

Define the following function

$$H_n=\frac{1}{2}z^2-\frac{\gamma^2}{2}\sum_{j=1}^{n}(\Delta_j^2+\sigma_j^2)+\dot{V}_n(S_1,\cdots,S_n). \quad (44)$$

Then, by using (43) and (41) with $i = n - 1$, it can be proved that

$$\begin{aligned} H_n \quad \leq \quad & \Psi_1-\sum_{j=2}^{n}\left[\frac{\gamma^2}{4}\Delta_j^2+\left(\frac{\gamma}{2}\Delta_j-\frac{S_j}{\gamma}\right)^2\right] \\ & -\sum_{j=2}^{n-1}k_jS_j^2+S_n\left(\frac{S_n}{\gamma^2}+f_n(\bar{x}_n)+g_n(\bar{x}_n)u\right) \\ & +S_n\left(g_{n-1}(\bar{x}_{n-1})S_{n-1}-\dot{x}_{i,out}\right). \end{aligned} \quad (45)$$

Choose the control input u as follows:

$$\begin{aligned} u \quad = \quad & -\frac{1}{g_n(\bar{x}_n)}\left(k_nS_n+\frac{S_n}{\gamma^2}+f_n(\bar{x}_n)\right) \\ & -\frac{1}{g_n(\bar{x}_n)}\left(g_{n-1}(\bar{x}_{n-1})S_{n-1}-\dot{x}_{n,out}\right), \end{aligned} \quad (46)$$

where k_n is a positive design parameter. It follows from substituting u into (45) that

$$H_n\leq\Psi_1-\sum_{j=2}^{n}\left[\frac{\gamma^2}{4}\Delta_j^2+\left(\frac{\gamma}{2}\Delta_j-\frac{S_j}{\gamma}\right)^2\right]-\sum_{j=2}^{n}k_jS_j^2\leqslant 0. \quad (47)$$

Select $V_N(S_1,\cdots,S_n)=2V_n(S_1,\cdots,S_n)$. Then it follows from (44) that the derivative of V_N satisfies

$$\dot{V}_N(S_1,\cdots,S_n)=2H_n-(||z||^2-\gamma^2\,||\Delta||^2-\gamma^2\,||\sigma||^2). \quad (48)$$

Because of $H_n \leqslant 0$, the following inequality is obtained

$$\dot{V}_N(S_1, \cdots, S_n) \leq (\gamma^2 \left|\left|\bar{\Delta}\right|\right|^2 - ||z||^2). \tag{49}$$

By integrating both sides of inequality (49), the inequality (14) in Definition 1 can be obtained with the initial condition $V_N(0) = 2V_n(0)$, which indicates that the L_2 gain from uncertainties $\bar{\Delta}_i$ to output z is smaller than or equal to a positive constant γ.

Theorem 1. Suppose that Assumptions 1-2 are satisfied. If the initial condition $e(0)$ satisfies (9), then the H_∞ control problem is solvable. All signals are bounded in the closed loop system, and the output tracking error $e(t)$ satisfies the prescribed performance with the desired trajectory $y_r(t)$.

Proof.

(1) By substituting $\Psi_1 = -\frac{\gamma^2}{4}\Delta_1^2 - \left(\frac{\gamma}{2}\Delta_1 - \frac{\Xi}{\gamma}S_1\right)^2 - \frac{k_1}{(1-\varepsilon_1)^2}S_1^2$ into (47), the following relation can be derived

$$H_n \leq -\frac{k_1}{(1-\varepsilon_1)^2}S_1^2 - \sum_{j=2}^{n} k_j S_j^2. \tag{50}$$

Replacing H_n by (44), it is easy to derive the following

$$\dot{V}_n(S_1, \cdots, S_n) \leq -\frac{k_1}{(1-\varepsilon_1)^2}S_1^2 - \sum_{j=2}^{n} k_j S_j^2 + \frac{\gamma^2}{2}\sum_{j=1}^{n}(\Delta_j^2 + \sigma_j^2). \tag{51}$$

Select the control gains $k_1 = (1-\varepsilon_1)^2\Lambda_1$ and $k_j = \Lambda_j, j = 2, \cdots, n$. (51) becomes

$$\dot{V}_n(S_1, \cdots, S_n) \leq -2\Lambda V_n(S_1, \cdots, S_n) + \vartheta, \tag{52}$$

where $\Lambda = \min[\Lambda_1, \cdots, \Lambda_n]$ and $\vartheta = \frac{\gamma^2}{2}\sum_{j=1}^{n}(\Delta_j^2 + \sigma_j^2)$.

Solving inequality (52) gives

$$V_n(t) \leq V_n(0)e^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}, \forall t > 0. \tag{53}$$

From (53), it is easy to see that $V_n(t) \geq 0$ and $V_n(t)$ is bounded by $\frac{\vartheta}{2\Lambda}$, which implies that $\frac{\vartheta}{2\Lambda}$ can be made arbitrarily small by selecting appropriate design parameters. Therefore, all signals in the closed-loop system are semiglobally, uniformly and ultimately bounded.The objective (O1) is achieved.

(2) According to (53), it satisfies

$$V_1 = \frac{1}{2}S_1^2 = \frac{1}{2}\frac{\varepsilon_1^2}{(1-\varepsilon_1)^2} \leq V_n(0)e^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}. \tag{54}$$

Substituting $\varepsilon_1(t) = \frac{e_1(t)}{\zeta_1(t)}$ into the above inequality, it gives

$$|e_1(t)| \leq |\zeta_1(t)|\sqrt{2}(1-\varepsilon_1)\sqrt{V_n(0)e^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}}. \tag{55}$$

For $t \to \infty$, $V_n(0)e^{-2\Lambda t} = 0$, then it follows from (55) that

$$|e_1(t)| \leq |\zeta_1(t)|\sqrt{2}(1-\varepsilon_1)\sqrt{\frac{\vartheta}{2\Lambda}}. \tag{56}$$

According to the conclusion $0 < \varepsilon_1 < 1$, (56) becomes

$$|e_1(t)| \leq |\zeta_1(t)|\sqrt{\frac{\vartheta}{\Lambda}}. \tag{57}$$

If the selected design parameters satisfies $\Lambda \geq \vartheta$, (57) yields

$$|e_1(t)| \leq |\zeta_1(t)|. \tag{58}$$

Therefore, the output tracking errors are smaller than the prescribed bounds and the errors can be arbitrarily small by selecting appropriate design parameters. The objective (O2) is achieved.

(3) The objective (O3) was proved before the theorem.

4 Simulation

Consider the following second-order strict-feedback nonlinear system:

$$\begin{cases} \dot{x}_1 = 0.1x_1^2 + x_2 + \Delta_1, \\ \dot{x}_2 = 0.1x_1x_2 - 0.2x_1 + (1 + x_1^2)u + \Delta_2, \\ y = x_1. \end{cases} \tag{59}$$

According to Theorem 1, the robust controller for system (59) is designed, and the initial conditions are $x_1(0) = 0.4$ and $x_2(0) = 0$. The desired trajectory is $y_r(t) = \sin(t) + \sin(2t)$. $\Delta_1 = 0.01\cos(t)$ and $\Delta_2 = 0.05\sin(t)$. The design parameters are as follows: $\rho_0 = 0.8$, $\rho_\infty = 0.01$, $l = 2$, $\delta = 0.5$, $\tau = 0.01$, $k_1 = k_2 = 1$ and $\gamma = 0.5$.

The simulation results are shown in Figs. 1-4, which show that the proposed control method guarantees the transient and steady state performance for the tracking errors of nonlinear system.

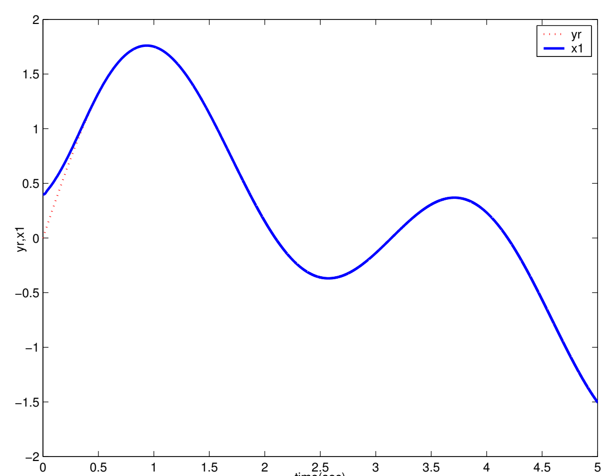

Figure 1: Tracking performance of H_∞ control with IPPC.

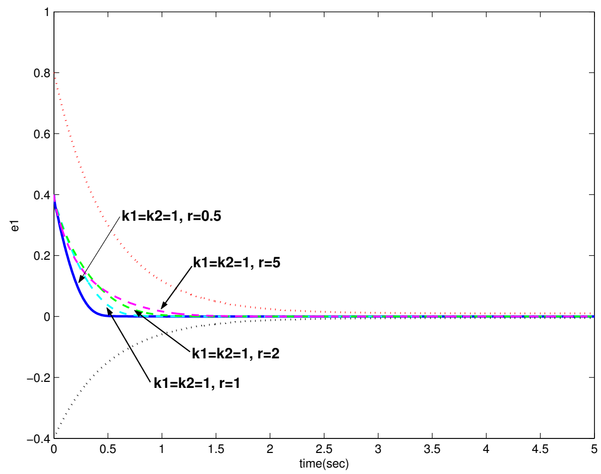

Figure 2: Comparisons of the proposed method with different values of γ.

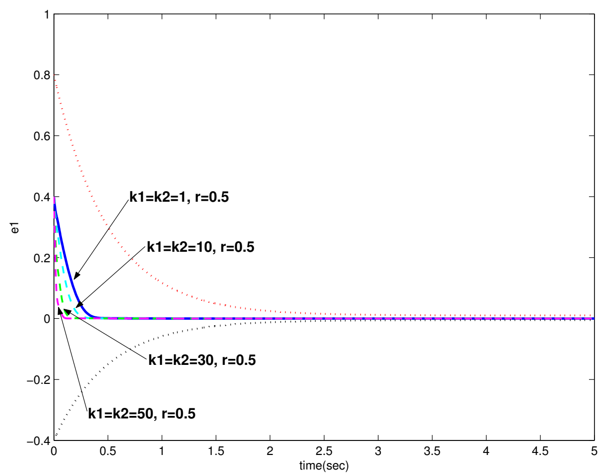

Figure 3: Comparisons of the proposed method with different control gains.

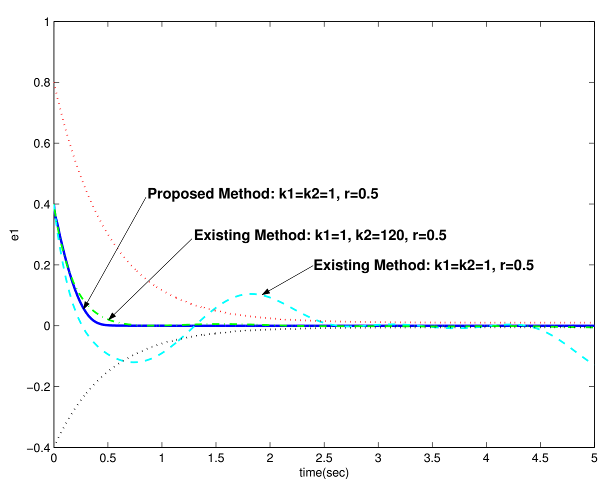

Figure 4: Comparisons of tracking errors between proposed method and existing method with H_∞ control.

5 Conclusion

In this paper, an improved prescribed performance constraint method has been adopted to achieve prescribed performance bounds on the tracking errors. Surface control has been used to avoid the differentiation of virtual control in each recursive step of backstepping design, H_∞ robust control has been introduced to attenuate the impacts of the unknown disturbances and modeling errors. By selecting appropriate parameters, a better tracking performance has been obtained by simulation. The simulation results have shown that the proposed design scheme is effective and feasible.

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