An Iterative Learning Control Approach for Two Dimensional Discrete Fornasini–Mrchesini Model

Kai Wan[1], Xiao-Dong Li[1,*]
1. School of information Science and Technology, Sun Yat-Sen University, 510006, Guangzhou, China
E-mail: wankai606815@163.com E-mail: lixd@mail.sysu.edu.cn

Abstract: Almost all existing Iterative Learning Control (ILC) algorithms have focused on One-Dimensional (1-D) dynamical systems, and seldom are used in multidimensional systems. In this article, a two-gain ILC law is presented to deal with ILC issue of Two-Dimensional (2-D) linear discrete systems described by the First Fornasini–Marchesini Model (FMMI). Convergence of the proposed ILC law under identical boundary condition is theoretically investigated. A super-vector technique is used to transfer the ILC process of 2-D FMMI into a 2-D Roessor model such that a sufficient convergence condition of the proposed ILC law is derived. Also, robustness of the proposed ILC law against iteration-variant boundary condition is illustrated by simulation.

Keywords: Iterative Learning Control (ILC), Two-Dimensional (2-D) linear discrete systems, The First Fornasini–Marchesini model, Roessor model.

1. INTRODUCTION

Iterative Learning Control (ILC) [1] is an effective and intelligent control strategy which constantly updates the control input for dynamical system operating on a given task repetitively over a fixed time interval. The information of error and control input in the preceding trial is used to generate a new control input in the next trial, such that the tracking performance of the plant is greatly improved. One of the significant features of ILC is that less a prior knowledge about the controlled system is required in design process, which has made ILC strategy increasingly prevalent in application with repetitive cases, such as disk drive systems, robotic manipulators, etc.[2]-[3].

However, most of the reported research achievements on ILC hitherto have been made for One-Dimensional (1-D) dynamical systems [1]-[6]. A brief survey with categorization on these ILC studies for 1-D dynamical systems was reported in [4], and a general introduction to the contraction mapping based ILC designs for 1-D dynamical systems can be found in [5]. As far as the Two-Dimensional (2-D) dynamical systems are concerned, they exist widely in practical life and industry, such as multidimensional robotic manipulators, image data processing, and gas absorptions, etc, and are often required to operate in repetitive ways [7]-[8]. As a result, a broad engineering blueprint of ILC theories for 2-D dynamical systems has been presented. However, compared with the 1-D dynamical systems, the ILC process of 2-D dynamical systems is very complicated. And the existing ILC designs for 1-D dynamical systems cannot be directly transplanted to 2-D dynamical cases, such that few ILC literatures on 2-D dynamical systems are produced. To the best of authors' knowledge, the ILC research for 2-D dynamical systems began in [9]. The authors in [10] presented a novel ILC method for 2-D linear Single-Input Single-Output (SISO) discrete systems described by the First Fornasini–Marchesini Model (FMMI). And optimal ILC algorithms for 2-D linear discrete systems with the Second Fornasini–Marchesini Model (FMMII) were discussed in [11]. Then, the ILC technique with the fuzzy logic algorithms for 2-D FMMI was presented in [12], which can reduce the trajectory error in far less number of iterations. Recently, a novel P-type ILC algorithm was proposed in [13] for a class of

This work is supported by National Science Foundation under Grants U1135005 and 61573385.
* Corresponding author.

2-D linear discrete systems with Roessor model [14]. And an adaptive ILC approach was proposed in [15] for 2-D linear discrete systems represented by 2-D FMMI to deal with the ILC issue with iteration-varying reference trajectory and random initial error. However, the number of the inputs and the number of outputs in FMMI of [15] are required to be identical. Based on these very limited ILC results, it is essential to exploit more ILC techniques for 2-D dynamical systems.

The main objective of this paper is to investigate the convergence of a two-gain ILC law for 2-D linear discrete FMMI systems under identical boundary condition. The used strategy is based on the super-vector technique, and transfers the ILC process of 2-D FMMI into a 2-D Roessor model. A sufficient convergence condition for the proposed ILC law is derived from the convergent property of 2-D Roessor model. Under identical boundary condition, the proposed ILC law can drive the ILC tracking error of 2-D FMMI systems to zero at the iteration domain except for the boundary. As the boundary condition is iteration-variant, the bound of the robust ILC tracking error with the proposed ILC law will be addressed.

The remainder of this brief is organized as follows. The problem formulation is provided in Section 2. In Section 3, convergence analysis of ILC design for 2-D FMMI is presented. In Section 4, an illustrative example is introduced. Section 5 concludes this paper.

Notations: Let R^n denote the n-Dimensional (n-D) Euclidean space, $R^{n\times m}$ denotes the set of all $n\times m$ real matrices. Also the n-D identity matrix is denoted by I_n. $\rho(\cdot)$ is the spectral radius of its matrix argument.

2. PROBLEM FORMULATION

Consider the ILC issue of the following 2-D linear discrete FMMI systems [16], performing a given task repeatedly over a finite region

$$\begin{aligned} x_k(i+1,j+1) &= A_1\cdot x_k(i+1,j)+A_2\cdot x_k(i,j) \\ &\quad + A_3\cdot x_k(i,j+1)+B\cdot u_k(i,j), \quad (1)\end{aligned}$$

$$y_k(i,j)=C\cdot x_k(i,j), \quad (2)$$

where $k=0,\ 1,\ 2,\ \cdots$ denotes the number of operation cycle, and i, j are integer-valued coordinates running from $i=0$ to $i=M$ and $j=0$ to $j=N$ to complete a cycle. For all integer pairs (i,j), $x_k(i,j)\in R^n$, $u_k(i,j)\in R^r$ and $y_k(i,j)\in R^m$ are system state, control input, and system output, respectively; $A_1\in R^{n\times n}$, $A_2\in R^{n\times n}$, $A_3\in R^{n\times n}$, $B\in R^{n\times r}$, and $C\in R^{m\times n}$ are real matrices.

For convenience of discussing the ILC issue on 2-D FMMI (1)-(2), the following definition on integer pair (i,j) is given.

Definition 1: The relations "$\le$, $<$ and $=$" on integer pair (i,j) are defined as follows

- $(i_1,j_1)\le(i_2,j_2)$ iff $i_1\le i_2$ and $j_1\le j_2$,
- $(i_1,j_1)=(i_2,j_2)$ iff $i_1=i_2$ and $j_1=j_2$,
- $(i_1,j_1)<(i_2,j_2)$ iff $(i_1,j_1)\le(i_2,j_2),(i_1,j_1)\ne(i_2,j_2)$

Given a reference output trajectory $y_d(i,j)\in R^m$, $(0,0)\le(i,j)\le(M,N)$, which is achievable for 2-D FMMI (1)-(2), and an initial control input $u_0(i,j)$, $(0,0)\le(i,j)\le(M-1,N-1)$, the objective of ILC is to iteratively determine a control input sequence $\{u_k(i,j)\}$, $(0,0)\le(i,j)\le(M-1,N-1)$, such that the system output $y_k(i,j)$ can well track the reference trajectory $y_d(i,j)$ as the cycle number k goes to infinity. Let the ILC tracking error $e_k(i,j)$ at the k-th iteration be denoted as

$$e_k(i,j)=y_d(i,j)-y_k(i,j). \quad (3)$$

In this paper, the following two-gain ILC law is proposed

$$u_{k+1}(i,j)=u_k(i,j)+\Gamma_1\cdot[e_k(i+1,j+1)-K_1\cdot e_k(i+1,j)], \quad (4)$$

where $\Gamma_1\in R^{r\times m}$ and $K_1\in R^{m\times m}$ are the learning gain matrices. For convergence analysis of the ILC law (4), an identical boundary condition on 2-D FMMI (1)-(2) is assumed as following,

Assumption 1: The boundary condition of $x_k(i,0)$ and $x_k(0,j)$ on 2-D FMMI (1)-(2) satisfies that

$$x_k(0,j)=x_0(0,j) \quad j=1,\ 2,\ \dots,\ N,$$

$$x_k(i,0)=x_d(i,0) \quad i=0,\ 1,\ 2,\ \dots,\ M,$$

where $x_0(0,j)$ and $x_d(i,0)$ denote the fixed boundary function and the desired boundary function for $i=0,\ 1,\ 2,\ \dots,\ M$, respectively.

3. CONVERGENCE ANALYSIS of ILC DESIGN

In this section, the convergence of the proposed ILC law with sufficient condition for 2-D FMMI is provided in Theorem 1.

For 2-D FMMI (1)-(2), let

$$\hat{A}_1=\begin{bmatrix} I_n & & & 0 \\ -A_3 & I_n & & \\ & \ddots & \ddots & \\ 0 & & -A_3 & I_n \end{bmatrix}\in R^{nM\times nM},$$

$$\hat{A}_2=\begin{bmatrix} A_1 & & & 0 \\ A_2 & A_1 & & \\ & \ddots & \ddots & \\ 0 & & A_2 & A_1 \end{bmatrix}\in R^{nM\times nM},$$

$$\hat{B}=\begin{bmatrix} B & & 0 \\ & B & \\ & & \ddots & \\ 0 & & B \end{bmatrix}\in R^{nM\times rM},\ \hat{C}_2=\begin{bmatrix} A_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\in R^{nM\times n},$$

$$\hat{C}=\begin{bmatrix} C & & 0 \\ & C & \\ & & \ddots & \\ 0 & & C \end{bmatrix}\in R^{mM\times nM},\ \hat{C}_1=\begin{bmatrix} A_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\in R^{nM\times n},$$

$$X_k(j)=\begin{bmatrix} x_k^T(1,j) & x_k^T(2,j) & \cdots & x_k^T(M,j)\end{bmatrix}^T\in R^{nM}, \quad (5)$$

$$Y_k(j)=\begin{bmatrix} y_k^T(1,j) & y_k^T(2,j) & \cdots & y_k^T(M,j)\end{bmatrix}^T\in R^{mM}, \quad (6)$$

$$Y_d(j)=\begin{bmatrix} y_d^T(1,j) & y_d^T(2,j) & \cdots & y_d^T(M,j)\end{bmatrix}^T\in R^{mM}, \quad (7)$$

$$U_k(j)=\begin{bmatrix} u_k^T(0,j) & u_k^T(1,j) & \cdots & u_k^T(M-1,j)\end{bmatrix}^T\in R^{rM}, \quad (8)$$

Using the super-vector technique, the 2-D FMMI (1)-(2) in ILC process can be rewritten as

$$\hat{A}_1\cdot X_k(j+1)=\hat{A}_2\cdot X_k(j)+\hat{B}\cdot U_k(j)$$

$$+\hat{C}_1\cdot x_k(0,j+1)+\hat{C}_2\cdot x_k(0,j), \quad (9)$$

$$Y_k(j)=\hat{C}\cdot X_k(j). \quad (10)$$

Furthermore, since the matrix $\hat{A}_1$ is non-singular, we can obtain the following system

$$X_k(j+1)=\widetilde{A}\cdot X_k(j)+\widetilde{B}\cdot U_k(j)$$

$$+\widetilde{C}_1\cdot x_k(0,j+1)+\widetilde{C}_2\cdot x_k(0,j) \quad (11)$$

$$Y_k(j)=\widetilde{C}\cdot X_k(j), \quad (12)$$

where

$$\widetilde{A}=\hat{A}_1^{-1}\hat{A}_2,\ \widetilde{B}=\hat{A}_1^{-1}\hat{B},\ \widetilde{C}_1=\hat{A}_1^{-1}\hat{C}_1,$$

$$\widetilde{C}_2=\hat{A}_1^{-1}\hat{C}_2,\ \widetilde{C}=\hat{C}.$$

At each iteration of the ILC process, the ILC tracking error can be represented as

$$E_k(j)=Y_d(j)-Y_k(j) \qquad j=0,1,2,\cdots,N, \quad (13)$$

where

$$E_k(j)=\begin{bmatrix} e_k^T(1,j) & e_k^T(2,j) & \cdots & e_k^T(M,j)\end{bmatrix}^T\in R^{mM}. \quad (14)$$

Using (13), we have

$$E_{k+1}(j)-E_k(j)=Y_k(j)-Y_{k+1}(j). \quad (15)$$

Then, let

$$\eta_k(j+1)=X_{k+1}(j)-X_k(j).$$

From (11), it yields

$$\eta_k(j+1)=\widetilde{A}\cdot X_{k+1}(j-1)+\widetilde{B}\cdot U_{k+1}(j-1)$$

$$+\widetilde{C}_1\cdot x_{k+1}(0,j)+\widetilde{C}_2\cdot x_{k+1}(0,j-1)$$

$$-\widetilde{A}\cdot X_k(j-1)-\widetilde{B}\cdot U_k(j-1)$$

$$-\widetilde{C}_1\cdot x_k(0,j)-\widetilde{C}_2\cdot x_k(0,j-1)$$

$$=\widetilde{A}\cdot\eta_k(j)+\widetilde{B}\cdot\Delta U_k(j-1)$$

$$+\widetilde{C}_1\cdot[x_{k+1}(0,j)-x_k(0,j)]$$

$$+\widetilde{C}_2\cdot[x_{k+1}(0,j-1)-x_k(0,j-1)], \quad (16)$$

where

$$\Delta U_k(j-1)=U_{k+1}(j-1)-U_k(j-1). \quad (17)$$

On the other hand, for each $j=0,1,2,\cdots,N$, the two-gain ILC law (4) can be rewritten as the following super-vector form,

$$\begin{bmatrix} u_{k+1}(0,j) \\ u_{k+1}(1,j) \\ \vdots \\ u_{k+1}(M-1,j)\end{bmatrix}=\begin{bmatrix} u_k(0,j) \\ u_k(1,j) \\ \vdots \\ u_k(M-1,j)\end{bmatrix}+\begin{bmatrix} \Gamma_1 & & 0 \\ & \Gamma_1 & \\ & & \ddots \\ 0 & & \Gamma_1\end{bmatrix}$$

$$\times\begin{bmatrix} e_k(1,j+1) \\ e_k(2,j+1) \\ \vdots \\ e_k(M,j+1)\end{bmatrix}-\begin{bmatrix} \Gamma_1 & & 0 \\ & \Gamma_1 & \\ & & \ddots \\ 0 & & \Gamma_1\end{bmatrix}$$

$$\times\begin{bmatrix} K_1 & & & 0 \\ & K_1 & & \\ & & \ddots & \\ 0 & & & K_1 \end{bmatrix} \cdot \begin{bmatrix} e_k(1,j) \\ e_k(2,j) \\ \vdots \\ e_k(M,j) \end{bmatrix}. \quad (18)$$

According to the definitions of $U_k(j)$ and $E_k(j)$ in (8) and (14), the equation (18) becomes

$$U_{k+1}(j) = U_k(j) + \Gamma \cdot [E_k(j+1) - K \cdot E_k(j)], \quad (19)$$

where

$$\Gamma = diag\{\Gamma_1, \Gamma_1, \cdots, \Gamma_1\}, \quad K = diag\{K_1, K_1, \cdots, K_1\}.$$

In order to present the convergence result of the proposed ILC law (4) or (19) under Assumption 1, Lemma 1 on 2-D linear discrete Roessor model is firstly introduced as follows.

Lemma 1 [17]: For the Roesser model of 2-D linear discrete system

$$\begin{bmatrix} \eta_k(j+1) \\ \zeta_{k+1}(j) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} \eta_k(j) \\ \zeta_k(j) \end{bmatrix}, \quad (20)$$

where $\eta_k(j) \in R^{n_1}$, $\zeta_k(j) \in R^{n_2}$, $A_{11} \in R^{n_1 \times n_1}$, $A_{12} \in R^{n_1 \times n_2}$, $A_{21} \in R^{n_2 \times n_1}$ and $A_{22} \in R^{n_2 \times n_2}$, the boundary conditions are $\eta_k(1) = 0$ for $k = 0, 1, 2, \cdots,$ and finite $\zeta_0(j)$ for $j = 1, 2, \cdots, N$. If there is

$$\rho(A_{22}) < 1,$$

then, for $j = 1, 2, \cdots, N$, we have

$$\lim_{k \to +\infty} \begin{bmatrix} \eta_k(j) \\ \zeta_k(j) \end{bmatrix} = 0. \quad (21)$$

Based on Lemma 1 on 2-D Roessor model, the convergence result of the proposed ILC law (4) in Theorem 1 can be proved.

Theorem 1: For 2-D linear discrete system described by the FMMI (1)-(2), Assumption 1 on boundary condition and the ILC law (4) are provided. If there is $\rho(I_m - CB\Gamma_1) < 1$, then the ILC tracking error $e_k(i,j)$ satisfies that.

$$\lim_{k \to +\infty} e_k(i,j) = 0, \quad (1,1) \le (i,j) \le (M,N). \quad (22)$$

Proof: Substituting (19) into (16), and according to Assumption 1, we obtain

$$\eta_k(j+1) = \widetilde{A} \cdot \eta_k(j) + \widetilde{B}\Gamma \cdot [E_k(j) - K \cdot E_k(j-1)]. \quad (23)$$

Let

$$\zeta_k(j) = E_k(j) - K \cdot E_k(j-1). \quad (24)$$

From (12), (13) and (24), we have

$$\begin{aligned} \zeta_{k+1}(j) - \zeta_k(j) &= E_{k+1}(j) - E_k(j) - K \cdot [E_{k+1}(j-1) - E_k(j-1)] \\ &= -\widetilde{C} \cdot [X_{k+1}(j) - X_k(j)] + K\widetilde{C} \\ &\quad \times [X_{k+1}(j-1) - X_k(j-1)]. \end{aligned} \quad (25)$$

Inserting (23) into (25), we get

$$\zeta_{k+1}(j) = \left(-\widetilde{C}\widetilde{A} + K\widetilde{C}\right) \cdot \eta_k(j) + \left(I_{mM} - \widetilde{C}\widetilde{B}\Gamma\right) \cdot \zeta_k(j). \quad (26)$$

Then, (23) and (26) can be rewritten as

$$\eta_k(j+1) = \widetilde{A} \cdot \eta_k(j) + \widetilde{B}\Gamma \cdot \zeta_k(j), \quad (27)$$

$$\zeta_{k+1}(j) = \left(-\widetilde{C}\widetilde{A} + K\widetilde{C}\right) \cdot \eta_k(j) + \left(I_{mM} - \widetilde{C}\widetilde{B}\Gamma\right) \cdot \zeta_k(j). \quad (28)$$

Essentially, (27) and (28) constitute a 2-D linear discrete system with Roessor model [14], from Assumption 1 and the definitions on $\eta_k(j)$ and $\zeta_k(j)$, it is obtained that $\eta_k(1) = 0$ for $k = 0, 1, 2, \cdots,$ and finite $\zeta_0(j)$ for $j = 1, 2, \cdots, N$. According to **Lemma 1**, if there is

$$\rho\left(I_{mM} - \widetilde{C}\widetilde{B}\Gamma\right) < 1,$$

equivalently, $\rho(I_m - CB\Gamma_1) < 1$, then for $j = 1, 2, \cdots, N$,

$$\lim_{k \to +\infty} \zeta_k(j) = 0. \quad (29)$$

Furthermore, according to (24), it yields

$$E_k(j) = K^j \cdot E_k(0) + \sum_{h=1}^{j} K^{j-h} \cdot \zeta_k(h). \quad (30)$$

From Assumption 1 and the definition (14) on $E_k(j)$, it is obtained that $E_k(0) = 0$, $k = 0, 1, 2, \cdots,$ Then, taking limitation on both sides of (30) as $k \to +\infty$, we have

$$\lim_{k \to +\infty} E_k(j) = 0, \quad j = 1, 2, \cdots, N. \quad (31)$$

From (14), it yields

$$\lim_{k \to +\infty} e_k(i,j) = 0, \quad (1,1) \le (i,j) \le (M,N). \quad (32)$$

Theorem 1 is proved.

Remark 1: Theorem 1 shows that the convergence of the ILC rule (4) depends on the learning gain Γ_1 only, and is irrelevant of the learning gain K_1. Specifically, when the learning gain K_1 is set as $K_1 = 0$, a simpler form of P-type ILC law for 2-D discrete system is produced. However, as the 2-D FMMI system (1)-(2) is subject to the iteration-variant boundary condition, the selection of the learning gain K_1 will greatly affect the robust property of the ILC rule (4). This point will be better illustrated by simulation studies in next section.

Remark 2: Although the ILC law (4) with convergence analysis is presented for 2-D FMMI, it is well known that there exist transformation relations among classical models of 2-D linear discrete system [7]. For example, the Attasi model is a special case of the 2-D FMMI, and the Roesser model with some particular coefficient matrices can be transferred into the 2-D FMMI. Therefore, it is straightforward to see that the results in this paper can be easily extended to 2-D linear discrete systems described by the Attasi model and the Roesser model, etc.

4. ILLUSTRATIVE EXAMPLES

To show the effectiveness of the proposed ILC algorithm for 2-D FMMI, computer simulations are conducted for mathematical systems in this section.

Example: Consider the ILC problem of 2-D FMMI system (1)-(2) with

$$A_1=\begin{bmatrix}0.0139 & 0.01\\ 0.01 & 0.03\end{bmatrix},\quad A_2=\begin{bmatrix}0.0139 & 0.02\\ 0.01 & 0.04\end{bmatrix},$$

$$A_3=\begin{bmatrix}0.01 & 0.02\\ 0.0139 & 0.3\end{bmatrix},\quad B=\begin{bmatrix}2\\ 0.015\end{bmatrix},\quad C=\begin{bmatrix}-1 & -0.05\end{bmatrix}.$$

For different discrete indexes i and j, the desired reference trajectory $y_d(i,j)$ is described by

$$y_d(i,j)=\sin(2\pi(i+j)/10),\quad (0,0)\le(i,j)\le(20,20),$$

In this example, the ILC rule (4) with initial control input $u_0(i,j)=0$ for $(0,0)\le(i,j)\le(19,19)$ is applied, In order to evaluate the accuracy of ILC tracking beyond the boundary, the following tracking error index EE_k is proposed,

$$EE_k=\sum_{i=1}^{20}\sum_{j=1}^{20}\left|y_d(i,j)-y_k(i,j)\right|.$$

Corresponding to Assumption 1, the boundary conditions $x_k(i,0)$ and $x_k(0,j)$ of the simulated 2-D FMMI system are given by

$$x_k(i,0)=\begin{bmatrix}-\sin(0.2\pi i)\\ 0\end{bmatrix},\quad i=0,1,2,\cdots,20,$$

$$x_k(0,j)=\begin{bmatrix}0.5\\ \sin(j)\end{bmatrix},\quad j=1,2,\cdots,20.$$

As the learning gains Γ_1 and K_1 in ILC rule (4) are taken as $\Gamma_1=-0.3$ and $K_1=0.4$, which satisfy the convergence condition of Theorem 1, Fig. 1 presents the tracking error surfaces $e_k(i,j)$ at different iterations ($k=3,5,15,20$), respectively. Also, Fig. 2 shows the situation of the tracking error index EE_k along the iteration axis. Clearly, it is observed that the ILC tracking error $e_k(i,j)$ for $(1,1)\le(i,j)\le(20,20)$ converges to zero as the iteration number k goes to infinity. The convergence of the proposed ILC rule (4) under Assumption 1 is thus illustrated by Fig. 1 and Fig. 2.

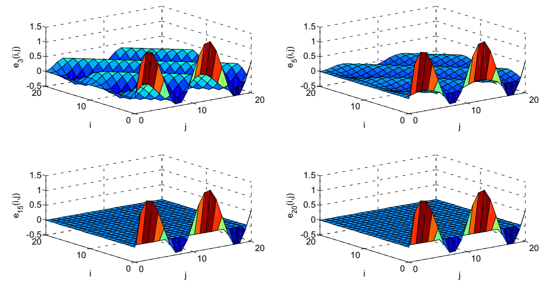

Fig.1. Under identical boundary condition, the ILC tracking error surfaces $e_k(i,j)$ at different iterations by using the ILC law (4).

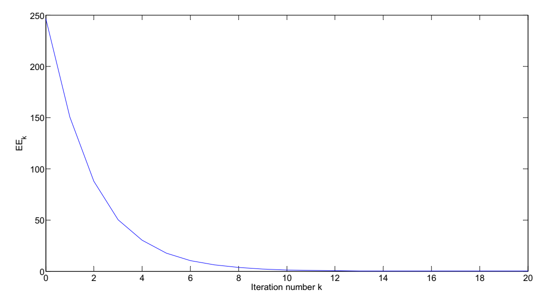

Fig.2. Under identical boundary condition, the ILC tracking error index EE_k by using the ILC law (4).

Next, under the iteration-variant boundary condition, let us illustrate the effects of learning gain K_1 on robustness of the proposed ILC law (4). Suppose that one boundary state of the simulated 2-D FMMI system is iteration-invariant and given as $x_k(0,j)=[0.5\quad \sin(j)]^T$, $j=1,2,\cdots,20$, and another iteration-variant boundary state is $x_k(i,0)=[-\sin(0.2\pi i)+m_k\quad n_k]^T$, $i=0,1,2,\cdots,20$, for $k=0,1,2,\cdots,$, where m_k and n_k vary randomly at the interval $[-1,1]$ and $[0,2]$, respectively. Fig. 3 presents the situation of the ILC tracking error index EE_k with different learning gain values of K_1. Obviously, it can be seen that the robust bound of the ILC tracking error $e_k(i,j)$ will reduce with decreasing of the learning gain K_1. The robust effects of learning gain K_1 in ILC law (4) against the iteration-variant boundary condition are thus well illustrated.

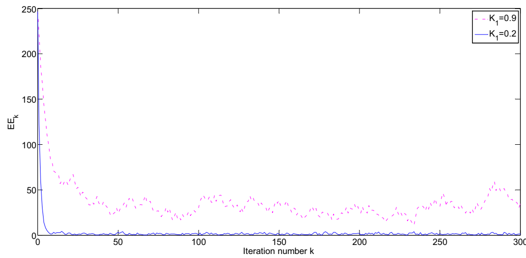

Fig. 3. Under iteration-variant boundary condition, the ILC tracking error index EE_k by using the ILC law (4) with the different learning gains $K_1 = 0.9, 0.2$, respectively.

5. CONCLUSION

The existing ILC algorithms have been mainly designed for 1-D dynamical systems. Compared to 1-D dynamical systems, the ILC process of 2-D dynamical systems is more complicated such that few ILC approaches hitherto have been related to 2-D dynamical systems. This paper has presented a two-gain ILC law for 2-D linear discrete FMMI systems. Using a super-vector technique and the convergence property of 2-D Roessor model, it has been theoretically proved that under identical boundary condition, the proposed ILC law can drive the ILC tracking error to zero at the iteration domain except for the boundary. In addition, our simulation studies have shown that the robust property of the proposed ILC law against the iteration-variant boundary condition can be greatly improved by adjusting the learning gain. In future work, the robustness of the proposed ILC law against the iteration-variant boundary condition will be further investigated in theory.

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