Brief Review on Sampled-Data Iterative Learning Control

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Abstract: The research on sampled-data iterative learning control (SDILC) is briefly reviewed in this note. The concerned system models are listed as an classification of SDILC papers. Two problems of SDILC are presented aiming to at-sample tracking performance and intersample behavior. The literature is reviewed following the research group category. Some discussions on the further study are also given.

Key Words: Sampled-Data System, Iterative Learning Control, Convergence

1 Introduction

In our daily lives, we could behave very well sooner or later if we could do some work again and again. This is because we could keep trying and learning, and by learning we find the drawbacks of our actions so that we can adjust our actions to make some improvements. This is a common experience for us. One is interested in whether this idea could be integrated into system control. This is the motivation of iterative learning control (ILC), which was started in the 80s of last century by Arimoto [1].

As is required to learn from previous experiences, repeatability is a basis requirement for the setup of ILC. To be specific, for ILC, it is assumed that the system could complete some given task in finite time interval and repeat it. Many practical systems are with this feature such as hard disk drive, chemical process, robot arm, and inductor motor. For such kinds of systems, the input signal and tracking errors from previous iterations/cycles/batches are used to formulate the input signal for the current iteration/cycle/batch, so that the tracking performance could be improved as the iteration number goes to infinity.

It is noticed that the structure of ILC is rather simple but its performance is very effective. To certain extent, ILC could be regarded as a kind of feedback control, but the feedback loop is made along the iteration axis rather than the operation time axis. Thus it has attracted many academy researchers and engineers to make efforts. Actually, ILC has been developed in depth in the past three decades and a lot of outstanding achievements have been made on this topic both from theory and application perspectives [2, 3]. Especially, the recent results and some promising directions of stochastic ILC are surveyed in [4]. For practical applications, the high-speed trains [5], permanent magnet step motors [6], robotic-assisted rehabilitation [7], industrial robots [8] are several typical cases.

It is a common setting in ILC that the reference trajectory, the input and output signal of previous iteration are stored in a memory to generate the input signal for the current iteration. The two issues arise in the practical applications. One is that the design and analysis of ILC would be much convenient if the discrete formulation is adopted. The other one is that most of the controlled plants are continuous-time systems in essence. Thus there is a gap between the actual systems and design requirements. In addition, it would cost much more memory to store the complete continuous signal. This motivates us to consider an interesting topic called sampled-data iterative learning control (SDILC).

For SDILC, the system is modeled by a continuous-time system while the controller is designed as a discrete-time implementation. Suppose only the tracking performance at the sampled points are considered, the technical analysis would be much similar between pure discrete-time ILC and SDILC. However, taking the interval behaviors between sampled points into account, the design and analysis of SDILC is in general more difficult and some novel issues arise for this direction. As a matter of fact, it is of great interest to investigate how to obtain a favorable performance for the original continuous-time system only using the tracking information from selected points.

The difference between SDILC and continuous-time ILC lies in that the former considers tracking performance with using step signals as only the information at sampling instants is computed, while the latter focuses on the precise tracking during the whole time interval since complete information is received for ILC updating. On the other hand, the difference between SDILC and discrete-time ILC is that the former studies both at-sampling and intersample performances as the controlled plant is continuous, while the latter only has to guarantee a well performance at discrete time instants since the controlled plant is discrete.

This paper gives a brief review on sampled-data iterative learning control. We first study the publications on SDILC from different research groups and make some comparisons among these results to detail the developments of SDILC. Then we try to present some discussions on the promising issues of SDILC from our viewpoint. In addition, an attempt on the connection of SDILC and interval behavior is also provided. It should be pointed out that we have tried our best to seek publications on SDILC; however, there must be some papers those are missed due to our limited capacity. Besides, we have omitted some conference papers whenever their results are covered by related journal papers for clarity.

The rest of the paper is arranged as follows: Section 2 gives the linear and nonlinear system models as well as the formulations of ILC and SDILC; Section 3 makes efforts

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to the literature review, while the promising issues are addressed in Section 4; Section 5 concludes the paper.

2 Problem Formulation of Sampled-Data ILC

2.1 System Models

The studies of SDILC are dispersed in different kinds of system models both linear and nonlinear. In order to make the following expressions and comparisons more convenient, here we first list the system models those have been considered.

The general linear system is formed as follows.

$$\begin{cases} \dot{x}_k(t) = A(t)x_k(t) + B(t)u_k(t) + w_k(t) \\ y_k(t) = C(t)x_k(t) + v_k(t) \end{cases} \tag{1}$$

where $k = 1, 2, \cdots$ denotes different iteration number, while t labels the time and $0 \leq t \leq T$ where T is the length of one iteration. $x_k(t)$, $u_k(t)$, and $y_k(t)$ are system state, input, and output, respectively. $w_k(t)$ and $v_k(t)$ denote the system and measurement disturbances or noises. $A(t)$, $B(t)$, and $C(t)$ are system matrices with suitable dimensions.

If we lift all the inputs and outputs in an iteration as supervectors U_k and Y_k, then the system (1) could be rewritten as the well-known lifted model

$$Y_k = PU_k + \xi_k \tag{2}$$

where P is system matrix constituted by $A(t)$, $B(t)$ and $C(t)$, while ξ_k is a combination of $w_k(t)$ and $v_k(t)$.

Special cases of (1) are listed as follows.

LM1 If both $w_k(t)$ and $v_k(t)$ are absent and $A(t) \equiv A$, $B(t) \equiv B$, $C(t) \equiv C$, then the model is a linear time invariant (LTI) deterministic system.

LM2 If both $w_k(t)$ and $v_k(t)$ are absent and $A(t)$, $B(t)$, $C(t)$ are not constant matrices, then the system is a linear time varying (LTV) deterministic system.

LM3 If both $w_k(t)$ and $v_k(t)$ are bounded disturbances and $A(t) \equiv A$, $B(t) \equiv B$, $C(t) \equiv C$, then the model is a linear time invariant (LTI) deterministic system with disturbances.

LM4 If both $w_k(t)$ and $v_k(t)$ are bounded disturbances and $A(t)$, $B(t)$, $C(t)$ are not constant matrices, then the system is a linear time varying (LTV) deterministic system with disturbances.

LM5 If both $w_k(t)$ and $v_k(t)$ are random noises and $A(t) \equiv A$, $B(t) \equiv B$, $C(t) \equiv C$, then the model is a linear time invariant (LTI) stochastic system.

LM6 If both $w_k(t)$ and $v_k(t)$ are bounded disturbances and $A(t)$, $B(t)$, $C(t)$ are not constant matrices, then the system is a linear time varying (LTV) stochastic system.

For the nonlinear systems, affine nonlinear models are adopted in many publications

$$\begin{cases} \dot{x}_k(t) = f(x_k(t)) + b(x_k(t))u_k(t) + w_k(t) \\ y_k(t) = g(x_k(t)) + v_k(t) \end{cases} \tag{3}$$

where $f(\cdot)$ and $b(\cdot)$ denote nonlinear functions.

Special cases of (3) are listed as follows.

AM1 If both $w_k(t)$ and $v_k(t)$ are absent, then the model is an affine nonlinear deterministic system.

AM2 If both $w_k(t)$ and $v_k(t)$ are bounded disturbances, then the model is an affine nonlinear deterministic system with disturbances.

AM3 If both $w_k(t)$ and $v_k(t)$ are random noises, then the model is an affine nonlinear stochastic system.

A general nonlinear system model could be formulated as

$$\begin{cases} \dot{x}_k(t) = f(x_k(t), u_k(t), w_k(t)) \\ y_k(t) = g(x_k(t), v_k(t)) \end{cases} \tag{4}$$

If both $w_k(t)$ and $v_k(t)$ are absent, it is denoted as NM1. Otherwise, it is denoted as NM2.

The reviewed papers on SDILC are classified in Table 2.1 from the perspective view of system models, where it is seen that most papers are based on time-variant linear and affine nonlinear systems. The cases with bounded disturbances are also addressed. The LTV systems and stochastic systems are with no papers.

Table 1: Classification of references

LM1	[9, 10, 12, 25–27, 30]
LM2	
LM3	[15, 16, 18]
LM4	
LM5	
LM6	
AM1	[19, 20, 22, 23, 29]
AM2	[13, 14, 17, 28]
AM3	
NM1	[21]
NM2	

2.2 Problems of SDILC

Let Δ_T be the sampling period of the digital control system and $N\Delta_T = T$, where N is a positive integer. Then for SDILC, only information on the sampling time $n\Delta_T$, $0 \leq n \leq N$ are available for controller design. The block diagram of SDILC is shown in Fig. 1, where a sampler is implemented at the output side to generate sampled output, the learning controller produces discrete input for the next iteration using the stored discrete input and sampled output as well as reference trajectory, and a holder is adopted to regain continuous signal for the controlled system.

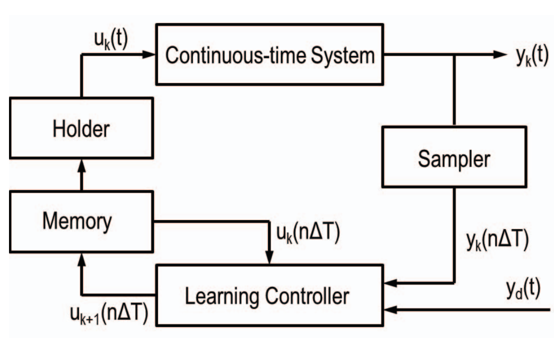

Fig. 1: Block diagram of Sampled-Data Iterative Learning Control.

There are two general problems associated with SDILC. The first one is how the behavior at the sampling instants is, while the other one is how the interval performance between sampling instants is. To be specific, the former aims to construct suitable learning algorithms to guarantee the convergence at the sampling instants, and the latter focuses on the quantitative analysis of the tracking performance between different sampling instants and possible solutions to reduce the tracking errors in the sampling interval. Roughly speaking, as long as only the former problem is considered, it is similar to design and analysis of ILC for discrete-time systems. The latter problem makes SDILC much different from the traditional ILC for discrete-time systems.

Several techniques are developed for these problems. Among these techniques, the major one is the conventional contraction mapping method combined with λ-norm or super norm definitions of the input error vectors. This method plays an important role in the early developments of ILC. To use this method, the nonlinear system is required to satisfy globally Lipschitz condition if considered. In addition, Bellman-Gronwall lemma is often used to prove the convergence property.

3 Literature Review

In this section, a brief literature review is given. Note that the amount of papers on SDILC is not a large number. In addition, these papers are published by several major research groups, which thus are with special interests. Therefore, in the following, the review follows the research group category. Four dimensions are principally focused, i.e., the system model, the update law, the convergence result, and the analysis technique. Some comparisons and remarks are also inserted.

3.1 Frequency Based SDILC

The frequency based design and analysis of SDILC is proposed by Prof. Xu's group [9–12]. The research focuses on the fundamental and synthetical problems of SDILC.

The paper [9] aims to present a framework for the design and analysis of SDILC from both time and frequency domains. The LTI system (LM1) is adopted. The design conditions are given for the monotonic convergence of P-type, D-type, D^2-type, and general filters. Some remarks are also provided on the relative degree relationship between continuous-time system and its sampled-data system as well as the influence on tracking performance of sampling period. The experimental investigation on piezoelectric motor is detailed in [10], where criteria for the selection of each type are presented.

In [11], a kind of SDILC algorithm is proposed in the frequency form to deal with the extreme precision motion tracking problem for piezoeletric positioning stage. The convergence condition and the robustness analysis under the inverse model in frequency field are expressed with an experimental validation. It is shown that the SDILC behaves much advantageous than traditional open-loop control and PI control. This problem is further studied in [12], where SDILC is added to a direct feedback control aiming to deal with repeatable and non-repeatable components simultaneously. The experiments verifies that the combination of SDILC and feedback control would improve the tracking precision and expedite the convergence speed.

In sum, the frequency-based design and analysis method is an interesting angle for SDILC and it is rarely studied by other academicians or engineers. Following this road, there are a lot of blanks to fill up.

3.2 Bounded Convergence under Bounded Disturbances

Prof. Chien and his co-workers contributed a series of papers on the bounded set convergence at the sampling instants for linear and nonlinear systems with bounded disturbances [13–18]. In all these papers, bounded system disturbances and/or measurement noises are added to the linear and nonlinear systems, that is, $\|w_k(t)\| \leq \epsilon_1$, $\|v_k(t)\| \leq \epsilon_2$, where ϵ_1 and ϵ_2 are some positive constants. In addition, the initial state error is also assumed to be bounded, i.e., $\|x_d(0) - x_k(0)\| \leq \epsilon_3$, where $x_d(0)$ denotes the desired initial state and ϵ_3 is a positive constant. Due to the existence of such unknown disturbances, it is hard to expect that the tracking error converges to zero no matter whether at the sampling instants or during the sampling interval. Therefore, it was shown that the tracking errors at the sampling instances converges to a set whose bound is a function of ϵ_i, $i = 1, 2, 3$ in [13–18]. In addition, a zero-order holder was adopted in all these papers, i.e., $u_k(t) = u_k(n\Delta_T)$, $n\Delta_T \leq t < (n+1)\Delta_T$. The inherent differences lie in the design of updating laws and analysis techniques.

In paper [13], the time-varying affine nonlinear model with bounded disturbances (AM2) is considered. The update law is defined as a traditional P-type form at the sampling instants

$$u_{k+1}(n\Delta_T) = u_k(n\Delta_T) + L_k(n\Delta_T)e_k((n+1)\Delta_T) \quad (5)$$

where $L_k(n\Delta_T)$ denotes the learning gain. Then by the well-known λ-norm technique in ILC, it is shown that the λ-norm of the tracking errors converges to a given bound if the sampling period Δ_T is small enough. Here, the λ-norm of a vector $p(n\Delta_T)$ is defined as $\|p(n\Delta_T)\|_\lambda = \sup_{0\leq n\leq N} a^{-\lambda n}\|p(n\Delta_T)\|$ with $\lambda > 1$, $a > 1$.

As is noticed in the ILC field, the convergence in λ-norm might result in a bad transient performance before it comes to convergence. Therefore, it is interesting to derive the convergence in the common norm sense. This is revealed in [14], where the system is model by (3) in absence of $v_k(t)$. The update law is in D-type

$$u_{k+1}(n\Delta_T) = u_k(n\Delta_T) + \frac{1}{\Delta_T}L_k(n\Delta_T)[e_k((n+1)\Delta_T) - e_k(n\Delta_T)] \quad (6)$$

A direct calculation on the norm inequalities of the input errors leads to a contraction mapping, and therefore the convergence of the norm of the tracking error is obtained. Due to the inherent nonlinearity, it is difficult to compute the learning gain $L_k(n\Delta_T)$, thus a fuzzy network is further introduced to approximate the original system based on *if-then* rules of T-S fuzzy type. Similar problem is taken into account in [17]. The difference between [14] and [17] is that the fuzzy approximation is made for the input signal directly, while the parameters of the approximation is iteratively updated.

The rest papers [15, 16, 18] study the LTI systems with bounded disturbances (LM3). In addition, the impact of the involving of errors in current iteration or feedback control is specialized. In [15], the update law follows

$$u_k(n\Delta_T) = (1-\beta)u_{k-1}(n\Delta_T) + Ke_k(n\Delta_T) \quad (7)$$

where K is the learning gain matrix and $0 < \beta < 1$ denotes a forgetting factor. A novel analysis technique, similar to the Lyapunov method, is proposed for the convergence proof. It is worthy pointing out that using the current error for learning would save much storage for practical applications. Another general formulation of the update law is given in [18] as follows

$$\begin{aligned} u_k(n\Delta_T) =&(1-\beta)u_{k-1}(n\Delta_T) + \beta u_0(n\Delta_T) + K_P e_k(n\Delta_T) \\ &+ K_D(e_k(n\Delta_T) - e_k((n-1)\Delta_T)) \end{aligned} \quad (8)$$

The analysis in [18] follow the conventional contract mapping method based on calculations of the input error inequality between successive iterations.

The combination of feedback control and ILC is done in [16], where the input is updated by

$$u_k(n\Delta_T) = u_k^b(n\Delta_T) + u_k^f(n\Delta_T) \quad (9)$$

where the feedback part is designed in dynamic form as

$$\begin{cases} z_k((n+1)\Delta_T) = p(z_k(n\Delta_T)) + q(z_k(n\Delta_T))e_k(n\Delta_T) \\ u_k^b(n\Delta_T) = r(z_k(n\Delta_T)) + s(z_k(n\Delta_T))e_k(n\Delta_T) \end{cases} \quad (10)$$

and the feedforward part is given as

$$\begin{aligned} u_k^f(n\Delta_T) =& u_{k-1}(n\Delta_T) \\ &+ L_{k-1}(n\Delta_T)(e_{k-1}((n+1)\Delta_T) - e_{k-1}(n\Delta_T)) \end{aligned} \quad (11)$$

Similar analysis technique and convergence to [18] is obtained.

Remark 1. *It is noticed that different update algorithms are investigated by Chien and his co-workers including P-type, D-type, and feedback of current error. The research mainly focuses on bounded convergence to some given set by letting the sampling period small enough under bounded disturbances.*

3.3 SDILC for Systems with Arbitrary Relative Degree

Prof. Sun and co-workers make a in-depth study on SDILC for nonlinear systems with arbitrary relative degree. Here the relative degree is a kind of description of the input-output relationship. Taken an SISO affine nonlinear model (3), in absence of disturbances $w_k(t)$ and $v_k(t)$, into account, the definition of relative degree used in [19–23] is given as follows.

Definition 1. *The SISO affine nonlinear system with input generated by a zero-order holder from sampled signals has extended relative η for $x_k(t) \in \mathbb{R}^n$, $t \in [0,T]$ if, for $0 \le j \le N-1$,*

$$\int_{j\Delta_T}^{(j+1)\Delta_T} L_b g(x(t_1))dt_1 = 0$$

$$\int_{j\Delta_T}^{(j+1)\Delta_T}\int_{j\Delta_T}^{t_1}\cdots\int_{j\Delta_T}^{t_i} L_b L_f^i g(x(t_{i+1}))dt_{i+1}\cdots dt_1 = 0$$

$1 \le i \le \eta - 2$, *and*

$$\int_{j\Delta_T}^{(j+1)\Delta_T}\int_{j\Delta_T}^{t_1}\cdots\int_{j\Delta_T}^{t_{\eta-1}} L_b L_f^{\eta-1} g(x(t_\eta))dt_\eta\cdots dt_1 \neq 0$$

Roughly speaking, the relative degree is larger than 1 means that the direct input-output coupling matrix is zero. In other words, the input signal could only be explicitly formulated in some differential of the output. Thus it is interesting to ask whether the traditional P-type update law could ensure the convergence of SDILC. This question is answered in [19–21].

In [19], the SISO affine nonlinear deterministic system (AM1) is considered and a zero-order holder is adopted to regain continuous signal from discrete input signals. The update law follows (5). Since no disturbances or initial error is assumed, it is proved that the tracking error at the sampling instants would converge to zero. This confirms the results in [13]. The corresponding MIMO case is discussed in [20] with similar dealing techniques and convergence results.

The P-type law is extended to a general case called SDILC with lower-order differentiations in [21] for general nonlinear model NM1. The paper [21] considers MIMO case, however, here the update law is given according to SISO case for simplicity.

$$\begin{aligned} v_{k+1}(n\Delta_T) =& u_k(n\Delta_T) + L_k(n\Delta_T)\big[e_k((n+1)\Delta_T) \\ &- \sum_{i=0}^{l}\frac{\Delta_T^i}{i!}e_k^{(i)}(n\Delta_T)\big] \\ u_{k+1}(n\Delta_T) =& \mathrm{Sat}(v_{k+1}(n\Delta_T)) \end{aligned}$$

where $0 \le i \le \eta - 1$ while η denotes the relative degree, and $\mathrm{Sat}(\cdot)$ is a saturation function. It is obvious that the update law is similar to (5) if $\eta = 1$. An initial state error is assumed in [21], thus the tracking error at sampling instants could only be proved to converge to a small set. As a special case, the affine deterministic system is also addressed in [21].

The initial rectifying problem is studied in [22, 23]. The paper [22] considers the fixed initial shift and shows that the conventional P-type sampled update algorithm needs much efforts to shift the actual output to the desired one after the first sampling instant. This may not be applicable due to the small sampling period. Then an initial rectifying action is added to improve the performance. It is proved that the output would follow the desired trajectory with a specified error bound. Then the initial error is extended to arbitrarily varying case and the so-called varying-order SDILC is designed. The inherent idea is to divide the error interval into several sub-intervals and design different ILC laws for different sub-intervals. Then the learning algorithm could select suitable one according to the initial shifting value for each iteration.

Remark 2. *In these studies, the convergence proofs are given based on a technical lemma, which is an extension of the contraction mapping. A bounded disturbance sequence is added to the contract inequality, and therefore the original sequence would converge to a finite value. For details, please refer to [19–23].*

3.4 Interval Performance of SDILC

It is observed that in most papers such as [13–23], only the performance at the sampling instants is considered while the intersample behavior is seldom discussed. However, achieving a good performance at the sampling instants can go at the expense of a poor intersample behavior [24]. However, how to guarantee a satisfied intersample tracking performance is a difficult problem for SDILC. Oomen *et al.* give a first attempt on this issue [25, 26].

In [25], the multirate ILC approach is proposed to balance the at-sample performance and the intersample behavior, where the key idea is to generate a command signal at a low sampling rate by using fast sampled measurements. The details of multirate systems and multirate ILC are given to enable an optimal SDILC in this paper.

The authors further develops an ILC framework for sampled-data systems by incorporating the system identification and a low-order optimal ILC controller in [26], which is an on-going study of [25]. The proposed system identification procedure delivers a model that encompasses the intersample in a multirate setting for the closed-loop system ad then the resulting model could be used for the optimal ILC synthesis, so that the computational burden is much less than common optimization-based algorithms for large systems.

In short, there still lack more in-depth studies on the intersample behavior of SDILC including novel design and analysis technique for improving the tracking performance between different sampling instants.

3.5 Other Contributions

The paper [27] presents a limiting property of the inverse of sampled-data systems. To be specific, for a continuous-time system with a relative degree of one or two, the inverse of the corresponding sampled-data system can approximate to the inverse of the original continuous-time system independently of the stability of the zeros as the sampling period Δ_T tends to zero.

Time-delay is introduced into the affine nonlinear model AM2 in [28] with other settings similar to Chien's publications. The following PD-type update law is used with a zero-order holder

$$u_{k+1}(n\Delta_T) = u_k(n\Delta_T) + Pe_k((n+1)\Delta_T) + D\dot{e}_k(n\Delta_T) \quad (12)$$

where P and D denote learning gain matrices. By using the contraction mapping method for the λ-norm technique, it is shown the at-sample output converges to a certain neighborhood of the desired trajectory points. However, the term $\dot{e}_k(n\Delta_T)$ is not suitable for sampled-data implementation.

The SDILC for singular systems is addressed in [29], where the P-type learning algorithm (5) is used. By following similar techniques of [20], the bounded set convergence of outputs at the sampling instants is proved.

An online optimal SDILC problem is dealt with in [30] for LTI system with bounded disturbances (LM3). The control objective is to find a sequence of control input to minimize some smooth objective function $Q(y_k(t), u_k(t))$. To solve this problem, the gradient descent method is used to generate the optimal solution iteratively. In addition, a feedback control is further added to handle various uncertainties.

4 Discussions

Based on the literature review in last section, we now give the following remarks.

- Much attention is paid to the SDILC for LTI and affine nonlinear system with/without bounded disturbances. Few paper aims to LTV systems, general nonlinear system, and stochastic systems. For these models, there are inherent difficulty to address. Thereto, it is an interesting and promising direction of SDILC.
- Most papers contribute to the at-sample performance while the intersample behavior is seldom considered. However, good tracking performance does not necessarily imply acceptable intersample behavior, and this has been revealed by some previous studies outside ILC field. Therefore, it is interesting to make more efforts on the assessment of intersample behavior for a given SDILC algorithm. For this topic, the monograph [31] may provide useful details.
- The traditional contraction mapping method and its general case for λ norm and super norm of the input error is the main technique used for convergence analysis. This technique has some limitations: 1) the convergence conditions derived from this technique is some conservative; 2) the nonlinear system, if considered, should satisfy global Lipschitz condition; and 3) it can not be applied for stochastic systems. Thus novel design and analysis techniques are warmly welcomed for SDILC.
- ILC is a control method built for practical applications, such as chemical processes, industrial robotics, and hard-disk drive. Thus the implementation of SDILC in practical applications is also of great interest and meaning. However, few publications are found on this direction. In addition, the experimental investigation is also rather rare. A recent book [32] can provide useful viewpoints.

5 Conclusions

A brief review on sampled-data iterative learning control is given in this note. The system models are first classified in linear, affine, and general nonlinear systems, following which the general problems of SDILC are then provided. A literature review is detailed in the research group category. Some discussions the future research are then presented.

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