Active Disturbance Rejection Based Iterative Learning Control

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Abstract: The traditional iterative learning control (ILC) algorithm improves the control performance by updating the control input to implicitly compensate the periodic uncertainties. In order to enhance the convergence rate of ILC, a new concept, iterative extended state observer (IESO), is presented which can estimate explicitly the periodic uncertainties during the process of iterations and be used to update the control input directly. The explicit estimation of the uncertainty by the linear IESO in the iteration domain is used to construct a new ILC algorithm based on active disturbance rejection (ADR). The ADR-based ILC algorithm and its corresponding theorem are given in detail and proven by using Lyapunov-like approach. Simulation results verify the effectiveness of the proposed ILC algorithm, and the iterative learning efficiency is improved greatly by using ADR-based ILC algorithm.

Key Words: Iterative Learning Control (ILC), Extended State Observer (ESO), Iterative Extended State Observer (IESO), Active Disturbance Rejection Based Iterative Learning Control (ADR-based ILC), Lyapunov-like Approach

1 INTRODUCTION

Iterative learning control (ILC) is a very effective control method in dealing with repeated tracking control and periodic disturbance rejection for nonlinear plants. The ILC system improves the control performance by updating the control input according to the previous control input and tracking error. In order to improve the convergence rate, the adaptive ILC (AILC) has been presented and studied widely in recent years [1]-[8]. The design and analysis of the AILC systems generally depend on the hypothesis of linear parameterization of the uncertainties which narrow the applications of AILC.

The traditional ILC overcomes the periodic uncertainties by adjusting the control iteration by iteration indirectly and implicitly. In time domain, active disturbance rejection control (ADRC) is a very effective control method for uncertain systems. The extended state observer (ESO) is central in ADRC, which can estimate directly and explicitly the uncertainties including inner disturbance and outer disturbance [9]-[13]. If the ESO can be generalized into the iteration domain, a new ILC method based on active disturbance rejection can be developed. In this paper, a iterative ESO (IESO) is presented to estimate system uncertainties explicitly according to the input and output data of controlled plant during the process of iterations. After that, a new ILC algorithm based on active disturbance rejection (ADR) technology is given by using linear IESO. The ADR-based ILC algorithm does not need the linear parameterization condition and has a wider applied field than traditional AILC. This paper is organized as follows. A typical ILC system is described in Section 2. In Section 3, the linear IESO is presented. After that, the corresponding ILC algorithm and theorem based on linear IESO is given and proven in detail. To demonstrate the effectiveness of the proposed new ILC method, a repetitive tracking control of an inverted pendulum system is used for digital simulations in Section 4. Finally, a conclusion is given in Section 5.

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2 PROBLEM FORMULATION

Consider the following nonlinear time-varying 2-D system which has been studied in [2] and [11]

$$\begin{cases} \dot{x}_i(t,k) = x_{i+1}(t,k); \quad i = 1,\cdots,n-1 \\ \dot{x}_n(t,k) = f(t,x) + b(t,x)u(t,k); \quad t \in [0,T] \end{cases} \tag{1}$$

where x_i is the measurable system state and the corresponding desired trajectory is $r_i(t)$, u is the control input, t is the time variable, k is the iteration variable, n is the system order, $f(t,x)$ and $b(t,x)$ are unknown continuous nonlinear functions of state and time. Denote $x^{\mathrm{T}} = [x_1, \ \cdots \ , x_{\mathrm{n}}]$ and $r^{\mathrm{T}} = [r_1, \ \cdots \ , r_n]$. The control objective for a repeatable system is to find the u to force the system state x to follow r such that $\lim_{k\to\infty}|x_i - r_i| = 0$. In order to achieve the above control objective, some assumptions on the nonlinear plant and desired trajectory are given as follows.

Assumption 1: The desired trajectory $r(t)$ and $\dot{r}_n(t)$ are bounded and satisfy

$$0 \le |\dot{r}_n(t)| \le r_M \tag{2}$$

Assumption 2: $f(t,x)$ and $b(t,x)$ are bounded and locally Lipschitz continuous, and $\dfrac{\partial f}{\partial t}$ is bounded. Suppose x, x^1 and x^2 are any three states, the following inequalities hold.

$$\left|f(t,x^1)-f(t,x^2)\right| \le L_f \left\|x^1-x^2\right\| \tag{3}$$

$$\left|f(t,x)\right| \le f_{\mathrm{M}} \tag{4}$$

$$\left|b(t,x^1)-b(t,x^2)\right| \le L_b \left\|x^1-x^2\right\| \tag{5}$$

$$0 < b_m \le \left|b(t,x)\right| \le b_M \tag{6}$$

Assumption 3: System (1) is with the identical initial condition. The initial errors can be expressed as

$$\begin{cases} e_i(t,k)=x_i(t,k)-r_i(t) \\ e_i(0,k)=0 \end{cases} \quad i=1,\cdots,n \tag{7}$$

The error form of system (1) can be written as

$$\begin{cases} \dot{e}_i(t,k)=e_{i+1}(t,k); \quad i=1,\cdots,n-1 \\ \dot{e}_n(t,k)=f(t,x)+bu(t,k)-\dot{r}_n \end{cases} \tag{8}$$

Assumption 4: $b_0(t,x)$ is a crude estimate of $b(t,x)$. Both $b(t,x)$ and $b_0(t,x)$ have the same sign and satisfy the inequality (5) and (6).

Then the error equation (8) becomes

$$\begin{cases} \dot{e}_i(t,k)=e_{i+1}(t,k); \quad i=1,\cdots,n-1 \\ \dot{e}_n(t,k)=f_{be}(t,x)+b_0u(t,k)-\dot{r}_n \end{cases} \tag{9}$$

where

$$f_{be}(t,x)=f(t,x)+(b-b_0)u(t,k) \tag{10}$$

The ILC is essentially a kind of data-driven control method [14]-[15]. For system (1), $f_{be}(t,x)$ can be estimated very well by ESO from the input and output data of controlled plant [11]-[12]. If we use linear ESO, the linear ESO of system (1) in the time domain can be expressed as

$$\begin{cases} \dot{\hat{x}}_i(t,k)=\hat{x}_{i+1}-\dfrac{a_{n+1-i}}{\varepsilon^i}(\hat{x}_1-x_1); \quad i=1,\cdots,n-1 \\ \dot{\hat{x}}_n(t,k)=\hat{x}_{n+1}-\dfrac{a_1}{\varepsilon^n}(\hat{x}_1-x_1)+b_0u(t,k) \\ \dot{\hat{x}}_{n+1}(t,k)=-\dfrac{a_0}{\varepsilon^{n+1}}(\hat{x}_1-x_1) \end{cases} \tag{11}$$

Assumption 5: The initial state of the above linear ESO satisfies the zero initial condition.

$$\hat{x}_i(0,k)=0 \qquad i=1,\ \cdots,n+1 \tag{12}$$

For linear ESO (11), there exists the following lemma.

Lemma 1: Suppose that Assumption 1 and 2 are satisfied, ε is a constant gain in the range of $0<\varepsilon\le 1$, $a_i>0 \ \ (i=0,\cdots,n)$ and the polynomial $s^{n+1}+a_ns^n+\cdots+a_1s+a_0=0$ is Hurwitz. Then system (11) is the linear ESO of system (1) and we have the following results.

(1) For every positive constant T_ε, the linear ESO's estimation error converges uniformly to limit zero.

$$\lim_{\varepsilon\to 0}\left|x_i(t,k)-\hat{x}_i(t,k)\right|=0 \quad t\in[T_\varepsilon,\infty) \tag{13}$$

(2) The limit superior of the LESO's estimation error satisfies

$$\overline{\lim_{t\to\infty}}\left|x_i(t,k)-\hat{x}_i(t,k)\right| \le O(\varepsilon^{(n+2-i)}) \tag{14}$$

where $i=1,\cdots,n+1$, and $x_{n+1}=f_{be}(t,x)$ is the extended state of system (1).

The remarks and proof of lemma 1 can be found in [9]-[12].

Remark 1: The estimation process of linear ESO in the time domain is not suitable for the iteration domain because its tracking accuracy will not be improved with the increase of iteration number. The estimation method of $f_{be}(t,x)$ given by Lemma 1 cannot be directly applied in ILC.

Remark 2: The ILC system (1) operates over the finite time interval $[0,T]$, while the linear ESO (7) operates over the infinite time interval $[0,\infty)$. When $T_\varepsilon \notin [0,T]$ because ε is not small enough, it is difficult to get the accurate estimate of $f_{be}(t,x)$ over the interval $[0,T]$. It is thus necessary to modify the linear ESO in the time domain and make it suitable for the iteration domain.

3 ADR-BASED ILC

This section first introduces the linear ESO in the iteration domain according to its form in the time domain. After that, the corresponding algorithm and theorem are presented.

3.1 Linear IESO

Taking the following coordinate transformation

$$\begin{cases} z_1(t,k)=\hat{x}_1(t,k) \\ z_i(t,k)=\hat{x}_i-\displaystyle\sum_{j=1}^{i-1}\frac{a_{n+1-j}}{\varepsilon^j}(z_{i-j}-x_{i-j}) \quad i=2,\cdots,n+1 \end{cases} \tag{15}$$

for the linear ESO (11) and defining

$$\begin{cases} z_0(t,k)=\int_0^t z_1(\tau,k)\mathrm{d}\tau \\ x_0(t,k)=\int_0^t x_1(\tau,k)\mathrm{d}\tau \end{cases} \tag{16}$$

we have

$$\begin{cases} \dot{z}_i(t,k)=z_{i+1}; \quad i=0,\cdots,n-1 \\ \dot{z}_n(t,k)=-\displaystyle\sum_{j=0}^{n}\frac{a_j}{\varepsilon^{n+1-j}}\delta_j(t,k)+b_0u \end{cases} \tag{17}$$

where

$$\begin{cases} \delta_j(t,k)=x_j(t,k)-z_j(t,k) \\ \delta_j(0,k)=0 \quad j=0,\cdots,n \end{cases} \tag{18}$$

From (1) we obtain

$$x_i-z_i=x_i-\hat{x}_i+\sum_{j=1}^{i-1}\frac{a_{n+1-j}}{\varepsilon^j}(x_{i-j}-z_{i-j}) \tag{19}$$

Therefore, it follows from the properties of the absolute value that

$$\left|x_i-z_i\right| \le \left|x_i-\hat{x}_i\right|+\left|\sum_{j=1}^{i-1}\frac{a_{n+1-j}}{\varepsilon^j}(x_{i-j}-z_{i-j})\right| \tag{20}$$

Moreover, according to (14) and (20), we obtain

$$\overline{\lim_{t\to\infty}}\left|x_i(t,k)-z_i(t,k)\right| \le O(\varepsilon^{(n+2-i)}) \tag{21}$$

Thus, both z_i and $\hat{x}_i$ can be considered as the approximations of x_i. The error equation of the transformed system (17) can then be written as

$$\begin{cases}\dot{\delta}_i(t,k)=\delta_{i+1}(t,k); \quad i=0,\cdots,n-1\\ \dot{\delta}_n(t,k)=f_{b\delta}(t,x)+b_0u(t,k)-\dot{x}_n\end{cases} \tag{22}$$

where

$$f_{b\delta}(t,x)=-\sum_{j=0}^{n}\frac{a_j}{\varepsilon^{n+1-j}}\delta_j \tag{23}$$

Obviously, both the error equation (9) and the linear ESO (22) have the same structure and parameters if the difference of the variable notation e_i and δ_i is not considered. ESO (22) employs the linear ESO's own state z_i to approximate the system state x_i and to estimate the extended state $f_{b\delta}(t,x)$ while equation (9) employs the current system state x_i to approximate the future system state r_i and to estimate the future extended system state $f_{be}(t,x)$. Therefore, the system error equation (9) itself has the ability to estimate the extended state $f_{be}(t,x)$. System (1) and system (17) are a pair of so-called dual systems, and so are the system (9) and system (22). According to (9) and (23), the following ILC algorithm (24) may be used to estimate $f_{be}(t,x)$, the extended state of system (1).

$$\begin{cases}w_r(t,0)=0, \quad k\in Z^+\\ w_r(t,k)=w_r(t,k-1)+\sum_{j=0}^{n}\frac{a_j}{\varepsilon^{n+1-j}}e_j(t,k)\end{cases} \tag{24}$$

where

$$e_0(t,k)=\int_0^t e_1(\tau,k)\mathrm{d}\tau \tag{25}$$

Thus, $w_r(t,k)$ can be considered as the estimate of $f_{be}(t,x)$. The estimation accuracy of $f_{be}(t,x)$ will increase rapidly by adding a correction term $\sum_{j=0}^{n}\frac{a_j}{\varepsilon^{n+1-j}}e_j(t,k)$ at each iteration. The system (17) can be used as the linear IESO of system (1).

3.2 ADR-based ILC Algorithm

Define the iterative estimation correction $\sigma(t,k)$ as

$$\sigma(t,k)=\frac{\varepsilon}{a_n}\sum_{j=0}^{n}\frac{a_j}{\varepsilon^{n+1-j}}e_j(t,k) \tag{26}$$

Thus, (24) becomes

$$\begin{cases}w_r(t,0)=\sigma(t,0)=0, \quad k\in Z^+\\ w_r(t,k)=w_r(t,k-1)+\frac{a_n}{\varepsilon}\sigma(t,k)\end{cases} \tag{27}$$

where $w_r(t,k)$ is the estimate of $f_{be}(t,x)$. Taking the derivative of $\sigma(t,k)$ with respect to *t* gives

$$\dot{\sigma}(t,k)=\frac{\varepsilon}{a_n}\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k)+\dot{e}_n(t,k) \tag{28}$$

Combining (8) and (28) yields

$$\dot{\sigma}(t,k)=\frac{\varepsilon}{a_n}\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}+f_{be}(t,x)+b_0u(t,k)-\dot{r}_n \tag{29}$$

Design the control law as

$$u(t,k)=b_0^{-1}(\dot{r}_n-w_x(t,k)-\beta\sigma(t,k)) \tag{30}$$

where $\beta\geq 0$ is feedback gain and $w_x(t,k)$ is defined by

$$w_x(t,k)=w_r(t,k)+\frac{\varepsilon}{a_n}\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k) \tag{31}$$

By substituting control law (30) and (10) into (29), we have

$$\begin{aligned}\dot{\sigma}(t,k)=&\frac{b}{b_0}(\hat{f}_b(t,x)-w_r(t,k))+\\ &(1-\frac{b}{b_0})\frac{\varepsilon}{a_n}\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k)-\frac{b}{b_0}\beta\sigma\end{aligned} \tag{32}$$

where

$$\hat{f}_b(t,x)=\frac{b_0}{b}f(t,x)+\frac{b-b_0}{b}\dot{r}_n \tag{33}$$

According to the boundedness and locally Lipschitz continuity of f, b, b_0 and r from (2)-(6), we have

$$\left|\hat{f}_b(t,x^1)-\hat{f}_b(t,x^2)\right|\leq L_{\hat{f}}\left\|x^1-x^2\right\| \tag{34}$$

where

$$L_{\hat{f}}=\frac{b_M}{b_m}L_f+\frac{2r_M}{b_m}L_b \tag{35}$$

Therefore, $\hat{f}_b(t,k)$ is bound and locally Lipschitz. Based on the above analysis, the solution to the ILC problem of system (1) is given by the following theorem.

Theorem 1: For the ILC of system (1) over the time interval of $[0,T]$, under Assumptions 1-4, the iterative learning law (27) and control law (30) ensures the perfect tracking performance of system (1), and the following limits hold.

$$\lim_{k\to\infty}e_i(t,k)=0 \qquad i=1,\cdots,n \tag{36}$$

$$\lim_{k\to\infty}w_r(t,k)=\hat{f}_b(t,r) \tag{37}$$

Proof: According to (6) and (26), we have

$$\dot{e}(t,k)=A_ce(t,k)+B_c\dot{\sigma}(\tau,k) \tag{38}$$

where

$$A_c=\begin{bmatrix}0 & 1 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \cdots & \vdots\\ \frac{-a_0}{\varepsilon^n a_n} & \frac{-a_1}{\varepsilon^{n-1}a_n} & \cdots & \frac{-a_{n-1}}{\varepsilon a_n}\end{bmatrix},\ B_c=\begin{bmatrix}0 & \cdots & 0 & 1\end{bmatrix}^{\mathrm{T}}$$

Considering the zero initial condition, integrating the both sides of (38) from 0 to *t*, we have

$$e(t,k)=\int_0^t A_ce(\tau,k)\mathrm{d}\tau+B_c\sigma(\tau,k) \tag{39}$$

Taking norms for both sides of (39), according to the compatibility of the matrix norm and the vector norm, we have

$$|e(t,k)|\leq\int_0^t\|A_c\|\cdot|e(\tau,k)|\mathrm{d}\tau+|\sigma(\tau,k)| \tag{40}$$

According to Gronwall's lemma, (40) becomes

$$|e(t,k)|\leq\|A_c\|\cdot\mathrm{e}^{\|A_c\|T}\int_0^t|\sigma(\tau,k)|\mathrm{d}\tau+|\sigma(\tau,k)|, t\in[0,T] \tag{41}$$

From (41) it is clear that $e(t,k)$ converges to zero when $\sigma(t,k)$ does. In order to prove the convergence of $\sigma(t,k)$ and $w_r(t,k)$, define the following Lyapunov-like functional [16]:

$$L(t,k)=\frac{b_0}{2b}\sigma^2(t,k)+\frac{\varepsilon}{2a_n}\int_0^t(\hat{f}_b(\tau,r)-w_r(\tau,k))^2\mathrm{d}\tau \quad (42)$$

We first prove the monotonicity of $L(t,k)$ and then the convergence of $\sigma(t,k)$ and $w_r(t,k)$. The difference between $L(t,k)$ and $L(t,k-1)$ is

$$\Delta L(t,k)=L(t,k)-L(t,k-1)=-\frac{b_0}{2b}\sigma^2(t,k-1)+\text{PartA}+\text{PartB} \quad (43)$$

where PartA is

$$\text{PartA}=\frac{b_0}{2b}\sigma^2(t,k)=\frac{b_0}{b}\int_0^t\sigma(\tau,k)\dot{\sigma}(\tau,k)\mathrm{d}\tau \quad (44)$$

because $\sigma(0,k)=0$ and PartB is

$$\text{PartB}=\frac{\varepsilon}{2a_n}\int_0^t((\hat{f}_b(\tau,r)-w_r(\tau,k))^2-(\hat{f}_b(\tau,r)-w_r(\tau,k-1))^2)\mathrm{d}\tau \quad (45)$$

Substituting (32) into (44) yields

$$\text{PartA}=-\beta\int_0^t\sigma^2(\tau,k)d\tau+\int_0^t\sigma(\tau,k)(\hat{f}_b(\tau,x)-w_r(\tau,k))\mathrm{d}\tau+(\frac{b_0}{b}-1)\frac{\varepsilon}{a_n}\int_0^t\sigma(\tau,k)\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k)\mathrm{d}\tau \quad (46)$$

Applying the algebraic relationship $(h\text{-}d)^2\text{-}(h\text{-}c)^2=\text{-}(d\text{-}c)^2\text{-}2(d\text{-}c)(h\text{-}d)$ to (45), we have

$$\text{PartB}=-\frac{\varepsilon}{2a_n}\int_0^t((w_r(\tau,k)-w_r(\tau,k-1))^2\mathrm{d}\tau-\frac{\varepsilon}{a_n}\int_0^t(\hat{f}_b(\tau,r)-w_r(\tau,k))\cdot(w_r(\tau,k)-w_r(\tau,k-1))\mathrm{d}\tau \quad (47)$$

Substituting the learning law (27) into (47) yields

$$\text{PartB}=-\frac{a_n}{2\varepsilon}\int_0^t\sigma(\tau,k)^2\mathrm{d}\tau-\int_0^t\sigma(\tau,k)(\hat{f}_b(\tau,r)-w_r(\tau,k))\mathrm{d}\tau \quad (48)$$

Substituting (48) and (46) into (43) yields

$$\Delta L(t,k)\le-\frac{b_0}{2b}\sigma^2(t,k-1)-(\beta+\frac{a_n}{2\varepsilon})\int_0^t\sigma^2(\tau,k)\mathrm{d}\tau+\text{PartC} \quad (49)$$

where

$$\text{PartC}=\int_0^t\sigma(\tau,k)\cdot(\hat{f}_b(\tau,x)-\hat{f}_b(\tau,r))\mathrm{d}\tau+(\frac{b_0}{b}-1)\frac{\varepsilon}{a_n}\int_0^t\sigma(\tau,k)\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k)\mathrm{d}\tau \quad (50)$$

Taking the absolute value of the right-hand side of (50), we have

$$\text{PartC}\le\int_0^t|\sigma(\tau,k)|\cdot|f(\tau,x)-f(\tau,r)|\mathrm{d}\tau+\left|\frac{b_0}{b}-1\right|\frac{\varepsilon}{a_n}\int_0^t|\sigma(\tau,k)|\left|\sum_{j=0}^{n-1}\frac{a_j}{\varepsilon^{n+1-j}}e_{j+1}(t,k)\right|\mathrm{d}\tau \quad (51)$$

Considering Assumption 2, we have

$$\text{PartC}\le(L_f+\left|\frac{b_0}{b}-1\right|\frac{\varepsilon}{a_n}|a|)\int_0^t|\sigma(\tau,k)|\cdot|e(\tau,k)|\mathrm{d}\tau\le L_{\hat{f}_b}\int_0^t|\sigma(\tau,k)|\cdot|e(\tau,k)|\mathrm{d}\tau \quad (52)$$

where

$$a=[\frac{a_0}{\varepsilon^{n+1}},\frac{a_1}{\varepsilon^n},\cdots,\frac{a_{n-1}}{\varepsilon^2},\frac{a_n}{\varepsilon^1}] \quad (53)$$

$$L_{\hat{f}_b}=L_f+\left|\frac{b_0}{b}-1\right|\frac{\varepsilon}{a_n}|a| \quad (54)$$

Combining (52) and (39) yields

$$\text{PartC}\le L_{\hat{f}_b}\int_0^t|\sigma(\tau,k)|^2\mathrm{d}\tau+L_{\hat{f}_b}\cdot\|A_c\|\mathrm{e}^{\|A_c\|T}\int_0^t|\sigma(\tau,k)|(\int_0^\tau|\sigma(\tau_1,k)|\mathrm{d}\tau_1)\mathrm{d}\tau \quad (55)$$

Because $\tau\in[0,t]$ we have

$$\text{PartC}\le L_{\hat{f}_b}\int_0^t|\sigma(\tau,k)|^2\mathrm{d}\tau+L_{\hat{f}_b}\cdot\|A_c\|\mathrm{e}^{\|A_c\|T}\int_0^t|\sigma(\tau,k)|(\int_0^t|\sigma(\tau_1,k)|\mathrm{d}\tau_1)\mathrm{d}\tau \quad (56)$$

Applying the properties of double integrals to (56), we have

$$\text{PartC}\le L_f\int_0^t|\sigma(\tau,k)|^2\mathrm{d}\tau+L_f\|A_c\|\mathrm{e}^{\|A_c\|T}(\int_0^t|\sigma(\tau,k)|\mathrm{d}\tau)^2 \quad (57)$$

Applying Cauchy-Schwarz inequality to (57), we have

$$\text{PartC}\le L_{\hat{f}_b}\int_0^t\sigma^2(\tau,k)\mathrm{d}\tau+L_{\hat{f}_b}\|A_c\|\mathrm{e}^{\|A_c\|T}t\int_0^t\sigma^2(\tau,k)\mathrm{d}\tau \quad (58)$$

Considering $0\le t\le T$, we have

$$\text{PartC}\le L_{\hat{f}_b}(1+\|A_c\|\mathrm{e}^{\|A_c\|T}T)\int_0^t\sigma^2(\tau,k)\mathrm{d}\tau \quad (59)$$

Substituting (59) into (49) yields

$$\Delta L(t,k)\le-\frac{b_0}{2b}\sigma^2(t,k-1)-(\beta+\frac{a_n}{2\varepsilon}-L_{\hat{f}_b}(1+T\|A_c\|\mathrm{e}^{\|A_c\|T}))\int_0^t\sigma^2\mathrm{d}\tau \quad (60)$$

Choose β and ε so that the following relation holds

$$\beta+\frac{a_n}{2\varepsilon}>L_{\hat{f}_b}(1+T\|A_c\|\mathrm{e}^{\|A_c\|T}) \quad (61)$$

Thus, from (60), it can be derived that

$$\Delta L(t,k)\le-\frac{b_0}{2b}\sigma^2(t,k-1) \quad (62)$$

When $\sigma=0$, the equality sign in (62) holds. Hence, $L(t,k)$ is a strictly monotonic decay function in the iteration domain when $\sigma\ne0$. When $\sigma=0$, (36) in Theorem 1 holds directly. The following discussion is about $\sigma\ne0$.

The boundedness of $L(t,k)$ can then be proved as follows. The first iteration of $L(t,k)$ is

$$L(t,1)=\Delta L(t,1)+L(t,0) \quad (63)$$

Substituting (43) when $k=1$ into the above equation and considering $w_r(t,0)=0$, we have the following relation by utilizing the same derivations as from (43) to (60)

$$L(t,1)\le\frac{\varepsilon}{2a_n}\int_0^t(\hat{f}_b(\tau,r))^2\mathrm{d}\tau-(\beta+\frac{a_n}{2\varepsilon}-L_{\hat{f}_b}(1+T\|A_c\|\mathrm{e}^{\|A_c\|T}))\int_0^t\sigma^2(\tau,1)\mathrm{d}\tau \quad (64)$$

Furthermore, according to (60) and the boundedness of $\hat{f}_b(t,r)$, we have

$$L(t,1) \le \frac{\varepsilon}{2a_n}\int_0^t (\hat{f}_b(\tau,r))^2 \mathrm{d}\tau \le \frac{\varepsilon}{2a_n}\int_0^T (\hat{f}_b(\tau,r))^2 \mathrm{d}\tau < \infty \quad (65)$$

Therefore, $L(t,1)$ is bounded. Because $L(t,k)$ is nonnegative and strictly monotonically decreases with k, the boundedness of $L(t,k)$ guarantees its limit exists.

$$\lim_{k\to\infty} L(t,k) = 0 \quad (66)$$

Using (62) repeatedly, we have

$$L(t,k) = \sum_{j=2}^{k}(L(t,j) - L(t,j-1)) + L(t,1) = L(t,1) + \sum_{j=2}^{k}\Delta L(t,j) \le L(t,1) - \frac{b_0}{2b}\sum_{j=2}^{k}\sigma^2(t,j-1) \quad (67)$$

Taking the limits of both sides of (67), we have

$$\sum_{j=2}^{\infty}\sigma^2(t,j-1) \le \frac{2b}{b_0}L(t,1) \quad (68)$$

According to the necessary condition for convergent series, we have

$$\lim_{k\to\infty}\sigma(t,k) = 0 \qquad t\in[0,T] \quad (69)$$

Considering (41), we have

$$\lim_{k\to\infty} e_i(t,k) = 0 \qquad t\in[0,T]$$

Combining (42)、(66) and (69), we have

$$\lim_{k\to\infty} w_r(t,k) = \hat{f}_b(t,r)$$

Hence, (36) and (37) hold.

Combining (30), (31) and (37), we have

$$u(t,\infty) = b^{-1}(\dot{r}_n - f(t,r)) \quad (70)$$

According to (14) and (33), we have

$$f_{be}(t,r) = f(t,r) + (b-b_0)u(t,\infty) \quad (71)$$

and

$$\hat{f}_b(t,r) = \frac{b_0}{b}f(t,r) + \frac{b-b_0}{b}\dot{r}_n \quad (72)$$

Substituting (70) into (70), we have

$$f_{be}(t,r) = \hat{f}_b(t,r) \quad (73)$$

This means $w_r(t,k)$ is an iterative estimate of $f_{be}(t,x)$ and converges to $f_{be}(t,r)$. □

Remark 3: Theorem 1 gives explicitly an iterative estimation method for $f_{be}(t,x)$ which includes the system uncertainties from unknown $f(t,x)$ and $b(t,x)$. This estimation method does not need the structure information of $f(t,x)$ and $b(t,x)$ and has stronger adaptability than general adaptive ILC which requires the uncertainty to satisfy the linear parameterization condition.

4 NUMERICAL SIMULATIONS

Consider the following second-order nonlinear uncertain system

$$\begin{cases} \dot{x}_1(t,k) = x_2(t,k) \\ \dot{x}_2(t,k) = f(t,x) + u(t,k) \end{cases} \quad (74)$$

Suppose $f(t,x)$ and $b(t,x)$ satisfy the assumptions of Theorem 1 and their structure and parameters are unknown. During simulation, they are set as

$$f(t,x) = \frac{g\sin x_1 - \dfrac{ml(x_2)^2\cos x_1 \cdot \sin x_1}{m_c+m}}{l(\dfrac{4}{3} - \dfrac{m(\cos x_1)^2}{m_c+m})} \quad (75)$$

$$b(t,x) = \frac{\dfrac{\cos x_1}{m_c+m}}{l(\dfrac{4}{3} - \dfrac{m(\cos x_1)^2}{m_c+m})} \quad (76)$$

where $g=9.8$, $m_c=1$, $m=0.1$ and $l=0.5$. Actually, the above equation (74) is the dynamic equation of the inverted pendulum system. Suppose the desired trajectory is

$$\begin{cases} r_1(t) = \sin t + \sin 2t \\ r_2(t) = \cos t + 2\cos 2t \\ r_3(t) = \sin t - 4\sin 2t \end{cases} \quad (77)$$

The initial system states are $x_1(0,k)=0$ and $x_2(0,k)=1$. Utilize the following indices to evaluate the control performance.

$$J_1(k) = \log_{10}(\max_{t\in[0,T]}|e_1(t,k)|),$$

$$J_2(k) = \log_{10}(\max_{t\in[0,T]}|e_2(t,k)|)$$

Choose the parameters of (27) and (30) as $a_0=1$, $a_1=3$, $a_2=3$, $\varepsilon=0.05$, $\beta=5$. The nice tracking performances of both states after 7 iterations are provided from Figure 1 to Figure 5.

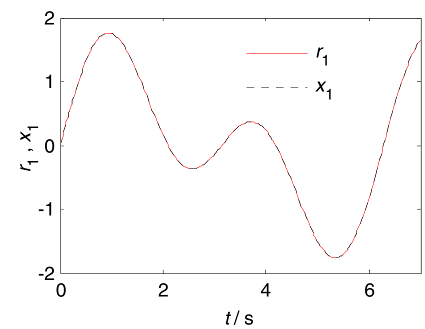

Fig. 1. r_1 and x_1 after 7 iterations

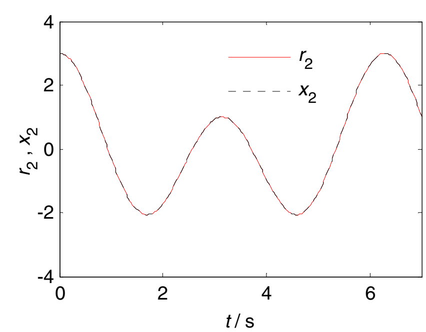

Fig. 2. r_2 and x_2 after 7 iterations

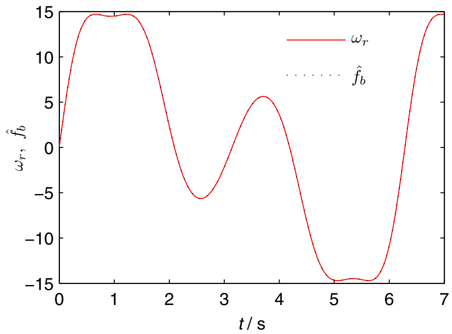

Fig. 3. $w_r(t,k)$ and $\hat{f}_b(t,r)$ after 7 iterations

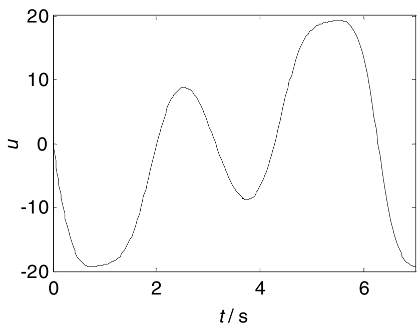

Fig. 4. Control input u after 7 iterations

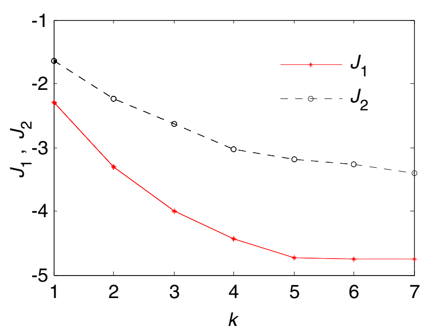

Fig. 5. $J_1(k)$ and $J_2(k)$ over iterations

From the simulation results, it is clear that $J_1(k)$ and $J_2(k)$ decrease rapidly, x_i and r_i nearly coincide with each other and $w_r(t,k)$ is the accurate estimate of $f(t,r)$ after 7 iterations.

5 CONCLUSION

ADR-based ILC algorithm is presented for uncertain time-varying system. Rigorous theoretical proof and numerical simulation of this new algorithm demonstrates its effectiveness. This new method does not require the linear parameterization condition and has a wider field of applications than traditional adaptive ILC methods.

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