Previous best input based first order D-type iterative learning control for nonlinear system with high order relative degree

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Abstract: In general, high-order D-type iterative learning control (ILC) law should be taken for the controlled system with high order relative degree to ensure the convergence, but the high order differential operation is seriously influenced by measurement noise. This paper proposes a first order D-type ILC law combined with the previous best control input for the system with high order relative degree. The form of the ILC with forgetting factor is utilized to guarantee its convergence and the previous best control input is introduced to improve its convergence performance. Its sufficient convergence condition is given based on contraction mapping method. Finally, several numerical results illustrate the efficiency of the proposed method.

Key Words: Iterative learning control, Previous best input, First order D-type, Contraction mapping method

1 INTRODUCTION

Iterative learning control (ILC) as an effective control technology has been widely applied in repeatable control processes, such as robotic manipulator, hard disk drives, and heat flux control[1-3]. In these applications, D-type ILC law is one of the most popular and effective iterative learning law, which depends crucially on the relative degree of the controlled system. It is well known that the control errors will be divergent if the classical first-order D-type learning law is applied in the system with the high order relative degree r, so for the system with $r \geq 2$, in general, high-order D-type ILC law (HDILC) should be taken as follow[4]

$$u_{k+1}(t) = u_k(t) + We_k^{(r)} \quad (1)$$

where W is the learning gain, $e_k(t)$ is the system output error at the iteration k, i.e. $e_k(t) = y_d(t) - y_k(t)$, $y_d(t)$ and $y_k(t)$ are the desired output and system output respectively, $e_k^{(r)}$ denotes the r-th derivative of $e_k(t)$. But the high order differential operation in HDILC will augment the influence of the output measurement noise. In addition, sometimes it is hard to determine the relative degree of the uncertain nonlinear systems.

Sun et al.[10, 11] proposed a P-type ILC law for nonlinear system with arbitrary relative degree by introducing the concept of extended relative degree. But it is difficult to determine the extended relative degree if the structure or parameters of the model of the nonlinear system is uncertain[4]. Thus, it is necessary to extend the classical first-order D-type ILC law to solve the aforementioned issues.

Song et al.[4] proposed a first-order D-type ILC law based on a dummy model with relative degree 1 for a class of nonlinear systems with unknown relative degree. Heinzinger et al.[5] had proposed a D-type ILC law with forgetting factor, which can efficiently overcome the impact of the state disturbances and the output noise:

$$u_{k+1}(t) = (1-\gamma)u_k(t) + \gamma u_0(t) + L(y_k(t), t)\dot{e}_k \quad (2)$$

where γ is the forgetting factor, $u_0(t)$ is the initial control input, and learning gain L is bounded related to the output $y_k(t)$. Obviously, the convergence of the learning algorithm (2) relies on the initial control input. Subsequently, Xie et al.[6] proposed a new ILC with forgetting factor by substituting the control input signal of last iteration $u_{k-1}(t)$ for the initial control input signal, which is presented as follow:

$$u_{k+1}(t) = (1-\gamma)u_k(t) + \gamma u_{k-1}(t) + L(y_k(t), t)\dot{e}_k \quad (3)$$

For the system with one relative degree, the learning laws (2) and (3) can guarantee the tracking error $e_k(t)$ converges to zero if and only if the learning factor γ decreases to zero in direction of the iterative domain. But, if the relative degree r of the system is bigger than one, the learning laws (2) and (3) cannot guarantee the tracking error $e_k(t)$ converges to zero. For the law (2), the control input $u_k(t)$ will ultimately converge to the neighborhood of the initial control input $u_0(t)$, not the desired control input $u_d(t)$; for the law (3), the parameter L has to decrease to zero as the iteration k tends to infinite so as to guarantee the convergence, and the unfeasible selection of the corresponding parameters will cause the vibration of performance index along the iterative domain.

Motivated by the form of ILC with forgetting factor, which can guarantee that the ILC law convergence, this paper proposes a new first-order D-type ILC law to improve the first order D-type ILC law to deal with the tracking problem of the controlled system with high order relative degree. To eliminate the influence of measurement noise and suppress the vibration of performance index along the iterative domain, in the proposed method, the previous

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best control input signal u^{best} until current iteration k is added to improve the convergence performance of the classical first order D-type ILC law. Compared with the method in reference [4], the proposed ILC law is simpler, independence on the system model and easier for realization; compared with the learning law (3), the information of previous best control input signal u^{best} can effectively suppress the vibration of performance index along the iterative domain during the iterative process. In addition, the sufficient convergence condition of the proposed method based on contraction mapping method is given. Eventually, several numerical results prove the efficiency of the proposed method.

2 PRELIMINARIES

Consider a single-input single-output (SISO) nonlinear system:

$$\dot{x}(t) = f(x(t)) + g(x(t)) \cdot u(t)$$
$$y(t) = h(x(t)) \quad (4)$$

where $t \in [0, \mathrm{T}]$ denotes the time index, $x \in \mathrm{R}^n$, $u \in \mathrm{R}^1$ and $y \in \mathrm{R}^1$ are the state, input and output vectors of the controlled system, respectively. The nonlinear functions $f(\cdot)$, $g(\cdot)$, $h(\cdot)$ are sufficiently smooth in their domain of definition and have appropriate dimensions.

Definition 1. The system (4) is said to have relative degree r, $1 \leq r \leq n$, at x_0, if

$$L_g L_f^{i-1} h(x) = 0, i = 1,2, \ldots, r-1$$
$$L_g L_f^{r-1} h(x) \neq 0 \quad (5)$$

for all x in a neighborhood of x_0.

The system (4) is assumed to satisfy the following assumptions.

Assumption 1. A continuous desired output $y_d(t)$ for $t \in [0, \mathrm{T}]$ is available with a unique input $u_d(t)$, i.e.

$$\dot{x}_d(t) = f(x_d(t)) + g(x_d(t)) \cdot u_d(t)$$
$$y_d(t) = h(x_d(t)) \quad (6)$$

where $x_d(t)$ is the desired state. And, the desired input $u_d(t)$ is bounded, $\|u_d(t)\| \leq b_{ud}$, and b_{ud} is a positive number.

Assumption 2. The nonlinear functions $f(x(t))$, $g(x(t))$, $h(x(t))$, $L_f\, h(x(t))$ and $L_g h(x(t))$ satisfy the global *Lipschitz* continuity condition in x, i.e.

$$\|f(x_1(t)) - f(x_2(t))\| \leq k_f \|x_1(t) - x_2(t)\|$$
$$\|g(x_1(t)) - g(x_2(t))\| \leq k_g \|x_1(t) - x_2(t)\|$$
$$\|h(x_1(t)) - h(x_2(t))\| \leq k_h \|x_1(t) - x_2(t)\|$$
$$\|L_f\, h(x_1(t)) - L_f\, h(x_2(t))\| \leq k_{lf} \|x_1(t) - x_2(t)\|$$
$$\|L_g\, h(x_1(t)) - L_g\, h(x_2(t))\| \leq k_{lg} \|x_1(t) - x_2(t)\| \quad (7)$$

where k_f, k_g and k_h are the positive *Lipschitz* constants.

Assumption 3. $g(x(t))$ is bounded in sense of $\|g(x(t))\| \leq b_g$ for x, and b_g is a positive number.

Assumption 4. $h(x(t))$ satisfies the following condition:

$$c_1 \|x(t)\| \leq \|h(x(t))\| \quad (8)$$

where c_1 is a positive number.

Assumption 5. All operations start from the initial condition $x_k(0) = x_d(0)$ for all $k = 1,2, \ldots, n$.

For example, the real robot manipulator systems satisfy the all aforementioned assumptions.

According to the assumptions 1-5, it is easy to get the lemma 1 for the system's convergence[7].

Lemma 1. Given the desired output $y_d(t), t \in [0, \mathrm{T}]$, let $D(x_k(t)) = L_g L_f^{r-1} h(x_k(t))$. If the iterative learning law (1) is applied to the system (4), and the following inequality

$$\|1 - W \cdot D(x_k(t))\| \leq \rho < 1 \quad (9)$$

holds for all $x_k(t)$, the output errors $e_k(t)$ will tend to zero as $k \to \infty$.

3 MAIN RESULTS

3.1 First order D-type ILC based on previous best input

A new ILC based on the previous best control input is proposed in this section. Let us define the performance index

$$J_k = log_{10}(max\,|e_k(t)|)$$

where k is the iteration number, and let the previous best control input u^{best} is the corresponding control input of the minimum performance J_{min} up to k, i.e. $J_{min} = min\{J_1, J_2, \ldots, J_k\}$, y^{best} is the corresponding output signal of the previous best control input u^{best}.

The first order D-type ILC based on previous best input is given as follows:

$$u_{k+1}(t) = u_k(t) + \mathrm{r}\dot{e}_k(t) + \alpha\left(u^{\text{best}} - u_k(t)\right) \quad (10)$$

where α denotes the learning factor, $\dot{e}_k(t)$ denotes the derivative of $e_k(t)$. If $\alpha = 0$, the learning law (10) is turn into the classical first-order D-type learning law.

Lemma 2. For the system (4), the previous best control input u^{best} is bounded under the assumptions 1-4.

Proof: the output y^{best} is bounded because $0 \leq J_{min} \leq J_1$ and the desired trajectory $y_d(t)$ is bounded. In addition, $f(\cdot)$, $g(\cdot)$, $h(\cdot)$ are sufficiently smooth and satisfy the global *Lipschitz* continuity condition, which are also bounded. Thus, for the system (4), the state x^{best} and $\dot{x}^{\text{best}}$ with a finite time interval $t \in [0, \mathrm{T}]$ is also bounded. Therefore, u^{best} is bounded.

The convergence condition of the proposed learning law (10) is presented in the next subsection.

3.2 Convergence analysis

Definition 1. The λ-norm for a function $b(t)$ is

$$\|b\|_\lambda \triangleq \sup e^{-\lambda t} \|b\|, t \epsilon [0, T] \quad (11)$$

where λ is a positive scalar.

Lemma 3[7]**.** Let $\{a_k\}$ be a real sequence defined as

$$a_k \leq \rho a_{k-1-j} + b_k, 0 \leq j \leq M-1, \quad (12)$$

where b_k is a real sequence satisfying $\lim_{k\to\infty} \sup b_k \leq b_\infty$. If $0 \leq \rho < 1$, then $\lim_{k\to\infty} sup\, a_k \leq \frac{b_\infty}{1-\rho}$.

According to the above lemma, the sufficient condition to ensure the convergence of the proposed ILC is established in the following theorem.

Theorem 1. Under the Assumptions 1-5, the iterative learning law (10) is applied to the system (4). If the following inequality

$$\left\|1-\alpha-\mathrm{r}L_g h(x_k(t))\right\| \le \rho < 1,\ t\in[0,\mathrm{T}] \quad (13)$$

holds for all $x_k(t)$, then

1) For the system (4) with relative degree one, the output error $e_k(t)$ will converge to zero if the parameter α decreases to zero as $k\to\infty$.

2) For the system (4) with relative degree $\mathrm{r}\ge 2$, the output error $e_k(t)$ will converge to a bounded domain if the parameter α decreases to a constant ϵ as $k\to\infty$.

Proof. From the learning law (10), we can get that

$$\begin{aligned}\Delta u_{k+1}(t) &= u_d(t)-u_{k+1}(t)\\ &= u_d(t)-u_k(t)-\mathrm{r}\dot{e}_k(t)-\alpha(u^{\text{best}}-u_k(t))\\ &= \Delta u_k(t)-\mathrm{r}[L_f h(x_d(t))-L_f h(x_k(t))+\\ &\quad L_g h(x_d(t))\cdot u_d(t)-L_g h(x_k(t))\cdot u_k(t)]-\\ &\quad \alpha[u^{\text{best}}-u_d(t)+\Delta u_k(t)]\\ &= [(1-\alpha)I-\mathrm{r}L_g h(x_k(t))]\Delta u_k(t)\ -\\ &\quad \mathrm{r}\Big[L_f h(x_d(t))+\big(L_g h(x_d(t))-L_g h(x_k(t))\big)\cdot\\ &\quad u_d(t)\Big]-\\ &\quad \alpha\left(u^{\text{best}}-u_d(t)\right)-L_f h(x_k(t))\end{aligned} \quad (14)$$

Taking the norms on both sides of (14), we get

$$\begin{aligned}\|\Delta u_{k+1}(t)\| &\le \left\|1-\alpha-\mathrm{r}L_g h(x_k(t))\right\|\cdot\|\Delta u_k(t)\|\ +\\ &\quad \mathrm{r}(k_{lf}+k_{lg}b_{ud})\cdot\|\Delta x_k(t)\|+\alpha\left\|u^{\text{best}}-u_d(t)\right\|\end{aligned} \quad (15)$$

Moreover, we can also get

$$\begin{aligned}\Delta x_k(t) &= x_d(t)-x_k(t)\\ &= \int_0^t [f(x_d(t))+g(x_d(t))\cdot u_d(t)-\\ &\quad f(x_k(t))-g(x_k(t))\cdot u_k(t)]d\tau\end{aligned} \quad (16)$$

Taking the norms on both sides of (16), we get

$$\begin{aligned}\|\Delta x_k(t)\| &\le \int_0^t [(k_f+k_g b_{ud})\cdot\|\Delta x_k(t)\|+\\ &\quad b_g\|\Delta u_k(t)\|]d\tau\end{aligned} \quad (17)$$

According to the Bellman-Cronwall Lemma[8], the (17) can be rewritten by

$$\|\Delta x_k(t)\| \le b_g\int_0^t e^{b_2(t-\tau)}\,\|\Delta u_k(t)\|d\tau \quad (18)$$

where $b_2=k_f+k_g b_{ud}$. Substituting (18) into (15) to obtain:

$$\begin{aligned}\|\Delta u_{k+1}(t)\| &\le \rho\cdot\|\Delta u_k(t)\|\ +\alpha\left\|u^{\text{best}}-u_d(t)\right\|+\\ &\quad b_1 b_g\int_0^t e^{b_2(t-\tau)}\,\|\Delta u_k(t)\|d\tau\end{aligned} \quad (19)$$

where, $b_1=k_{lf}+k_{lg}b_{ud}$ and let the parameter ρ satisfies the following condition:

$$\left\|1-\alpha-\mathrm{r}L_g h(x_k(t))\right\| \le \rho,$$

and considering that u^{best} and $u_d(t)$ are bounded, then we can get

$$\left\|(u^{\text{best}}-u_d(t))\right\|_\lambda \le k^{\text{best}} \quad (20)$$

k^{best} is a positive parameter. Multiplying $e^{-\lambda t}$ on both sides of (17) to obtain:

$$\begin{aligned}e^{-\lambda t}\|\Delta u_{k+1}(t)\| &\le \rho e^{-\lambda t}\|\Delta u_k(t)\|\ +(\alpha k^{\text{best}})+\\ &\quad b_1 b_g\int_0^t e^{(b_2-\lambda)\cdot(t-\tau)}\,e^{-\lambda\tau}\|\Delta u_k(t)\|d\tau\end{aligned} \quad (21)$$

To simplify (21) as:

$$\|\Delta u_{k+1}(t)\|_\lambda \le (\rho+\rho_1)\cdot\|\Delta u_k(t)\|_\lambda+\alpha k^{\text{best}} \quad (22)$$

where $\rho_1=b_1 b_g\cdot\frac{1-e^{(b_2-\lambda)\cdot T}}{\lambda-b_2}$, and ρ_1 will be small enough to be neglected if λ is big enough. Thus, if the inequality (13) is satisfied, according to the Lemma 3, then yield

$$\lim_{k\to\infty} sup(\|\Delta u_k(t)\|_\lambda) \le \frac{\alpha k^{best}}{1-\rho-\rho_1},\ t\in[0,\mathrm{T}] \quad (23)$$

If the relative degree r of the system is one, the inequality (13) can be guaranteed by adjusting the parameter value of learning gain r when parameter α decreases to zero as $k\to\infty$, then we can get that

$$\|\Delta u_k(t)\|_\lambda \to 0\ \ (k\to\infty) \quad (24)$$

Consequently, according to (18), we can also get

$$0 \le \lim_{k\to\infty}\|e_k(t)\|_\lambda \le k_h\cdot\lim_{k\to\infty}\|\Delta x_k(t)\|_\lambda = 0 \quad (25)$$

Since u^{best} is the current previous best control input of $u_k(t)$ found so far, thus u^{best} will converge to the desired control input $u_d(t)$ as $u_k(t)$ tends to u_d when $k\to\infty$.

If the relative degree r of the controlled system is bigger than two, then $L_g h(x_k(t))$ will always be zero. In order to guarantee the convergence of the proposed ILC law (10), the parameter α should be smaller than one, but bigger than zero. If the parameter α finally converges to a very small positive number ϵ, then $\Delta u_k(t)$ will converge to a small bounded domain,

$$\lim_{k\to\infty}\sup(\|\Delta u_k(t)\|_\lambda) \le \frac{\epsilon k^{\text{best}}}{1-\rho-\rho_1} \quad (26)$$

And, according to (18),

$$\begin{aligned}0 &\le \lim_{k\to\infty}\|e_k(t)\|_\lambda \le k_h\cdot\lim_{k\to\infty}\|\Delta x_k(t)\|_\lambda\\ &\le \frac{k_h\rho_1\epsilon k^{\text{best}}}{1-\rho-\rho_1}\end{aligned} \quad (27)$$

This completes the proof.

Remark 1. For the ILC law (2), there is the similar conclusion as theorem 1, that is, if the parameter γ converges to a small constant ϵ, the control input $u_k(t)$ is convergent and will finally converge to the neighborhood of the initial control input $u_0(t)$ as the iteration k tends to infinite. However, it is hard to determine a satisfactory initial control input $u_0(t)$ at first in the ILC law (2).

Remark 2. It is difficult to determine whether the control input $u_{k-1}(t)$ is bounded or not when the ILC law (3) is utilized to deal with the tracking problem of the controlled system with relative order $r\ge 2$. And, the tracking performance will be very poor if the control input $u_{k-1}(t)$ is unbounded.

Remark 3. The classical first-order ILC law dissatisfies the convergence condition (13) because $L_g h(x_k(t))$ are all equal to zero, thus it is difficult to guarantee that the satisfactory tracking performance when the relative degree of the controlled system is greater than or equal to two.

Remark 4. If the performance index along the iterative domain is monotonically decreasing during the iterative process in some cases, the proposed ILC is equivalent to the ILC law (3). However, the performance index oscillates up and down in many cases, and then the proposed method has better ability to suppress this vibration because the previous best control input u^{best} in the ILC law (10) ensures that the performance index is bounded by the minimum performance J_{min}.

Remark 5. According to the ILC law (10) and its sufficient convergence condition, the control input u_k is convergent, so is the previous best input u^{best}, and u_k and u^{best} will finally converge to the same limit if the learning gain r damps to zero as iteration k tends to infinite. In other words, it means u_k is getting closer to u^{best} during the iterative process, so the ILC law (10) can guarantee that the control input $u_k(t)$ converges to the neighborhood of the previous best control input u^{best}, Because the performance index corresponding to the u^{best} is monotonically decreasing, it reflects the vibration of the performance index along the iterative orientation is waning.

Remark 6. In the ILC law (10) and ILC law (2), the initial value of the parameter r can be set as a bigger positive number and then decreases to zero as the iteration k tends to infinite. This action is beneficial to have better convergence performance. However, in the ILC law (2), it will cause the vibration of performance index during iterative process. For the proposed ILC law (10), according to remark 5, the previous best control input u^{best} can suppress the vibration of performance index. So the proposed ILC law (10) has better convergence performance than the ILC law (2).

Remark 7. For the system with high order relative degree, the parameter α can be set to be a constant small enough, which is also satisfied the sufficent convergence condition in Theorem 1.

4 SIMULATION AND RESULTS

In this section, the simulations on a single-link manipulator are implemented to illustrate the performance of the proposed method. The state equation of the single-link manipulator is shown as follow[9]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{vx_2(t)+(0.5m+M)glsin(x_1(t))}{J_m} + \frac{u(t)}{J_m} \qquad (28)$$

where $x_1(t)$ is the vector of the hub angle vectors, $x_2(t)$ is the vector of the hub angle velocity, $u(t)$ is the vector of input torques, m, M, l and v are the mass, tip load, length and the damping coefficient of the manipulator, respectively, and $J_m = (Ml^2 + \frac{1}{3}ml^2)$. The specific parameters are shown in Table 1. Moreover, note that the relative degree r of the system (28) is equal to 2, so only the hub angles are assumed to be the system output, i.e. $y(t) = x_1(t) + v(t)$, $v(t)$ is the measurement noise.

Table 1 The specific parameters of the single-link manipulator.

parameter	value
m	1 kg
M	2 kg
l	0.5m
v	1 kg*m²/s
g	9.8m/s²

The desired tracking trajectory is given as

$$x_d = \frac{\pi}{2} \cdot (6t^5 - 15t^4 + 10t^3), t \in [0,1] \qquad (29)$$

Let the sampling period be 0.005s, and the max iteration is set as 200.

In addition, the comparisons are done with other existing methods. The parameters to all methods are set to be the best ones corresponding to the almost best performances, which are listed in table 2.

Table 2 The corresponding parameters of the five different methods

Number	Method	Parameters
1	First order D-type ILC	Learning gain $W = 0.3$
2	High order D-type ILC	Learning gain $W = 0.3$
3	Heinzinger et al.[5]	Learning gain $L = 4\exp\left(-\frac{k-1}{450}\right)$ Forgetting factor $\gamma = 0.4\exp\left(-\frac{k-1}{600}\right) + 0.01$
4	Xie et al.[6]	Learning gain $L = 4\exp\left(-\frac{k-1}{450}\right)$ Forgetting factor $\gamma = 0.4\exp\left(-\frac{k-1}{600}\right) + 0.01$
5	The proposed method	$\mathrm{r} = 4\exp(-\frac{k-1}{450})$ $\alpha = 0.4\exp\left(-\frac{k-1}{600}\right) + 0.01$

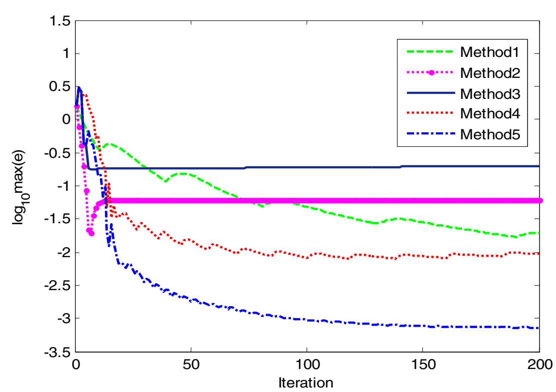

Fig.1 Iteration histories of the performance index by using method 1-5 when $v(t)$ is equal to zero.

First of all, the measurement noise $v(t)$ is zero, and the simulation results are shown in Fig.1, and the performance indexes of method 1-5 at iteration 200 are 0.0182, 0.0004, 0.1972, 0.0059, and 0.0003, respectively. Compared with method 3 and method 4, the proposed method (method 5) has rapid convergence and better performance index. Moreover, the proposed method has the same final performance index as the high-order D-type ILC law (method 2), though the former's convergence speed is slower than the latter's.

Then, the measurement noise is set as $v(t) = 0.01sin(2\pi t)$. And, the corresponding simulation results of the five methods are presented in Fig.2, and the performance indexes of method 1-5 at iteration 200 are 0.0197, 0.0618, 0.2010, 0.0095, and 0.0007, respectively. It can be obviously seen that the tracking performance of the high-order D-type ILC law (method 2) is very poor in the influence of measurement noise. Meanwhile, the proposed method (method 5) can get the ideal performance index.

In summary, the proposed method (method 5) can deal with the tracking task of the system with high order relative degree well and has certain robustness against the measurement noise.

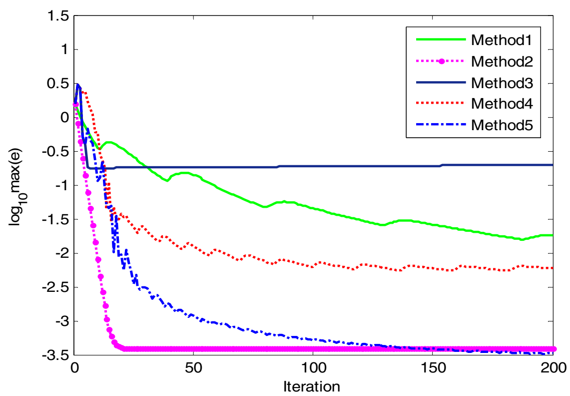

Fig.2 Iteration histories of the performance index by using method 1-5 when $v(t) = 0.01sin(2\pi t)$.

5 CONCLUSIONS

The high order differential operation will enlarge the influence of the measurement noise in the system output. Sometimes it is hard to determine the relative degree of the uncertain nonlinear systems. And, the classical first order D-type ILC law is unable to cope the system with high order relative degree. Motivated from the form of iterative learning control with forgetting factor, this paper proposes a new first order D-type ILC law by introducing the previous best control input to extend the tracking performance of the classical first order D-type ILC law, which can cope with the tracking tasks of the system with high order relative degree. The convergence analysis of the proposed method is presented based on contraction mapping method. Finally, several numerical simulations are implemented to illustrate the tracking performance of the proposed method.

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