Interval Kalman Filter Based RFID Indoor Positioning

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Abstract: Radio Frequency Identification (RFID) technology has shown its potential in the field of indoor positioning in recent years, and has been widely used in large establishments such as the airport lobby, supermarkets, libraries, underground parking and so on. However, the complexity of the indoor environments make positioning a challenging problem in terms of accuracy, stability, reliability, and interference immunity. In this paper, we develop an algorithm of passive RFID indoor positioning based on interval Kalman filter, according to the geometric constraints of responding tags, combined with the target motion information. The interval Kalman filter is adopted to integrate target position of one preceding time point and reference tags position to estimate the current position. Advantage of this algorithm is to use passive tags to reduce hardware costs, and on the other hand the introduction of filtering algorithm improves the positioning accuracy.

Key Words: RFID, Indoor positioning, Interval Kalman Filter, Interval arithmetic

1 Introduction

In recent years, with the increase of data businesses and multimedia business data, people are more and more demanding for positioning and navigation in complex indoor environments, such as airport terminal, supermarkets, libraries, underground parking garages, mines and other environments [1, 2]. In those environments, the indoor location information of mobile terminals or its owners, devices and goods shall be determined frequently. With the development of communication technology and mobile internet technology, smart phones, tablets and other mobile terminals are becoming more and more popular and they are being widely used. Accordingly, location-based services increasingly bring great conveniences to people, which show great vitalities and business opportunities [3]. Additionally, new applications emerge one after another. There is no doubt that the importance of indoor positioning technology is self-evident and its development will bring profound changes and influences to people's work and life [4].

Currently, more common indoor positioning technologies have emerged such as image positioning technology, Global Positioning System (GPS) technology, infrared technology, ultrasonic technology, laser radar positioning technology, Wi-Fi technology and so on [5, 6]. These technologies largely meet the consumer demand for indoor positioning, but each individual technology has its own advantages and limitations for different scenarios. Image positioning, which need to process large numbers of image information, has the problem of poor real-time performance and is not suitable to sheltered environment. GPS, as it is satellite-dependent, cant be applied inside the building because of signal strength and multipath effects [7]. Infrared technology is limited by the obstructions, indoor objects will block the infrared signal and lead the positioning effect worse. Ultrasonic shares a significant advantage of overall accuracy, but it requires a large number of infrastructure to assure high effectiveness and accuracy, and the cost is so expensive that it is inaccessible to normal users [8]. Laser radar positioning technology also has similar problems above. Wi-Fi or Bluetooth positioning system, which is easy to form the network and communicate with each other, but its high energy consumption and low precision limits its application [9, 10]. Comparatively speaking, due to its low cost and the ability of high speed contactless identification in non-lone of sight shared medium, RFID has recently been widely used to track physical objects and navigate in various indoor environments such as smart houses, hospitals, schools and shopping malls.

Indoor environment can be very complex, since there are various obstacles such as tables, chairs, appliances, etc, which can interfere radio frequency signals. Besides, reader antenna is difficult to achieve the isotropy and it will increase the positioning errors [11]. Therefore, the key to improve the positioning accuracy is how to effec-

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[†] This work is partially supported by National Natural Science Foundation (NSFC) under Grant 61473038 and Engineering and Physical Sciences Research Council (EPSRC) under Grant EP/L026856/1. Also partially supported by Beijing Outstanding Talents Programme (2012D009011000003).

tively eliminate the interference from the environment and the hardware itself. This paper develops a new algorithm of passive RFID indoor positioning based on interval Kalman filter. The remarkable trait of our research compared with previous work is the introduction of the interval Kalman filter in data processing stage to reduce the positioning errors.

The rest of the paper is organized as follows: Section 2 describes related work about RFID indoor positioning. In Section 3, we have provided the details of the positioning algorithm we have implemented. Section 4 would give a detailed simulation and experimental results. The last part concludes our work in this paper.

2 Related work

Over these years, many RFID-based systems have been established to deal with the problem of indoor positioning. As early as in 2000, Jerffey Hightower and Roy Want had proposed the SpotON system which is based on signal strength analysis. The algorithm is an iterative algorithm, making estimates gradually approach the true value through the cycle iteration and finally getting results in the minimum error [12]. However, this algorithm is greatly affected by step size and initial value. Worse, it has a huge amount of calculation so that it is hard to achieve real-time positioning.

In 2003 LANDMARC system was proposed by Lionel Ni et al. which was based on received signal strength indication (RSSI). In order to reduce the cost and improve positioning accuracy, the system introduces the concept of reference tag to offset some environmental interference and use weighted algorithm to get target position [13]. However, LANDMARC system still has some problems. First, hardware devices can only provide energy level of tag, which would increase the positioning error. Second, all reference tags are assumed to be candidates when selecting neighbor tags, but not all information is really useful so that it would generate a lot of redundant computations. Despite of these disadvantages, the LANDMARC system is the most widely used, many domestic and international research in this field is based on its improvements [14]. In 2007 Lionel M. Ni proposed Virtual Reference Elimination (VIRE) algorithm based on the LANDMARC system [15]. Wang et al. improved the LANMARC weight formula to enhance positioning accuracy in harsh environments. In 2008 LANDMARC algorithm was extended to three-dimensional space by Indian scholar M Ayoub Khan [16]. Then French scholar Mathieu Bouet and Guy Pujolle proposed indoor three-dimensional localization algorithm based on virtual landmarks. The idea of the algorithm is that the length and breadth of the interior space are considered as a known hexahedral and RFID readers are installed at a certain interval on the ground and roof [17]. However, the algorithm can hardly be applied due to the high cost.

Due to limitations of hardware performance or cost, some ideas can only be simulated so that it is difficult to be widely applied. In this contribution, we focus on the localization algorithm, to seek a breakthrough to guarantee positioning accuracy, while taking into account the costs.

3 Interval Kalman filter for RFID indoor positioning

3.1 The description of positioning scene and issue

In this paper, we need to locate the target in an ordinary room in which there are some tables, chairs and other obstacles. RFID reference tags will be posted on the ground according to certain rules which ensure that there are at least two tags will responding when the reader is located in tag array as shown Fig. 1. When the target carrying the reader goes into the room, there will be a different sequence of tags responding at each moment and the target is positioning based on it.

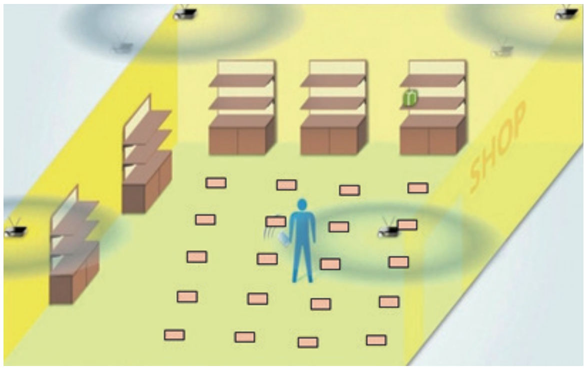

Figure 1: the arrangement of RFID tags in the room

It should be noted that the positioning device is general Ultra High Frequency (UHF) reader with omnidirectional antenna and passive tags in the market. The reader is unable to measure tag's signal intensity, that is, we can only know which tag has responded and can't calculate the distance between the reader and response tag in the positioning process. Besides, the interference from the environment and deficiencies of hardware itself will affect the positioning result. In theory the antenna is isotropic antenna, but in practice the detection range in each direction is different [18]. Obstructions will affect signal transmission, which may make the reader to receive the wrong response sequence. All these will lead to increasing positioning error. Hardware performance is limited, environment impact can't be changed, hence we can only focus on the positioning algorithm and make use of mathematical optimization to improve the positioning accuracy. According to the description of positioning problem, the estimation result is an interval. It should be considered that how to deal with interval data in positioning algorithm.

3.2 Interval Kalman filter

Due to external noise interference, the positioning results will have the fluctuation which can be regarded as interval data. The basic rules of interval arithmetic should be understood and interval Kalman filter algorithm is introduced to deal with interval data [19,20].

For completeness, the interval arithmetic or interval mathematics is first introduced. An interval number $I = [a, b]$ is defined as a set of x which may take value from a to b, where a and b can be any real number with only one exception that $[-\infty, \infty]$ refers an interval covering the whole set of real numbers. Roughly speaking, the interval arithemetic is defined as the simplest way to calculate the upper and lower endpoints for the range of an expression regarding an interval. For example, let $I = [a, b]$, then $f(I)$ can be defined as the smallest interval covering $f(x)$ for all $x \in [a, b]$. In practical applications, we may have the following rules:

- $[a_1, b_1] + [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$;
- $[a_1, b_1] - [a_2, b_2] = [a_1 - b_2, b_1 - a_2]$;
- $[a_1, b_1] \cdot [a_2, b_2] = [\alpha, \beta]$ where

  $$\alpha = \min(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2)$$

  and

  $$\beta = \max(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2);$$

- $[a_1, b_1]/[a_2, b_2] = [a_1, b_1] \cdot (1/[a_2, b_2])$ where

  $$1/[a_2, b_2] = [1/b_2, 1/b_1] \text{if } 0 \notin [a_2, b_2].$$

In interval arithmetic, the upper and lower endpoints may not necessarily be the supremum or infimum, since the precise calculation of those values can be difficult or impossible for a nonlinear function.

An interval matrix A^I means that each entry of the matrix takes value from an interval, whose upper and lower endpoints can be described. Upper endpoints and lower endpoints for all entries of an interval matrix for two ordinary matrices, denoted by $\bar{A}^I$ and $\underline{A}^I$. For simplicity, A^I can be written as $[\underline{A}^I, \bar{A}^I]$.

Consider the following class of uncertain linear interval system [21]:

$$\begin{cases} x_{k+1} = A_k^I x_k + B_k^I \zeta_k \\ y_k = C_k^I x_k + \eta_k \end{cases} \tag{1}$$

where A_k^I, B_k^I and C_k^I are interval matrices which describe the uncertainty of model parameter. Interval matrices are expressed as follows.

$$A_k^I = A_k + \Delta A_k = [A_k - |\Delta A_k|, A_k + |\Delta A_k|] \tag{2}$$

$$B_k^I = B_k + \Delta B_k = [B_k - |\Delta B_k|, B_k + |\Delta B_k|] \tag{3}$$

$$C_k^I = C_k + \Delta C_k = [C_k - |\Delta C_k|, C_k + |\Delta C_k|] \tag{4}$$

where $k = 0, 1, 2, 3, \cdots$, A_k^I, B_k^I and C_k^I are $n \times n$, $n \times p$ and $q \times p$ coefficient matrices, $|\Delta A_k|, |\Delta B_k|$ and $|\Delta C_k|$ are on behalf of the disturbance boundary of corresponding interval matrices. The expected value of the noise signal is known to be zero.

Obviously, the standard Kalman filter algorithm is not suitable for the system that equation (1) describes. Interval Kalman filter is proposed by Guanrong Chen who introduces the idea of interval arithmetics into standard Kalman filter. The derivation process of interval Kalman filter is similar with standard Kalman filter and both have the same part of nature [21,22].

3.3 Mathematical model and positioning algorithm

As described in the previous paragraph, the target can be located in a certain area according to response tags if the tags are laid as the way of rectangular as show in Fig. 2. The coordinate we obtain is in fact a range, however it is noisy. We need to deal with a series of interval data and filter out noise to optimize positioning results. To this end, a mathematical model of positioning process needs to be established with interval information addressed interms of the so-called interval Kalman filter for the purpose of estimating the location coordinates.

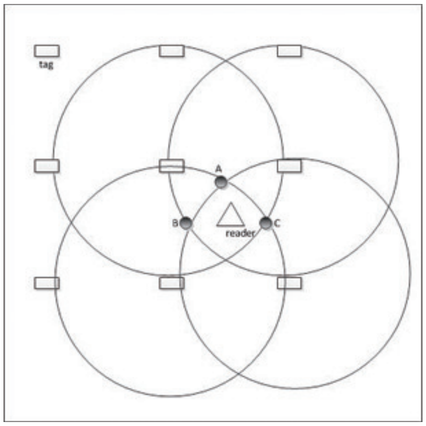

Figure 2: the intersection of response tag sequence

Suppose at time k, the reader's abscissa, the speed on abscissa direction and the acceleration on abscissa direction are respectively denoted as $x(k)$, $\dot{x}(k)$, $\ddot{x}(k)$, the vertical axis, the ordinate, the speed on ordinate direction and the acceleration on ordinate direction are respectively denoted as $y(k)$, $\dot{y}(k)$, $\ddot{y}(k)$.

From kinematic state equation, we can obtain

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \\ \ddot{x}(k+1) \\ y(k+1) \\ \dot{y}(k+1) \\ \ddot{y}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & t & \frac{t^2}{2} & 0 & 0 & 0 \\ 0 & 1 & t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t & \frac{t^2}{2} \\ 0 & 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \\ y(k) \\ \dot{y}(k) \\ \ddot{y}(k) \end{bmatrix} + V(k) \tag{5}$$

According to equation (1), define

$$A_k^I = \begin{bmatrix} 1 & t & \frac{t^2}{2} & 0 & 0 & 0 \\ 0 & 1 & t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t & \frac{t^2}{2} \\ 0 & 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$V(k) = B_k^I \zeta_k \tag{7}$$

We take B_k^I as unit matrix and obtain the system equation, then the observation equation is given based on the relationship between reader coordinate and response tag coordinate. There is a set of response tag sequence at each time, and the reader can be positioning in an area according to the coordinates of response tags. We can calculate the boundary points through the intersection operation. Suppose the number of all tags is n and reading distance is r. At time k, there are m tags responding to the reader and the ith response tag coordinate is (x_i, y_i), the boundary point coordinate is (x_b, y_b).

$$\begin{cases} (x_b - x_1)^2 + (y_b - y_1)^2 = r^2 \\ (x_b - x_2)^2 + (y_b - y_2)^2 = r^2 \\ \vdots \\ (x_b - x_m)^2 + (y_b - y_m)^2 = r^2 \\ (x_b - x_{m+1})^2 + (y_b - y_{m+1})^2 > r^2 \\ \vdots \\ (x_b - x_n)^2 + (y_b - y_n)^2 > r^2 \end{cases} \tag{8}$$

Through the above equation, some boundary point coordinates such like points A, B and C in Fig. 2 can be calculated. Here we suppose that the average values of boundary point coordinates are regarded as the center of the region, approximately valid in most situations. Suppose that the center coordinate is (x_c, y_c), then The observation equation can be expressed as follows:

$$\begin{bmatrix} x_c(k) \\ y_c(k) \end{bmatrix} = \begin{bmatrix} 1+s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+s & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \\ y(k) \\ \dot{y}(k) \\ \ddot{y}(k) \end{bmatrix} + U(k) \tag{9}$$

Among the above equation, $(x_c(k), y_c(k))$ is regional center coordinate, and s is the adjustment range which represents the approximation degree between reader and center point. Define

$$C_k^I = \begin{bmatrix} 1+s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+s & 0 & 0 \end{bmatrix} \tag{10}$$

$$U(k) = \eta_k \tag{11}$$

Eventually we obtain the basic equations of interval Kalman filtering and then we are able to filter the data with the interval Kalman filter. To ensure unbiasedness of estimation, we take the following initial values

$$\hat{x}_0^I = E(x_0^I) \tag{12}$$

$$P_0^I = \mathrm{Cov}(x_0^I) \tag{13}$$

with the following one step prediction covariance matrix M_k^I and interval Kalman gain matrix G_{k+1}^I

$$M_k^I = A_k^I P_k^I [A_k^I]^T + Q_k^I \tag{14}$$

$$G_{k+1}^I = M_k^I [C_{k+1}^I][C_{k+1}^I M_k^I [C_{k+1}^I]^T + R_{k+1}^I]^{-1} \tag{15}$$

Then the interval states and covariance matrices are updated by

$$\hat{x}_{k+1}^I = A_k^I \hat{x}_k^I + G_{k+1}^I [y_{k+1}^I - C_{k+1}^I A_k^I \hat{x}_k^I] \tag{16}$$

$$\begin{aligned} P_{k+1}^I = [I - G_{k+1}^I C_{k+1}^I] M_k^I [I - G_{k+1}^I C_{k+1}^I]^T \\ + [C_{k+1}^I] R_{k+1}^I [C_{k+1}^I]^T \end{aligned} \tag{17}$$

This shows that interval Kalman filter is similar with standard Kalman filter, except that the interval matrix operation in filter process. In the equation (15) the calculation of gain matrix G_{k+1}^I contains interval matrix inversion which takes a long time and is not conducive to real-time filtering. In order to improve computational efficiency, we simplify the inverse process and substitute conventional matrix $[C_{k+1} M_k [C_{k+1}]^T + \Delta Rk + 1]^{-1}$ for interval matrix $[C_{k+1}^I M_k^I [C_{k+1}^I]^T + R_{k+1}^I]^{-1}$ [21].

4 Simulation and result

In this paper, we use the 920Mhz UHF reader with an omni-directional antenna and rectangular passive tags with side length 2cm. Now we make a simulation experiment of positioning moving object in a $5m \times 5m$ indoor room. As shown in Fig. 2, arrange the tags 20cm apart, and every tag's coordinate is known *a priori*. The target is equipped with the reader and its reading distance is set as 25cm. Besides, noise signal will be added to simulate the influence of environment change and antenna performance.

We conduct extensive simulation experiments according to the above assumptions in MATLAB©. The average value of upper and lower bounds represents estimated coordinates in interval Kalman filter algorithm. The position tracking is shown in Fig. 3, and the positioning error is plotted in Fig. 4.

Blue circle represents the position estimation after the interval Kalman filter algorithm, and the green is generated

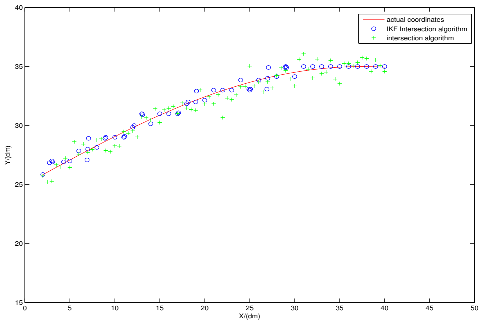

Figure 3: the comparison chart of position tracking

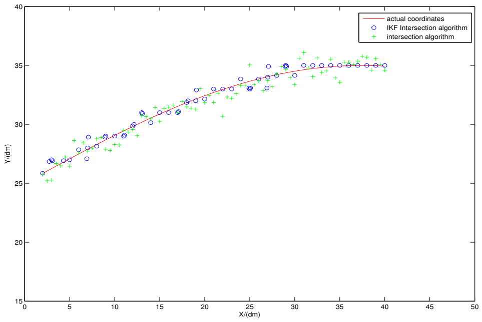

Figure 4: the comparison chart of positioning error

by intersection. Compared with the green point, the blue track is closer to actual coordinates curve, and the positioning error becomes smaller and stable with time goes on. Then we change object's motion path, correspondingly, the simulation result is shown in Fig. 5. Obviously, comparing Fig. 3 with Fig. 5, the positioning effect is better when object is in linear motion .

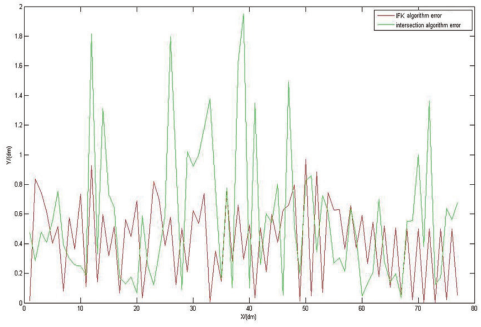

Figure 5: position tracking at linear motion

Different tag arrangements will also affect the positioning accuracy. Two arrangements are adopted for doing simulation experiments, among which one is the arrangement of square, another is the arrangement of equilateral triangle, where the distance between adjacent tags is 20cm. The simulation result is shown in Fig. 6 and Fig. 7.

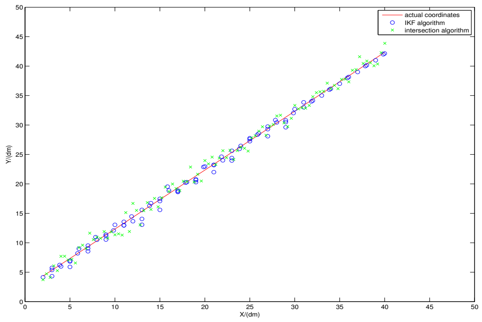

Figure 6: position tracking in different arrangement

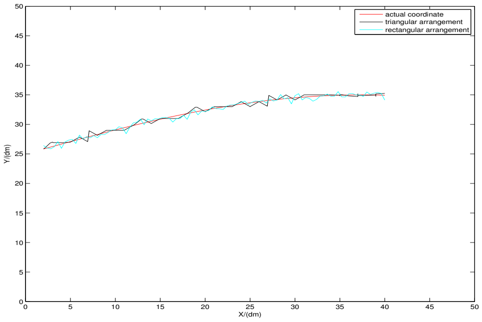

Figure 7: positioning error in different arrangement

From the two figures, we can know that the positioning accuracy of two kinds of tags arrangement is similar. Relatively speaking the positioning effect in triangle arrangement is slightly better than that in square arrangement.

5 Conclusion

In this paper, we discussed a few preliminary issues concerning RFID indoor positioning. Chip performance, environmental factors and cost constraint restrict the improvement of the positioning accuracy. This paper introduces a new kind of positioning algorithm based on interval Kalman filter, which is employed to filter noise signal and improve positioning accuracy, such that high positioning accuracy at low cost can be achieved. Its keyidea is to take the bounds of the measurement from the sensors into consideration, and the RFID sensors have unique property of nondistinguishness in the limited sensing range, which makes it natural to use interval arithmetic to describe the uncertainties due to the sensors. And geometric relationship of detecting ranges of sensors in fact form multiple constraints for the target to be localised, hence using the idea of "information concentration" [23], we can significantly reduce the uncertainty of positioning the target through combination of geometric constraints. And the motion model with the interval Kalman filter makes it possible to successfully track the moving target.

To verify the effectiveness of the algorithm, we have carried out simulated experiments under different conditions, such as changing the target trajectory or adjusting tags arrangement. Simulation results show that the proposed algorithm performs better when the target moves along a straight line and positioning error will increase in the corner. Moreover, the positioning error is smaller as tags are in triangular arrangement compared with that in the case of rectangular arrangement, which needs further investigation. Some existing work [24] may shed a light in this issue, despite that range sensors are more often addressed in the literature.

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