Convergence Properties of PDD-Type Iterative Learning Control for Discrete-Time Systems in Frequency Domain

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Abstract: This paper investigates the sufficient and necessary conditions for monotone convergence of a proportional-derivative-derivative-type iterative learning control scheme for a class of discrete linear time-invariant systems in frequency domain. First, the discrete-time Fourier pair, properties of the Fourier coefficients and the Parseval-type's Energy Formula are deduced. Next, regarding to the first-order proportional-derivative-derivative-type iterative learning control scheme, the spectrum of the tracking errors is derived. Sufficiently and necessarily monotonous convergence of the first-order iterative learning control algorithm is analyzed. Numerical simulations are illustrated to demonstrate the validity and the effectiveness of the theory and results.

Key Words: Iterative learning control, monotone convergence, discrete-time system, discrete frequency-domain spectrum, discrete-time Parseval-type's Energy Formula

1 INTRODUCTION

Since iterative learning control (ILC) was proposed by Arimoto in 1984 for a robot manipulator that repeats the same task in a finite time interval [1], it has been extensively studied and lots of ILC research results have been made in both theory and applications over the last three decades. The fundamental mechanism of the ILC is to utilize the proportional, integral and/or derivative tracking error(s) at the current iteration to compensate for its control input so as to iteratively construct the control input of the next iteration. The goal is that the constructed iterative control input enables to drive the system to track the desired trajectory as precisely as possible as the iteration index goes to infinity. In the field of the ILC convergence investigations, the techniques are time-domain mode, 2D mode and frequency-domain mode and so on. In time domain, there are several kinds of tracking error estimation methods, such as lambda-norm [2-4], sup norm [5], Lebesgue-*p* norm [6], etc. In the view of a rigorously mathematical point, Lebesgue-*p* norm may have more rigorous mathematical theory basis. For the frequency domain investigations, the techniques are ranging from Fourier series expansion to Laplace transform [7-9]. Currently, the most majority of the ILC works on the frequency domain convergence analysis considers continuous-time systems [10-13]. Literatures [14, 15] adopted Fourier series expansion and Parseval's Energy Equality for analyzing the convergence of ILC for continuous systems in Frequency domain. With the advancement of high-quality digital computers, discrete-time techniques have been emerged in many fields, such as cybernetics, communication systems, digital signal processing, stochastic time series analysis, discrete neural networks. It is no doubt that, with the continuous improvement of discrete-time systems theory, discrete-time technique will play an important role in control theory, applied mathematics, application analysis [16-20]. To some extent, as the discrete-time systems are essentially different from the time continuous systems, numerous continuous-time methods cannot be directly migrated to discrete-time system. From an engineering point of view, the frequency domain technique is sometimes preferable because it may exhibit the spectrum feature of a signal and may take advantage of its lower computation complexity in a multiplication form of the transfer functions [21-23]. This motivates the paper to investigate sufficiently and necessarily convergent assumptions in frequency domain for a proportional-derivative-derivative-type iterative learning control (PDD-type ILC) algorithm for a class of discrete linear time-invariant systems. The main idea is to express a discrete-time signal within finite numbers of sampling instants as a combination of a set of finite Fourier basis functions and deduce the convergence characteristics of PDD-type ILC by means of evaluating the tracking error in the form of discrete spectrum.

The remaining of the paper is organized as follows. Section 2 exhibits preliminaries including the well-known Dirichlet Theorem, properties of the Fourier coefficients and the discrete-time Parseval-type's Energy Formula. Section 3 derives the Fourier coefficients of the frequency-domain tracking errors. In Section 4, sufficiently and necessarily monotonous convergence of the proposed ILC algorithm is analyzed in frequency domain. Numerical Simulations are displayed in Section 5 and the last Section 6 concludes the paper.

This work is supported by National Nature Science Foundation of China under Grant (No. F010114-60974140 and 61273135).

978-1-4673-9714-8/16/$31.00

2 PRELIMINARIES

Dirichlet Theorem[24]. If a periodic function $g(t)$, $t \in (-\infty, +\infty)$ with a period T is piecewise monotone on the interval $[0, \mathrm{T}]$ and is continuous except possibly for a finite number of discontinuous points of the first type, then the function $g(t)$ can be decomposed as a Fourier series in a complex form as

$$S(t) = \sum_{n=-\infty}^{+\infty} C_n \mathrm{e}^{jn\omega t} \tag{1}$$

where

$$\omega = \frac{2\pi}{\mathrm{T}},\ j^2 = -1,\ \mathrm{e}^{jm\omega t} = \cos(m\omega t) + j\sin(m\omega t)$$

$$C_0 = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} g(t)dt,\ C_m = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} g(t)\mathrm{e}^{-jm\omega t}dt,$$

$$m = 0, \pm 1, \pm 2, \cdots,$$

$$S(t) = \begin{cases} \mathrm{g}(t), & \text{if } t \text{ is a point of continuity,} \\ \frac{1}{2}\left[\lim\limits_{\Delta s \to 0^+} g(t+\Delta s) + \lim\limits_{\Delta s \to 0^-} f(t+\Delta s)\right], & \text{if } t \text{ is a point of discontinuity,} \\ \frac{1}{2}\left[\lim\limits_{\Delta s \to 0^+} f(0+\Delta s) + \lim\limits_{\Delta s \to 0^-} f(\mathrm{T}+\Delta s)\right], & \text{if } t = 0 \text{ or T.} \end{cases}$$

In the summation (1), the terms $\sin(\omega t)$ and $\cos(\omega t)$ produced by $C_{-1}\mathrm{e}^{-j\omega t} + C_1\mathrm{e}^{j\omega t}$ are called fundamental sinusoidal and cosine waves, respectively, whilst the terms $\sin(m\omega t)$ and $\cos(m\omega t)$ produced by $C_{-m}\mathrm{e}^{-jm\omega t} + C_{+m}\mathrm{e}^{jm\omega t}$ for $m = 2, 3, \cdots$, are named as higher-frequency harmonic waves. In usual, the summation $S(t) = \sum_{m=-\infty}^{+\infty} C_m \mathrm{e}^{jm\omega t}$ is called as a Fourier series expansion of the function $g(t)$. The above function $g(t)$ is called as a Dirichlet-type function.

In specific, for a real discrete-time sequence $\{g(0), g(1), \cdots, g(\mathrm{N})\}$, it is reasonable to conjecture that

$$\begin{cases} g(n) = \sum\limits_{m=0}^{\mathrm{N}-1} G(m)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}n},\quad n = 1, 2, \cdots, \mathrm{N}-1, \\ \frac{1}{2}(g(0) + g(\mathrm{N})) = \sum\limits_{m=0}^{\mathrm{N}-1} G(m)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}0} = \sum\limits_{m=0}^{\mathrm{N}-1} G(m)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}\mathrm{N}}. \end{cases} \tag{2}$$

Denote the coefficients $G(m)$ for $m = 0, 1, 2, \cdots, \mathrm{N}-1$ as

$$F(g(n)) = G(m) = \frac{1}{2\mathrm{N}}(g(0) + g(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} g(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n},\ m = 0, 1, \cdots, \mathrm{N}-1. \tag{3}$$

Thus, the formulae (2) and (3) can be regarded as a discrete-time Fourier pair.

Lemma 1.

(Property 1) $g(n \pm \mathrm{N}) = g(n)$, $n = \pm 1, \pm 2, \cdots, \pm(\mathrm{N}-1)$.

(Property 2)

$$F(\lambda_1 g(n) + \lambda_2 h(n)) = \lambda_1 F(g(n)) + \lambda_2 F(h(n)),\ n = 1, 2, \cdots, \mathrm{N}-1,$$

where λ_1 and λ_2 are constant.

(Property 3)

$$F(g(n+1)) = \mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}G(m) + \frac{1}{2\mathrm{N}}(g(\mathrm{N}) - g(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}},$$

$$m = 0, 1, \cdots, \mathrm{N}-1,\ n = 1, 2, \cdots, \mathrm{N}-1.$$

Proof:

$$\begin{aligned} F(g(n+1)) &= \frac{1}{2\mathrm{N}}\left(g(n+1)\big|_{n=0} + g(n+1)\big|_{n=\mathrm{N}}\right) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} g(n+1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n} \\ &= \frac{1}{2\mathrm{N}}(g(1) + g(\mathrm{N}+1)) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} g(n+1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}(n+1-1)}, \\ &= \mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\Big(\frac{1}{2\mathrm{N}}(g(0) + g(\mathrm{N})) + \frac{1}{\mathrm{N}}(g(1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}} \\ &\quad + \sum_{n=1}^{\mathrm{N}-2} g(n+1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}(n+1)})\Big) + \frac{1}{2\mathrm{N}}(g(\mathrm{N}) - g(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} \\ &= \mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}G(m) + \frac{1}{2\mathrm{N}}(g(\mathrm{N}) - g(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}. \end{aligned}$$

(Property 4)

$$F(g(n-1)) = \mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}G(m) - \frac{1}{2\mathrm{N}}(g(\mathrm{N}) - g(0))\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}},$$

$$m = 0, 1, \cdots, \mathrm{N}-1,\ n = 1, 2, \cdots, \mathrm{N}-1.$$

Proof:

$$\begin{aligned} F(g(n-1)) &= \frac{1}{2\mathrm{N}}\left(g(n-1)\big|_{n=0} + g(n-1)\big|_{n=\mathrm{N}}\right) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} g(n-1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n} \\ &= \frac{1}{2\mathrm{N}}(g(-1) + g(\mathrm{N}-1)) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} g(n-1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}(n-1+1)} \\ &= \mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}\Big(\frac{1}{2\mathrm{N}}(g(0) + g(\mathrm{N})) + \frac{1}{\mathrm{N}}(\sum_{n=1}^{\mathrm{N}-1} g(n-1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}(n-1)} \\ &\quad + g(\mathrm{N}-1)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}(\mathrm{N}-1)})\Big) - \frac{1}{2\mathrm{N}}(g(0) - g(\mathrm{N}))\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}} \\ &= \mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}G(m) - \frac{1}{2\mathrm{N}}(g(0) - g(\mathrm{N}))\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}. \end{aligned}$$

(Property 5) Discrete-time Parseval-type's Energy Formula

$$\sum_{m=0}^{\mathrm{N}-1}\left|G(m)\right|^2 = \frac{1}{4\mathrm{N}}(g(0) + g(\mathrm{N}))^2 + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1}(g(n))^2.$$

Proof: Denote $G(m)^*$ be the conjugation of the complex value $G(m)$. Then

$$G(m)^* = \frac{1}{2\mathrm{N}}(g(0) + g(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{l=1}^{\mathrm{N}-1} g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}l}$$

Therefore

$$4\mathrm{N}^2\left|G(m)\right|^2 = 4\mathrm{N}^2 G(m)G(m)^*$$

$$= ((g(0)+g(\mathrm{N})) + 2\sum_{n=1}^{\mathrm{N}-1} g(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n})((g(0)+g(\mathrm{N}))$$

$$+2\sum_{l=1}^{\mathrm{N}-1} g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}l})$$

$$= (g(0)+g(\mathrm{N}))^2 + 2(g(0)+g(\mathrm{N}))(\sum_{n=1}^{\mathrm{N}-1} g(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n}$$

$$+\sum_{l=1}^{\mathrm{N}-1} g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}l})) + 4\sum_{n=1}^{\mathrm{N}-1}\sum_{l=1}^{\mathrm{N}-1} g(n)g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}(l-n)} .$$

Then

$$4\mathrm{N}^2\sum_{m=0}^{\mathrm{N}-1}\left|G(m)\right|^2$$

$$= \mathrm{N}(g(0)+g(\mathrm{N}))^2 + 2(g(0)+g(\mathrm{N}))(\sum_{m=0}^{\mathrm{N}-1}\sum_{n=1}^{\mathrm{N}-1} g(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n}$$

$$+\sum_{m=0}^{\mathrm{N}-1}\sum_{l=1}^{\mathrm{N}-1} g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}l}) + 4\sum_{m=0}^{\mathrm{N}-1}\sum_{n=1}^{\mathrm{N}-1}\sum_{l=1}^{\mathrm{N}-1} g(n)g(l)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}(l-n)}$$

$$= \mathrm{N}(g(0)+g(\mathrm{N}))^2 + 2(g(0)+g(\mathrm{N}))(\sum_{n=1}^{\mathrm{N}-1} g(n)\sum_{m=0}^{\mathrm{N}-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n}$$

$$+\sum_{l=1}^{\mathrm{N}-1} g(l)\sum_{m=0}^{\mathrm{N}-1}\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}l}) + 4\sum_{n=1}^{\mathrm{N}-1} g(n)\sum_{l=1}^{\mathrm{N}-1} g(l)\sum_{m=0}^{\mathrm{N}-1}\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}(l-n)}$$

$$= \mathrm{N}(g(0)+g(\mathrm{N}))^2 + 4\mathrm{N}\sum_{n=1}^{\mathrm{N}-1}(g(n))^2 .$$

So

$$\sum_{m=0}^{\mathrm{N}-1}\left|G(m)\right|^2 = \frac{1}{4\mathrm{N}}(g(0)+g(\mathrm{N}))^2 + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1}(g(n))^2 .$$

In particular, it is found that in contrast to the continuous-time case, there are no convergence issues with the discrete-time Fourier series in general. The reason for this stems form the fact that any discrete-time periodic sequence $g(n)$ is the sum with finite terms.

3 PDD-TYPE ITERATIVE LEARNING CONTROL SCHEMES

Considering a class of linear time-invariant SISO systems taking the form as

$$\begin{cases} x_k(n+1) = \mathrm{A}\,x_k(n) + \mathrm{B}\,u_k(n), \\ y_k(n) = \mathrm{C}\,x_k(n),\ \ x_k(0) = x_0,\ \ n\in \mathbf{S}; \end{cases} \tag{4}$$

where, $\mathbf{S} = \{0,1,2,\cdots,\mathrm{N}\}$ is an operation time interval, N is a positive integer the subscripts k refers to the operation number and hereafter. $x_k(n)\in \mathbf{R}^p$, $u_k(n)\in \mathbf{R}$ and $y_k(n)\in \mathbf{R}$ denote respective p-dimensional state vector, scalar control input and output at the k -th iteration. A , B and C are matrices with appropriate dimensions.

A first-order proportional-derivative-derivative-type (PDD-type) ILC is given as follows:

$L_{PDD}(1)$:

$u_1(n)$, $u_k(0)$: given arbitrarily,

$$u_{k+1}(n) = u_k(n) + \Gamma_{-1}e_k(n-1) + \Gamma_0 e_k(n) + \Gamma_1 e_k(n+1) ,$$

$$n\in \{1,2,\cdots,\mathrm{N}\}, k = 1,2,3,\cdots . \tag{5}$$

Here, Γ_{-1} is assigned as the first-order proportional learning gains, and Γ_0 and Γ_1 are the derivative learning gains, respectively, and hereafter. The expression $e_k(n) = y_d(n) - y_k(n)$ denotes the tracking error between the desired trajectory $y_d(n)$ and the system output $y_k(n)$ of the system (4) driven by $u_k(n)$ at the k-th iteration and hereafter.

Applying Property 2, Property 3 and Property 4 of Lemma 1 to both sides of the equation (4), we get

$$\begin{cases} X_k(m)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} + \frac{1}{2\mathrm{N}}(x_k(\mathrm{N}) - x_k(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} = \mathrm{A}X_k(m) + \mathrm{B}U_k(m), \\ Y_k(m) = \mathrm{C}X_k(m), \quad m\in \mathbf{S}; \end{cases}$$

where

$$X_k(m) = \frac{1}{2\mathrm{N}}(x_k(0) + x_k(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} x_k(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n} ,$$

$$U_k(m) = \frac{1}{2\mathrm{N}}(u_k(0) + u_k(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} u_k(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n} ,$$

$$Y_k(m) = \frac{1}{2\mathrm{N}}(y_k(0) + y_k(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} y_k(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n} .$$

$$m = 0,1,2,\cdots,\mathrm{N}-1,\ \ n = 1,2,\cdots,\mathrm{N}-1 .$$

Further

$$X_k(m) = \left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I} - \mathrm{A}\right)^{-1}\mathrm{B}U_k(m)$$

$$-\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I} - \mathrm{A}\right)^{-1}\frac{1}{2\mathrm{N}}(x_k(\mathrm{N}) - x_k(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} .$$

Thus

$$Y_k(m) = \mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I} - \mathrm{A}\right)^{-1}\mathrm{B}U_k(m)$$

$$-\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I} - \mathrm{A}\right)^{-1}\frac{1}{2\mathrm{N}}(x_k(\mathrm{N}) - x_k(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} . \tag{4a}$$

Analogously, the first-order PDD-type ILC law (5) leads to

$$U_{k+1}(m) = U_k(m) + \left(\Gamma_{-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}} + \Gamma_0 + \Gamma_1\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\right)E_k(m)$$

$$+(\Gamma_1 - \Gamma_{-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}})\frac{1}{2\mathrm{N}}(e_k(\mathrm{N}) - e_k(0))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}} . \tag{5a}$$

Here, $E_k(m) = \frac{1}{2\mathrm{N}}(e_k(0) + e_k(\mathrm{N})) + \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}-1} e_k(n)\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}n}$.

Owing to

$$e_{k+1}(n) = y_d(n) - y_{k+1}(n) = e_k(n) - \left(y_{k+1}(n) - y_k(n)\right) ,$$

then

$$E_{k+1}(m) = E_k(m) - \left(Y_{k+1}(m) - Y_k(m)\right)$$

$$= E_k(m) - \mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\mathrm{B}\left(U_{k+1}(m)-U_k(m)\right)$$

$$+\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\frac{1}{2\mathrm{N}}((x_{k+1}(\mathrm{N})-x_k(\mathrm{N}))$$

$$-(x_{k+1}(0)-x_k(0)))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}. \quad (6)$$

Substituting (5a) into (6) reduces

$$E_{k+1}(m)=\left(1-\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\mathrm{B}\left(\Gamma_{-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}+\Gamma_0+\Gamma_1\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\right)\right)\times$$

$$E_k(m)+\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\frac{1}{2\mathrm{N}}((x_{k+1}(\mathrm{N})-x_k(\mathrm{N})$$

$$-(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(\mathrm{N}))-(x_{k+1}(0)-x_k(0)$$

$$-(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(0)))\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}. \quad (7)$$

Let

$$G_{PDD}(m)=1-\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\mathrm{B}\left(\Gamma_{-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}+\Gamma_0+\Gamma_1\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\right),$$

$$\varPhi_k(\mathrm{N})=\frac{1}{2\mathrm{N}}\left(x_{k+1}(\mathrm{N})-x_k(\mathrm{N})-(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(\mathrm{N})\right),$$

$$\varPhi_k(0)=\frac{1}{2\mathrm{N}}\left(x_{k+1}(0)-x_k(0)-(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(0)\right).$$

Arranging (7), we get

$$E_{k+1}(m)=G_{PDD}(m)E_k(m)+\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\times$$

$$\left(\varPhi_k(\mathrm{N})\text{-}\varPhi_k(0)\right)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}, m=0,1,2,\cdots,\mathrm{N}-1. \quad (8)$$

Remark 1. It is found from (8) that, for the first-order PDD-type ILC algorithm (5), the spectrum of the tracking error at the next iteration is expressed by not only the spectrum of the tracking error at the current iteration, but also the values of the initial and terminal states and tracking errors of the current iteration and the values of the initial and terminal states at the next iteration explicitly. Find that the transfer function $G_{PDD}(m)$ transmitting $E_k(m)$ to $E_{k+1}(m)$ is dominated by the system state, input and output matrices as well as the proportional and derivative learning gains. This form is similar to the done in continuous systems as shown in literature [6].

4 CONVERGENCE ANALYSIS

Theorem 1. Assume that the first-order PDD-type ILC (5) is applied to the linear time-invariant systems (4), and the matrices A, B and C along with the learning gain Γ_{-1}, Γ_0 and Γ_1 satisfy the initial and terminal conditions $x_{k+1}(0)=x_k(0)+(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(0)$ and $\varPhi_k(\mathrm{N})=0$, Then, the propositions

$$\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2<\sum_{m=0}^{\mathrm{N}-1}\left|E_k(m)\right|^2 \quad (9)$$

and

$$\lim_{k\to+\infty}\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2=0. \quad (10)$$

hold if and only if

$$\rho=\max_{m=0,1,\cdots,\mathrm{N}-1}\left|G_{PDD}(m)\right|<1. \quad (11)$$

where

$$\rho=\max_{m=0,1,\cdots,\mathrm{N}-1}\left|1-\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\mathrm{B}\left(\Gamma_{-1}\mathrm{e}^{-jm\frac{2\pi}{\mathrm{N}}}+\Gamma_0+\Gamma_1\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\right)\right|.$$

Proof: It is obvious that the equality $\varPhi_k(\mathrm{N})=0$ leads the equation (8) to

$$E_{k+1}(m)=G_{PDD}(m)E_k(m)-\mathrm{C}\left(\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}}\mathrm{I}-\mathrm{A}\right)^{-1}\varPhi_k(0)\mathrm{e}^{jm\frac{2\pi}{\mathrm{N}}},$$

$$m=0,1,2,\cdots,\mathrm{N}-1.$$

Because the iteration-wise initial state satisfies $x_{k+1}(0)=x_k(0)+(\Gamma_1-\Gamma_{-1}\mathrm{e}^{-jm\frac{4\pi}{\mathrm{N}}})\mathrm{B}e_k(0)$, it is obvious that the equality $\varPhi_k(0)=0$ holds, by which the equation (13) becomes

$$E_{k+1}(m)=G_{PDD}(m)E_k(m),\ m=0,1,2,\cdots,\mathrm{N}-1. \quad (12)$$

Before derivation, it is worthy to mind that the energy $\sum_{m=0}^{\mathrm{N}-1}\left|E_k(m)\right|^2$ is finite for all iteration indices $k=1,2,3,\cdots$, supposing that the addressed learning control strategies (5) is executable for a practical engineering. The proof is done basing on the premise.

What follows is the proof of sufficiency.

$$\left|E_{k+1}(m)\right|^2=\left|G_{PDD}(m)\right|^2\left|E_k(m)\right|^2,\ m=0,1,2,\cdots,\mathrm{N}-1.$$

Due to the assumption $\rho=\max_{m=0,1,\cdots,\mathrm{N}-1}\left|G_{PDD}(m)\right|<1$, it is evident that

$$\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2=\sum_{m=0}^{\mathrm{N}-1}\left|G_{PD}(m)\right|^2\left|E_k(m)\right|^2<\sum_{m=0}^{\mathrm{N}-1}\left|E_k(m)\right|^2,$$

$$m=0,1,2,\cdots,\mathrm{N}-1.$$

Resultantly

$$\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2=\sum_{m=0}^{\mathrm{N}-1}\left|G_{PDD}(m)\right|^{2k}\left|E_1(m)\right|^2. \quad (13)$$

$$\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2<\sum_{m=0}^{\mathrm{N}-1}\left|E_1(m)\right|^2.$$

Taking limit on both sides of the above equation (13) with k, we get

$$\lim_{k\to+\infty}\sum_{m=0}^{\mathrm{N}-1}\left|E_{k+1}(m)\right|^2=\lim_{k\to+\infty}\sum_{m=0}^{\mathrm{N}-1}\left|G_{PDD}(m)\right|^{2k}\left|E_1(m)\right|^2$$

$$=\sum_{m=0}^{\mathrm{N}-1}\lim_{k\to+\infty}\left(\left|G_{PDD}(m)\right|^{2k}\left|E_1(m)\right|^2\right)=0.$$

This completes the proof of sufficiency.

Next is the proof of necessity.

Assume that the inequality $\rho = \max_{m=0,1,\cdots,N-1} |G_{PDD}(m)| < 1$ does not hold. Then there exists at least such a number m_0 that

$$|G_{PDD}(m_0)| = \left|1 - \mathrm{C}\left(\mathrm{e}^{\frac{jm 2\pi}{N}}\mathrm{I} - \mathrm{A}\right)^{-1}\mathrm{B}\left(\Gamma_{-1}\mathrm{e}^{-\frac{jm 2\pi}{N}} + \Gamma_0 + \Gamma_1 \mathrm{e}^{\frac{jm 2\pi}{N}}\right)\right| \geq 1 .$$

Then,

$$|E_{k+1}(m_0)| = |G_{PDD}(m_0)||E_k(m_0)| \geq |E_k(m_0)| .$$

and

$$|E_{k+1}(m_0)| = |G_{PDD}(m_0)|^k |E_1(m_0)| \geq |E_1(m_0)| .$$

Thus

$$\sum_{m=0}^{N-1} |E_{k+1}(m)|^2 \geq |E_{k+1}(m_0)|^2 \geq |E_1(m_0)|^2 .$$

$$\lim_{k\to\infty} \sum_{m=0}^{N-1} |E_{k+1}(m)|^2 \geq \lim_{k\to\infty} |E_1(m_0)|^2 = |E_1(m_0)|^2 .$$

It is possible to select $U_1(m)$ and $Y_d(m)$ such that $|E_1(m_0)| > 0$, which contradicts to the postulation (10).

This completes the proof.

Remark 2. It is observed that the theorem demands the assumption that $\Phi_k(0) = 0$ and $\Phi_k(\mathrm{N}) = 0$ hold. This is possible for the case when the system is stable with no steady-state error, specially, for batch industrial process. Theorem 1 can be understood as a sufficient and necessary condition for guaranteeing the strictly monotone convergence of the tracking error in terms of the energy.

5 NUMERICAL SIMULATIONS

To show the effectiveness of the learning control law (5) in discrete frequency domain, consider a discrete SISO linear time-invariant discrete system described as follows.

$$\begin{cases} \begin{bmatrix} x_1(n+1) \\ x_2(n+1) \\ x_3(n+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.002 & 0 \\ 0 & 1 & 0.002 \\ -0.0004 & -0.0001 & 0.9992 \end{bmatrix} \\ \qquad \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.002 \end{bmatrix} u(n), \\ y(n) = \begin{bmatrix} 0.0018 & 0.0272 & 0.0905 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix} . \end{cases} \quad (14)$$

The operation time interval of system (14) is set as $\{0,1,2,\cdots,500\}$. The desired trajectory is chosen as $y_d(n) = 4\times 10^{-6} n^2 (1 - 0.002n)$ and the initial control input is chosen as $u_1(n) = 1$, $u_k(0) = 1$. As $x_k(0) = 0$, it is obvious that $e_k(0) = 0$. For the PDD-type ILC algorithm (5), choose $\Gamma_{-1} = -0.35$, $\Gamma_0 = -2599.5$ and $\Gamma_1 = 2600$.

It is computed that $\rho = \max_{m=0,1,\cdots,N-1} |G_{PDD}(m)| = 0.5294 < 1$, which means the convergence condition $\rho = \max_{m=0,1,\cdots,N-1} |G_{PDD}(m)| < 1$ holds.

Fig.1 displays the tracking outputs driven by the first-order PDD-type ILC algorithm (5) at the 2nd and 8th implementations. Fig.2 depicts the spectrums of the frequency-wise tracking errors in the discrete frequency domain. It is seen that the tracking error spectrum at each fixed frequency is monotonously decreasing in the iteration direction. The monotone convergence regarding to the spectrums of the tracking errors made by the PDD-type ILC law (5) is illustrated in Fig.3 and the monotonicity in terms of the average power of the tracking error is manifested in Fig.4, respectively.

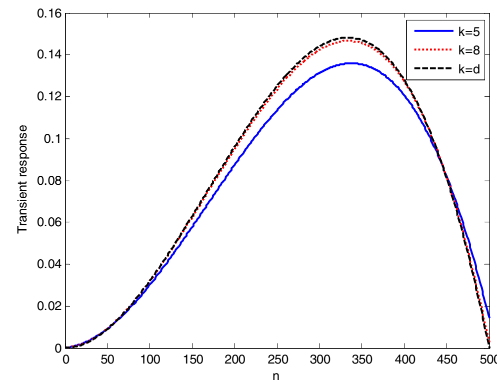

Fig 1. Outputs at the 2nd and 8th iterations

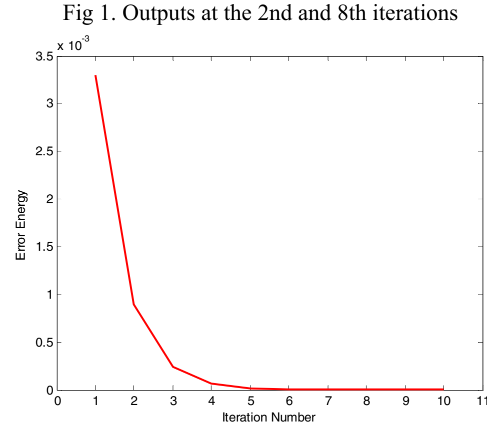

Fig 2. Error energy in discrete time domain

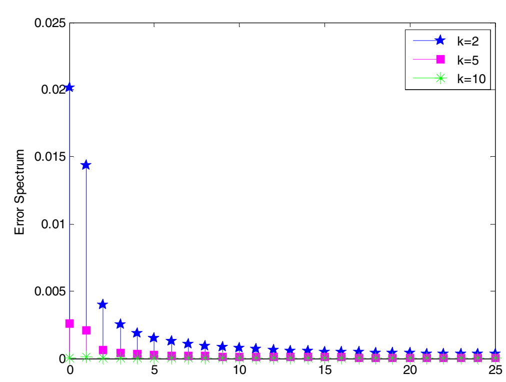

Fig 3. Error Spectrums at the 2nd, 5th and 10th iterations

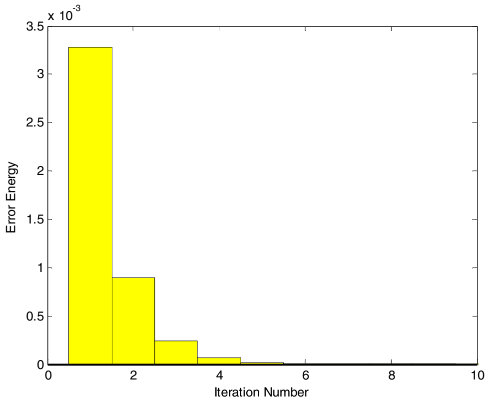

Fig 4. Error energy in discrete frequency domain

6 CONCLUSION

In this paper, the combination of Fourier basis functions is employed to express the discrete signal with finite sampling points. By deducing properties of discrete-time Fourier pair and the discrete-time Parseval's Energy Formula, the discrete frequency-wise spectrums of the system dynamics and the proposed PDD-type ILC algorithm are derived for a class of discrete linear time-invariant systems. Then, the convergence of the PDD-type first-order ILC law is clarified. Finally, a group of simulations testify the effectiveness of the proposed learning strategy. However, as time delay, noise as well as system initial state drift are unavoidable in real applications, it is very inspiring to investigate these issues for future work.

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