Consensus Tracking of Multi-Agent Systems with Constrained Time-delay by Iterative Learning Control

Yongil Ri, Yong Fang, Hongwen Yu

School of Communication and Information Engineering, Shanghai University, Shanghai, 200444, China
E-mail: riyongil@163.com yfang@staff.shu.edu.cn hongwenyu@shu.edu.cn

Abstract: This paper investigates the consensus tracking problem for the discrete multi-agent system (DMAS) with time-delay. By using iterative learning control (ILC) method and convergence in discrete system, the convergence properties for total system can be analyzed based on the time-domain analysis, frequency-domain analysis and matrix theory. The result shows that the proposed methods depend on each agent's self-delay time, the weights of the edges to each agent's neighbors, and the interconnection topology of the network. The iterative number of learning control relies on the sampling period and self-delay time for the discrete multi-agent system. Finally, numerical examples are presented to demonstrate the theoretical results.

Key Words: Consensus, Convergence , Multi-agent System, Self- Delay, Discrete System

1 INTRODUCTION

Multi-agent consensus has gained great attention in recent years, and it has a quite wide application, such as communication networks, sensor networks, multi-robot systems and so on [1-16]. Researchers have done a lot of work on multi-agent consensus. In [3], a simple model for phase transition of a group of self-driven particles was proposed and complex dynamics of the mode were numerically demonstrated. Moreover, a theoretical explanation for the consensus behavior of the Vicsek model using graph theory was provided [4]. Some results have been got in recent papers about multi-agent with different motion models and communication system [1, 6]. Ren et al studied the consensus problem of collection motions, and derived some conditions about spanning tree to guarantee asymptotical consensus of the undirected integrated systems [7–9]. Some other researches about consensus on robustness, nonlinearity, and averaging equilibrium in multi-agent systems can be found in [3].

A similar model was discussed in current papers was used in [3, 4]. The authors studied symmetrical, fixed and undirected network topology with time-delays, but the non-symmetrical directed network was less considered. In [9, 10], a non-smooth Lyapunov approach was proposed for the case of time-varying graphs with time-delays. Sometime-delayed consensus research details were also presented in [11–16]. However, the time-delays in the above-mentioned papers were time-invariant, and the time-varying delays are not taken into consideration. As is known to all, there are always sudden situations in its external environment for multi-agent, such as the topology link failures may occur by destructive attacks. Therefore, it is worthy to study the multi-agent consensus with switching topologies. Some research work on this problem has been regarded recent years. In [3], the average-consensus problem of multi-agent is investigated with fixed topologies and time-varying topologies. Reference [12, 15] studied the consensus problem with a time-varying reference state where the information states of all vehicles approach a time-varying reference state under the given condition. However, the case of time-varying (switching) topologies with time-delays is rarely considered.

Recently, many researchers have investigated the ILC problems in MAS, but the determining number of repetitions of the iteration (input delay) was not mentioned. In particular, input delay is one of the most important component of the total time-delay, so we focus on the self-delay (input delay) of agents, ILC rule and convergence condition by iterative learning control (ILC) in the MAS. In this paper, we consider the convergence criteria in discrete system, analyze its sample period and the time-delay. From the simulation results, it can be seen that the proposed method meet the requirements of the theoretical results.

This paper is organized as follows: Section 2 and Section 3 introduces the knowledge of algebra graph theory and ILC consensus rules. Convergence analysis of the discrete system with time-delays is given in Section 4. Simulations and conclusions are listed in Section 5 and Section 6.

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2 PRELIMINARIES ON GRAPH THEORY

Graph theory has been adopted in multi-agent coordination problem for many decades, and it will be used in this work to describe the communication among agents. Therefore, the basic terminologies in graph theory are briefly reviewed below. Let $G=(V,E)$ be a weighted undirected graph with the set of vertices $V=\{1,2,\cdots,N\}$ and the set of edges $E\subseteq V\times V$. Let V also be the index set representing the agents in the networked systems. A undirected edge from k to j can be denoted by an ordered pair $(k,j)\in E$, which means that agent j can receive information from agent k. In this case, the k is called the parent of j. The set of neighbors of the k th agent is denoted by $N_k=\{j\in V|(j,k)\in E\}$.

$A=(a_{k,j})\in R^{N\times N}$ is the weighed adjacency matrix of G with nonnegative entries.

In particular, $a_{k,k}=0, a_{k,j}=1$ if $(j,k)\in E$, otherwise $a_{k,j}=0$. The in-degree of vertex k is defined as, $d_k^{in}=\sum_{j=}^{N}\sum a_{k,j}$ and the Laplacian of G is defined as $L=D-A$, where $D=diag(d_1^{in},\cdots,d_N^{in})$. A spanning tree is a graph, and each vertex has exactly one parent except one vertex called the root which has no parent. A graph is said to have or contain a spanning tree if the vertices set V and a subset of the edges set E can form a spanning tree. Then, we introduce the λ-norm, which is essentially an exponentially time weighted norm.

Definition 1: Given a vector function f : $[0,T]\rightarrow R^N$, its λ-norm is defined by $\|f\|_\lambda=\max_{t\in[0,T]} e^{-\lambda t}|f(t)|$, where λ is a positive constant, and $|\bullet|$ is any generic vector norm. In the ILC convergence analysis, λ-norm can be used to suppress the effects of system dynamics and reveal the input/output relations directly, which will make the convergence is simpler to prove.

3 ITERATIVE LEARNING CONTROL

Consider a group of N homogeneous dynamic agents, and the jth agent is governed by the following linear time-invariant model,

$$\dot{x}_{i,j}(t)=u_{i,j}(t),\forall j\in V, \quad (1)$$

where i denotes the iteration number, $x_{i,j}\in R^n$ is the state vector, $u_{i,j}\in R^m$ is the control input. For simplicity, the time argument t is dropped when no confusion arises.

The leader's trajectory, or the desired consensus trajectory $x_d(t)$ is defined by a finite-time interval $[0,T]$, and generated by the following dynamics,

$$\dot{x}_d=u_d \quad (2)$$

where u_d is the continuous and unique desired control input. Due to communication and sensor limitations, the leader's trajectory is only accessible to a small portion of the followers. Let the communication among followers be described by the graph G. If the leader is labeled by vertex 0, then the complete information flow among all the agents can be characterized by a new graph $G=\{0\cup V,E\}$, where E is the new edge set.

The major task is to design a set of distributed ILC rules such that each individual agent in the network which is able to track the leader's trajectory under the sparse communication graph G.

To simplify the controller design and convergence analysis, following assumptions are imposed.

Assumption 1: The initial state of all agents are reset to desire initial state at every iteration, i.e., $x_{i,j}(0)=x_d(0)$ for all $i\geq 1$.

Assumption 1 is referred as the identical initialization condition (i.i.c.), which is widely used in the ILC literature [3]. Many efforts have been dedicated to the relaxation of this assumption, but trade-off always exists.

For example, if the initial state is manipulatable and some of system parameters are known, the initial state learning rule [15] can be applied to remove the i.i.c. assumption. The initial state learning rule is adopted in [9] for multi-agent systems consensus tracking. When the initial state cannot be manipulated, while a fixed position should be reset after each iteration, rectifying action [6] can be applied to ensure satisfactory performance. However, the desired trajectory is slightly modified.

Let $\xi_{i,j}$ be the distributed measurement by agent j at the ith iteration over the graph G, and it is defined as

$$\xi_{i,j}=\sum_{k\in N_j} a_{j,k}(x_{i,k}-x_{i,j})+d_j(x_d-x_{i,j}) \quad (3)$$

where $d_j=1$ if $(0,j)\in E$, otherwise $d_j=0$.

Note that the actual tracking error $e_{i,j}=x_d-x_{i,j}$ cannot be utilized in the controller design when only a small number of followers have access to the leader. Therefore, $e_{i,j}$ is not available some of the followers. It is natural to incorporate the distributed measurement $\xi_{i,j}$ in the ILC design. Hence, the following ILC rule is adopted in this work,

$$u_{i+1,j}=u_{i,j}+Q\dot{\xi}_{i,j}, u_{0,j}=0,\forall j\in v, \quad (4)$$

where Q is the learning gain. For simplicity, the initial control input $u_{0,j}$ is set to zero. However, in practical implementation, the initial control input can be generated by certain feedback mechanism so that the system is stable. This may improve the transient performance of the learning controller. Note that $\xi_{i,j}$ is already available at the $i+1th$ iteration. Therefore, the derivative of $\xi_{i,j}$ can be obtained by any sophisticated numerical differentiation that does not generate a large amount of noise. The initial state learning rule can be expressed as

$$x_{i+1,j}(0) = x_{i,j}(0) + \xi_{i,j}(0). \tag{5}$$

Reference [15] come to a conclusion that in multi-agent systems (1), under the communication graph G, and ILC rule (4), if the learning gain Q is chosen such that

$$\left| I_{mN} - H \otimes Q \right| \le \mu < 1 \tag{6}$$

where $I(\bullet)$ is the identity matrix with the subscript denoting its dimension, μ is a constant, $H = L + D$, L is the Laplacian matrix of G, $D = diag(d_1, d_2, \cdots, d_N)$, and $\otimes$ represents the Kronecker product, which makes the convergence proof simpler. It is more convenience when we discretize the plant by using zero-order hold circuit and impulse sampler, we can obtain the corresponding discrete-time formula as

$$\delta x_{i,j}[k] = x_{i,j}[k] + u_{i,j}[k],\ x_{i,j}[0] = x_0 \tag{7}$$

where T is the sampling period, $T \in R$ and $T > 0$. The index $k \in Z$ denotes discretized time, i.e., $t = kT$, and δ denotes the delta operator [7]

$$\delta x_{i,j}[k] = \frac{1}{T}(x_{i,j}[k+1] - x_{i,j}[k]) \tag{8}$$

Furthermore, $x[k]$ and $u[k]$ are the respective state which input at the discretized time k .Hence, the following ILC rule can be expressed in discrete-time system,

$$u_{i+1,j}[k] = u_{i,j}[k] + Q(\xi_{i,j}[k] - \xi_{i,j}[k-1]) / T \tag{9}$$

4 CONVERGENCE CONDITION IN DISCRETE SYSTEM

In practice, we usually consider the analog signal to digital signal in transmission, and time-delay can be considered separately in discrete system. zero-order multi-agent systems discrete sampling control model as follows

$$\begin{aligned} &x_i(kT+T) = x_i(kT) + Tu_i(kT) \\ &(k = 0,1,\cdots; i \in I) \end{aligned} \tag{10}$$

where sampling period $T > 0$; k is discrete index.

The consensus control protocol without time delay in [16] can be written

$$\begin{aligned} u_i(kT) = &-b_i \sum_{j=1}^{n} a_{ij}(x_i[kT] - x_j[kT]) - \\ &b_i a_{i0}(x_i[kT] - x_d[kT]) \end{aligned} \tag{11}$$

where control gain $b_i > 0$, If leader is the individual agent neighbors, then $a_{i0} > 0$, otherwise $a_{i0} = 0$.

While considering the effects of time delay $\tau, (\tau_s \in (0,T))$, this paper extends the consensus tracking protocol (11) to the following consensus tracking protocol:

$$u_i(kT) = \begin{cases} -b_i \sum_{j=1}^{n} a_{ij}(x_i[kT-T] - x_j[kT-T]) - b_i a_{i0} \\ (x_i[kT-T] - x_d[kT-T]), t \in [kT, kT+\tau_s) \\ -b_i \sum_{j=1}^{n} a_{ij}(x_i[kT] - x_j[kT]) - b_i a_{i0} \\ (x_i[kT] - x_d[kT]), t \in [kT+\tau_s, kT+T) \end{cases} \tag{12}$$

Definition 1: Protocol (12) is called a bounded consensus tracking protocol, if the system (10) using the protocol (12) has the property:

$$\lim_{k\to\infty} \left\| x_i[kT] - x_d[kT] \right\| \le C < \infty, \forall i \in I, \tag{13}$$

where C is a bounded positive constant independent on k .

Lemma 1: [15] take a fixed, undirected, connected network topological graph G which is combined into n multi-agent, at least exists one path between the agent and the virtual leader. When the multi-agent system (10) used consensus tracking protocol (12), if and only if the following two conditions are satisfied, the system can achieve consensus bounded tracking:

$$\tau_s < 1/\lambda_{\max}(H) \tag{14}$$

$$\tau_s < T < 2\tau_s + 2/\lambda_{\max}(H) \tag{15}$$

The following lemmas play an important role on the convergence analysis of bounded and consensus tracking.

Lemma 2: If and only if all features of formula roots in the left half open plane, the root of all features are located within the unit circle in the equation, among them $a, b \in R$.

Lemma 3 : The equation $s^2 + as + b = 0$ has all roots within the unit circle, if and only if all roots of $(1+a+b)t^2 + 2(1-b)t + 1 - a + b = 0$ are in open left half plane[11], where $a, b \in R$.

Lemma 4: This paper takes a fixed, undirected, connected network topological graph G which is combined with n multi-agent composition into consideration, there at least exists one path between the agent and the virtual leader, and the matrix $H = D + L$ will be definite, that the value of the smallest feature matrix $\lambda_{\min}(H) > 0$.

Lemma 5: [16]assuming that n agents of the communication network topology G composed is fixed, undirected and connected, at least one agent has the access to the virtual leader, and the time-varying reference state of the virtual leader satisfies

$$\left|x_d(kT+T) - x_d(kT)/T\right| \le \xi^0 < \infty \ \forall T > 0 \quad (16)$$

where $k = 0,1,\cdots,N$. Then the multi-agent system (10), applying the consensus tracking protocol (12), achieving the bounded consensus tracking if and only if all the roots of the equation

$$s^{m+2} - s^{m+1} + (T-\varepsilon)\lambda_i(H)s + \varepsilon\lambda_i(H) = 0,\ i \in I \quad (17)$$

are within the unit circle.

Obviously, Lemma 1 can be regarded as a special case of Lemma 5. When $m = 0$, the formula (17) reduces to

$$s^2 + [(T-\tau_s)\lambda_i(H) - 1]s + \tau_s\lambda_i(H) = 0\ (i \in I) \quad (18)$$

Lemma 3 shows that: if and only if all roots of the equation (19) are open in the left flat and all roots of the equation (18) are located inside the unit circle. The equation (19) can be expressed as:

$$\begin{aligned} &T\lambda_i(H)t^2 + 2[1-\tau_s\lambda_i(H)]t + \\ &[2 + (2\tau_s - T)\lambda_i(H)] = 0\,(i \in I) \end{aligned} \quad (19)$$

Because the system network topology G apparently satisfies the conditions of Lemma 4, and $\lambda_{\min}(H) > 0$.

Therefore, if and only if the following two inequality, all roots of the equation (18) are open in the left half plane, namely:

$$1 - \tau_s\lambda_i(H) > 0 \quad (20)$$

$$2 + (2\tau_S - T)\lambda_i(H) > 0 \quad (21)$$

Simplify inequalities (20) (21), and it can be obtained condition (14) (15).

5 ILLUSTRATIVE EXAMPLE

In this section we provide the numerical simulations to illustrate the effectiveness of the above results.

It is easy to verify that the complete information flow graph G contains a spanning tree with the leader being its root. The communication among follows is undirected, which means the information flow is bidirectional. We adopt 0-1 weighting, and the Laplacian for follows is

$$L = \begin{bmatrix} 1 & -0.2 & 0 & 0 & -0.4 \\ -0.2 & 1 & -0.1 & -0.7 & 0 \\ 0 & -0.1 & 1 & 0 & 0 \\ 0 & -0.7 & 0 & 1 & -0.5 \\ -0.4 & 0 & 0 & -0.5 & 1 \end{bmatrix} \quad (22)$$

and $D = diag(1,1,1,1,1)$ which represents the information flow from leader to the followers.

The leader's state is chosen as

$$x_d(t) = \sin(0.1t), t \in [0,100] \quad (23)$$

The initial conditions for followers are

$$\begin{aligned} &x_{i,1}(0) = 3.5, x_{i,2}(0) = 3, x_{i,3}(0) = 1.5, \\ &x_{i,4}(0) = 1.1, x_{i,5}(0) = 0.8 \end{aligned} \quad (24)$$

Obviously, initial state errors are nonzero. We apply the ILC rules (4) with learning gains $Q = 0.3$.

The eigenvalue of H are:

$$\begin{aligned} &\lambda_1 = 0.0461, \lambda_2 = 0.7997, \lambda_3 = 1.000, \\ &\lambda_4 = 1.2003, \lambda_5 = 2.0001 \end{aligned} \quad (25)$$

By calculating formula (14), the time-delay is $(0, 0.5)$. So choosing $\tau_s = 0.4$, by (15)we can get the sampling period T in the range of $(0.71, 1.8)$. Choosing $\tau_s = 0.4, T = 1.5$, the time-delay τ_s and sampling period T satisfy the Lemma 1 condition, each multi-agent states is shown in Fig 1. The state of each agent is gradually in line with the leader's trajectory. As shown in Fig.1, underlying satisfy conditions of the Lemma 1, the convergence gets faster when the time-delay and sampling period get smaller.

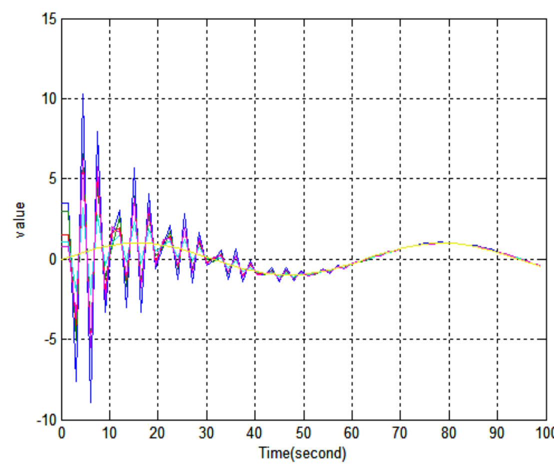

Fig 1: $\tau_s = 0.4, T = 1.5$ states of leader and each agent

In Fig 2 and Fig 3, we can find that when $\tau_s = 0.4, T = 1.8$ and $\tau_s = 0.4, T = 1.9$, which time-delay τ_s satisfies the condition of Lemma 1, while sample period

T does not satisfy the condition of the Lemma 1. Therefore, the state of each agent is gradually diverge and unable to trace the leader's trajectory.

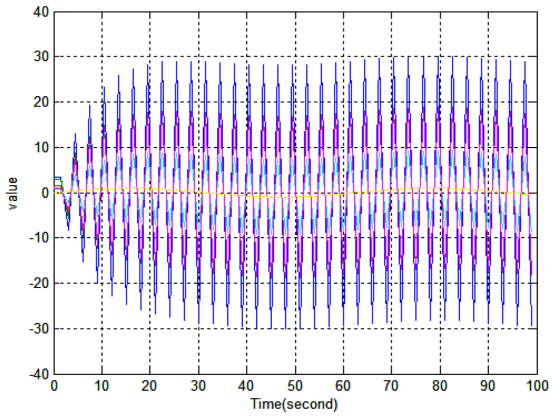

Fig 2: $\tau_s = 0.4, T = 1.8$ states of leader and each agent

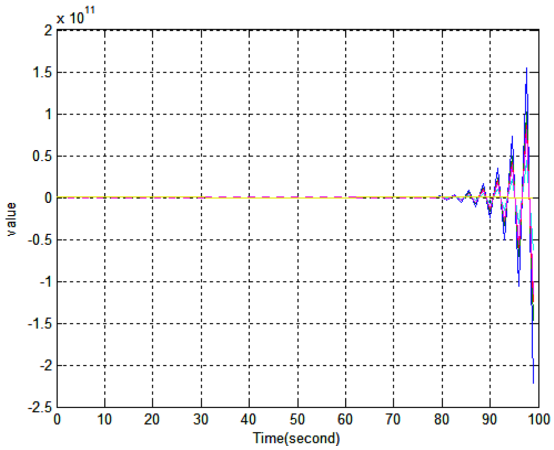

Fig 3: $\tau_s = 0.4, T = 1.9$ states of leader and each agent

Also, when $\tau_s = 0.5, T = 1.5$ and $\tau_s = 0.6, T = 1.5$, sample period satisfy the condition of Lemma 1, but the time-delay does not satisfy the condition of the Lemma 1. As shown in Fig 4 and 5, the state of each agent gradually diverge, cannot track the leader's trajectory.

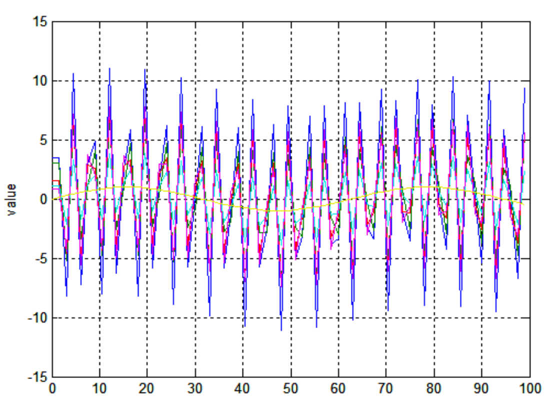

Fig 4: $\tau_s = 0.5, T = 1.5$ states of leader and each agent

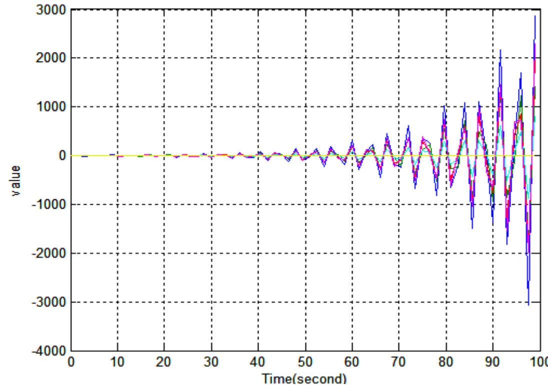

Fig 5: $\tau_s = 0.6, T = 1.5$ states of leader and each agent

Also, in order to consider time-delay by ILC rule (4), that considering the calculation time based on the ILC rule (4), convergence time and calculation time are shown in below table. Calculation time is the learning time based on the ILC rule (4) here.

Table 1: Convergence and calculations time (s)

iteration	convergence time(s)	calculations time (s)
10th	6.2	0.18
15th	5.7	0.24
20th	4.8	0.38
25th	4.1	0.42
30th	3.5	0.65
35th	1.2	0.72
50th	0.4	0.94

As shown in Table 1, as greater the number of iteration, the convergence time is shorter and the calculation time is longer.

In other words, the calculation time is a time-delay in the discrete system. Considering communication delay, the total time-delay is longer.

We choose the number of iteration that satisfies the calculation time, which is set to 25th.

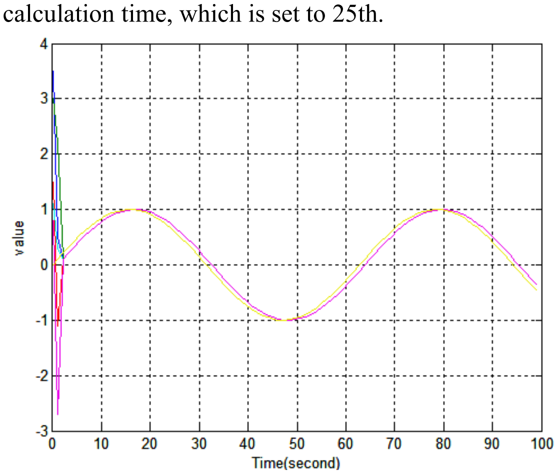

Fig 6: States at the 25th iteration

As shown in Fig 6, convergence time was significantly reduced. Thus, when ILC rules applied to discrete system, be sure to analyze the convergence of the system that determine the sample period and number of iteration.

Therefore, in practical applications should analyze the convergence properties of the system, it should determine the number of iterations.

6 CONCLUSION

In this paper, we analyzed the convergence of multi-agent systems by iterative learning control with constrained time-delay. Also, this paper analyzes the convergence properties of each agent in the discrete system and determines the number of reasonable iterations based on ILC rules.

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