A combined data-driven, robust-estimator and parameter-learning methodology for sensor fault compensation

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Abstract: This paper studies the sensor fault compensation problem on the basis of recorded process input/output data. The main contribution is to construct an unified framework for fault compensation law by combing the subspace model identification (SMI) technique, measurement disturbance estimator (MDE) theory and iterative learning approach (ILA). Firstly, the considered unknown process dynamic is modeled as a discrete-time state-space model via utilizing SMI. Secondly, based on the identified system, a simultaneous estimator with H_{∞} optimization performance is designed to monitor the system state and output disturbance that includes the possible measurement noise and sensor fault. Thirdly, a suitable online parameter update law is provided for the purpose of correcting the SMI identification error influence on disturbance estimation. Finally, the sensor fault compensation can be achieved by dismantling the iteratively appropriated disturbance estimation from the actual measured output signal. An numerical example is given to illustrate the effectiveness of the proposed compensation procedure.

Key Words: Sensor Fault, Subspace Model Identification, Measurement Disturbance Estimator, Signal Compensation

1 INTRODUCTION

In order to maintain and control the desired requirements on process performance, fault tolerant control technique is receiving considerably attentions in the research and application fields due to that it can weaken or even eliminate the unexpected influences from noise and fault [1, 2, 3]. In literature, fault tolerant control methods are classified into two types, i.e., passive fault tolerant control and active fault tolerant control [1]. More comprehensive studies can be founded in [4, 5].

Without control reconstruction, we will utilize the fault estimation and compensation to perform tolerant control in this study. As for this issue, the model-based techniques have been fully investigated such as in [6, 7] and the references therein. But the model-based results need to rely on first principle for accurate analytical or functional redundancy discussions. This design strategy will be unfeasible with the increasing complexity of modern industrial processes. Alternatively, data-driven fault diagnosis and control tuning via multi-variable statistics provide a suitable way to deal with such case [8]. Unfortunately, the generally insufficient information of dynamic description on studied process will lead to the ambiguous or misleading results of multi-variable statistics method [9]. Naturally, a fusion idea to balance above situations is to combine the large-scale process handling capacity of data-driven method and highly abnormal detection accuracy of modelbased approach. In line with such technical route, several researchers have proposed different combination schemes

to perform fault detection and isolation. Several representative work can be found in [9, 10, 11]. However, the fault estimation and compensation by combining data-driven and model-based methodology have not been well addressed.

On the other hand, note that the identified model by SMI inevitably leads to the estimation error of actual measurement disturbance, which further causes the inefficient fault compensation. In order to adjust such disequilibrium, a suitable online update law is constructed on the basis of the previous basic estimated disturbance. This update law is designed to learn the unknown output disturbance dynamic in real time via using adaptive parameter tuning. It can be predicted that this corrected estimation can significantly enhance the sensor fault compensation ability.

In this paper, we attempt to formulate a general framework for sensor fault compensation. This framework integrates the concepts from the data-driven, estimator-design and adaptive compensation methodologies via utilizing S-MI, MDE, and ILA techniques. Specifically, SMI is firstly used to model the state-space appropriate dynamic of the recorded process input/output data. Then, the MDE is analyzed through descriptor system remodelling with the H_{∞} performance evaluation. And the estimator is designed by convex optimization algorithm. In what follows, the estimated measurement disturbance is feeded into the learning update law via ILA, where the corrected estimation can be calculated in real time. Finally, the sensor fault compensation can be achieved by dismantling the such estimation from the actual measured output signal. An numerical example is given to verify the effectiveness of the proposed fault compensation procedure.

The notations are standard. \mathbb{R}^n and $\mathbb{R}^{m \times n}$ denote, respectively, the *n*-dimensional Euclidean space and the set of all

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 $m \times n$ real matrices. $Tr\{.\}$ stands for the matrix trace. Generally, $He\{A\}$ denotes $A + A^T$ and $diag\{X1, X2\}$ implies a diagonal matrix with elements (X1, X2).

2 PRELIMINARIES AND PROBLEM FOR-MULATION

This section presents a brief summary of SMI method and MDE strategy that will be used for state-space model identification and measurement output disturbance estimation.

2.1 SMI-Based State-Space Modeling

As illustrated by several researches (e.g. [12, 13, 10]), S-MI provides a feasible way to estimate the studied process dynamics using the recorded input/output data. The potential background to support such modeling is that practical experience has shown that many industrial processes can be approximated with sufficient accuracy by linear timeinvariant systems of finite dimensions [11, 14]. In what follows, set the recorded control input $u_r(k)$ and measured output $y_r(k)$ data sets over N horizon time window in the absence of fault situation be U and Y. Additionally for simplification, let the existing control law be static memoryless output feedback manner $u_r(k) = -Ky_r(k)$ with known gain matrix K. Finally, by SMI method with Akaike information criterion (AIC), the obtained fault-free statespace representation is assumed to have the following general form:

$$x(k+1) = Ax(k) + Bu(k) + W\omega(k),$$

$$y(k) = Cx(k) + Du(k) + V\nu(k),$$
(1)

where $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^{n_u}, y(k) \in \mathbb{R}^{n_y}$ are the state, input, and measured vectors at sampling time k. A, B, C, D, and W/V are state matrix, input matrix, output matrix, direct feed-through matrix and Kalman gain matrix with suitable dimensions, respectively. Traditionally, in order to ease the subsequent analysis and design for identified system $(1), \omega(k) \in \mathbb{R}^{n_\omega}$ and $\nu(k) \in \mathbb{R}^{n_\nu}$ are often defined to be zero-mean, normal distributed white noise sequences. Actually, as we know, the obtained system by SMI is indeed an appropriate model of studied process. Hence, it is better to assume that $\omega(k)$ and $\nu(k)$ are unknown input signals because they are more general for representing any bounded noises and modeling uncertainties.

In this study, we utilize the SMI method to model the LTI discrete-time system realization. Actually, the multimode, nonlinear and time-varying characteristics of considered process dynamics can also be captured by modified or combined SMI techniques. For example, [11, 15] constructed the unified frameworks to describe the hybrid multimode process systems by integrating the Gaussian mixture models and SMI theory; [16] applied CVA to modeling a nonlinear continuous stirred tank reactor; [17] used the Vector Regressive to establish a recursive subspace identification method for predicting time-varying stochastic systems. To facilitate the proposition of data-driven estimator-based fault compensation design concept, we only consider the LTI system modelling case. Note that, the fault compensation issues for complex dynamics including the above listed three aspects can also be handled if the related-type estimators are employed.

Remark 1 It should be mentioned that $y(k) \equiv y_r(k)$ will not be hold due to that the identification error will be not avoided.

2.2 Estimator-Based Sensor Fault Compensation Analysis

For the modeled system (1) by SMI method, an alarm can be created when a fault occurs through fault detection and isolation (FDI) phase in [11, 10]. But the magnitude of that fault will not be obtained only via FDI. Hence, it is indispensable to perform fault reconstruction to provide the shape of fault for the fault compensation that devotes to eliminating the fault influence on the original plant behavior without fault. In this paper, we consider the sensor fault case of system (1):

$$x(k+1) = Ax(k) + Bu(k) + W\omega(k),$$

$$y(k) = Cx(k) + Du(k) + V\nu(k) + Ff_s(k),$$
(2)

where $f_s(k) \in \mathbb{R}^{n_{f_s}}$ is the sensor fault and F is unknown fault distribution matrix. Plant (2) is assumed to run well in the absence of sensor fault $f_s(k)$. Now, for sensor fault system (2), define

$$d(k) = V\nu(k) + Ff_s(k), \tilde{x}(k) = \begin{bmatrix} x(k) \\ d(k) \end{bmatrix}, E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$
(3)

where d(k) is named as measurement disturbance which includes the noise $\nu(k)$ and sensor fault $f_s(k)$. Next, by descriptor system augmented approach we have

$$E\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}u(k) + \tilde{W}\omega(k),$$

$$y(k) = \tilde{C}\tilde{x}(k) + \tilde{D}u(k),$$
(4)

where

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$$\tilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \tilde{W} = \begin{bmatrix} W \\ 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & I \end{bmatrix}, \tilde{D} = D.$$

In what follows, the measurement disturbance estimator is taken in the form of

$$p(k+1) = (\tilde{A} - L_p \tilde{C})q(k) + (\tilde{B} - L_p \tilde{D})u(k) + L_p y(k),$$

$$q(k) = (E + L_q \tilde{C})^{-1}(p(k) - L_q \tilde{D}u(k) + L_q y(k)),$$
(5)

where $q(k) \in \mathbb{R}$ is the estimation of $\tilde{x}(k)$, and (L_p, L_q) are the designed parameters. Then define $L_q = \begin{bmatrix} 0 & G \end{bmatrix}^T$ with nonsingular matrix G, system (4) and estimator (5) can be respectively represented as follows

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$$(E + L_q C)\tilde{x}(k+1) = (A - L_p C)\tilde{x}(k) + (B - L_p D)u(k) + \tilde{W}\omega(k) + L_p y(k) + L_q(y(k+1)) - \tilde{D}u(k+1)), (E + L_q \tilde{C})q(k+1) = (\tilde{A} - Lp\tilde{C})q(k) + (\tilde{B} - L_p\tilde{D})u(k) + L_p y(k) + L_q(y(k+1) - \tilde{D}u(k+1))) (6)$$

Perform subtraction in (6) with definition $e(k) = q(k) - \tilde{x}(k)$, the estimation error system can be obtained

$$(E + L_q \tilde{C})e(k+1) = (\tilde{A} - L_p \tilde{C})e(k) - \tilde{W}\omega(k).$$
(7)

To facilitate the subsequent analysis, system (7) is rearranged in the following form with definition $M = (E + L_a \tilde{C})$

$$e(k+1) = M^{-1}(\tilde{A} - L_p \tilde{C})e(k) - M^{-1}\tilde{W}\omega(k).$$
 (8)

Now we are in a position to apply H_{∞} optimization theory to design robust estimator against noise $\omega(k)$. Namely, the current aim is to design estimator in the manner of finding minimum performance level γ that satisfies $|| e(k) ||_2 < \gamma ||$ $\omega(k) ||_2$.

Assume the measurement disturbance estimator has been designed, the disturbance signal and state observation can be estimated by $\hat{d}(k) = \begin{bmatrix} 0 & I \end{bmatrix} q(k)$ and $\hat{x}(k) = \begin{bmatrix} I & 0 \end{bmatrix} q(k)$, where $\hat{d}(k)$ and $\hat{x}(k)$ represent the estimations of d(k) and x(k) respectively. To the end, the fault compensation law can be realized by eliminating the effects of $\hat{d}(k)$ from the measurement y(k).

Remark 2 As state in [6], there is not need to estimate the fault $f_s(k)$ because the estimation $\hat{d}(k)$ is sufficient for achieving the fault compensation task. The detailed discussion and verification will be given in the following specific analysis and design.

3 SENSOR FAULT COMPENSATION DESIGN

Lemma 1 (Finsler Lemma) [18, 19, 20] For matrices $\Theta = \Theta^T, \mathfrak{B}$ and Υ , the following results are equivalent:

1.
$$\eta^T \Theta \eta < 0, \forall \eta \in \mathbb{R}^{n_\eta}, \quad \Upsilon \eta = 0, \eta \neq 0.$$

2. $\Theta + \mathfrak{B} \Upsilon + \Upsilon^T \mathfrak{B}^T < 0$

First, the estimator (5) can be designed on the basis of Theorem 1.

Theorem 1 For the identified state-space model with fault (2), assume positive definite matrix Q, nonsingular matrix G, and auxiliary matrix S_2 be the solutions to the following optimization problem:

$$\gamma^* := \min \quad \gamma$$

$$He\{S_2\} > 0,$$

$$s.t. \begin{cases} He\{S_2\} > 0, \\ diag\{I - Q, Q, -\gamma^2 I\} + He\{\left[\delta_1 I \quad I \quad \delta_2 I\right]^T \\ \times \left[R\tilde{A} - T\tilde{C} \quad -S_2 \quad -R\tilde{W}\right]\} < 0, \end{cases}$$
(9)

where $R = S_2 M^{-1}$, $T = RL_p$. Then, there exists a measurement disturbance estimator in the form of (5) with H_{∞} performance γ^* . Moreover, a suitable estimator realization is given by $L_q = ((S_2^{-1}R)^{-1} - E)\tilde{C}^T (\tilde{C}\tilde{C}^T)^{-1}$ and $L_p = R^{-1}T$.

Proof 1 The proof is briefly given. First for system (8), construct a Lyapunov functional candidate $\mathfrak{L}_e(k) = e^T(k)Qe(k)$. Then, based the definitions (R,T) and Lemma 1, the second inequality of constraint condition in (9) is equivalent to

$$\eta^{T}(k)diag\{I - Q, Q, -\gamma^{2}I\}\eta(k) < 0,$$
(10)

for
$$\eta(k) = \begin{bmatrix} e^T(k) & e^T(k+1) & \omega^T(k) \end{bmatrix}^T$$
 with

$$\begin{bmatrix} M^{-1}(\tilde{A} - L_p \tilde{C}) & -I & -M^{-1} \tilde{W} \end{bmatrix} \eta(k) = 0. \quad (11)$$

From (10), it is direct to deduce

$$e^{T}(k)e(k) - \gamma^{2}\omega^{T}(k)\omega(k) + e^{T}(k+1)Qe(k+1) - e^{T}(k)Qe(k) < 0.$$
(12)

Then, summing up (12) from k = 0 to $k = \infty$, there is $\sum_{k=0}^{\infty} (e^T(k)e(k)) < \gamma^2 \sum_{k=0}^{\infty} (\omega^T(k)\omega(k))$ under zero initial condition, which suggests that the H_{∞} performance can be guaranteed. In what follows, the minimum allowable performance level γ^* can be obtained by solving the optimization problem (9), which aims at enhancing the robustness of estimator (5) against $\omega(k)$.

Once the robust estimator has been designed by Theorem 1, the following phase is to perform fault compensation. First, the measurement disturbance estimation including the sensor fault estimation can be calculated by $\hat{d}(k) = \begin{bmatrix} 0 & I \end{bmatrix} q(k)$. Second, the sensor fault compensation is defined as $y^c(k) = y(k) - \mu \chi(\hat{d}(k))$, where $\chi(.)$ represents the utilization manner of estimation $\hat{d}(k)$ and parameter μ is the related weighted factor. Both of them are designed to weaken the identification error influence on disturbance estimation. Then, we have the following stability conclusion about sensor fault system (2).

Theorem 2 For sensor fault system (2), the compensated closed system is stable and approaches the original plant steady behavior without sensor fault.

Proof 2 For simplicity of narration, we suppose D = 0. Then, the compensated control command is

$$u = -Ky^{c}(k) = -K(y(k) - \mu \hat{d}(k))$$

= -K(Cx(k) + d(k) - \mu\chi(\dot{d}(k))), (13)

which implies that the closed-loop form of (2) is

$$\begin{aligned} x(k+1) &= (A - BKC)x(k) + W\omega(k) - BKd(k) \\ &+ \mu BK\chi(\hat{d}(k)), \\ \tilde{y}(k) &= Cx(k) + d(k) - \mu\chi(\hat{d}(k)). \end{aligned}$$
(14)

For compensated closed-loop system (14), it is intuitive to prove the stability, which is omitted here due to page limitation. Now, we focus on the discussions on the compensation functional of (13). Note that the closed-loop sensor fault system (2) without compensation is

$$x(k+1) = (A - BKC)x(k) + W\omega(k) - BKd(k),$$

$$y(k) = Cx(k) + d(k),$$
(15)

from where the sensor fault propagation can be seen in the dynamic equation. Namely, in the absence of compensation, the previous static output feedback controller will send the wrong command which obviously violates the current system operation conditions once the sensor fault occurs. However, as can be verified in (14), the fault compensation utilizes the estimated fault signal to eliminate any possible distortion in system output measurement, which further correct the deviation from the original system behavior without fault.

The above discussion illustrates the stability of closed-loop system by using the compensation signal $\chi(\hat{d}(k))$. Now, on the basis of the guaranteed stability and compensation effect by Theorem 2, we will focus on the design of function $\chi(.)$. In order to online appropriate the actual sensor fault by tuning estimation $\hat{d}(k)$, the following update law by ILA is utilized

$$\theta(k+1) = \theta(k) + \alpha \hat{d}(k) (e_{y^c}(k))^T Y$$
$$-\beta \parallel I - \alpha \hat{d}(k) \hat{d}^T(k) \parallel \theta(k), \qquad (16)$$
$$\chi(\hat{d}(k)) = \theta(k),$$

where $\alpha > 0$ is the learning rate, $\beta > 0$ and $Y(||Y|| \le \lambda, \lambda > 0)$ are the designed parameters. For (α, β) in (16), we have the following limitation.

Theorem 3 The determination of α and β should satisfy the following basic value relationship:

$$\alpha \in [0, \frac{\sqrt{3}\sqrt{3\beta^2 \parallel I - \alpha \hat{d}(k)\hat{d}^T(k) \parallel^2 - 2}}{3\lambda \parallel \hat{d}(k) \parallel}].$$
(17)

Proof 3 Define Lyapunov function for (16)

$$\mathfrak{L}_{\theta}(k) = \frac{1}{\alpha} Tr\{\tilde{\theta}^{T}(k)\theta(\tilde{k})\}$$
(18)

where $\tilde{\theta}(k) = \tilde{d}(k) - \theta(k)$. Note that, fault compensation works after the fault occurrence time instant. $\tilde{d}(k)$ is assumed to be the true process sensor fault, which is unknown in the analysis. Then the first difference of $\mathfrak{L}_{\theta}(k)$ is given by

$$\begin{aligned} \Delta \mathfrak{L}_{\theta}(k) &\leq \frac{1}{\alpha} Tr\{\tilde{\theta}^{T}(k)\tilde{\theta}(k) - \alpha\tilde{\theta}^{T}(k)\hat{d}(k)(e_{y^{c}}(k))^{T}Y \\ &+ \beta\tilde{\theta}^{T}(k) \parallel I - \alpha\hat{d}(k)\hat{d}^{T}(k) \parallel \theta(k) - \alpha(\hat{d}(k) \\ &\times (e_{y^{c}}(k))^{T}Y)^{T}\tilde{\theta}(k) + \alpha^{2}(\hat{d}(k)(y^{c}(k))^{T}Y)^{T}(\hat{d}(k) \\ &\times (e_{y^{c}}(k))^{T}Y) - \alpha\beta(\hat{d}(k)e_{y^{c}}^{T}(k)Y)^{T} \parallel I - \alpha\hat{d}(k) \\ &\times \hat{d}^{T}(k) \parallel \theta(k) + \beta \parallel I - \alpha\hat{d}(k)\hat{d}^{T}(k) \parallel \theta^{T}(k)\tilde{\theta}(k) \\ &- \alpha\beta \parallel I - \alpha\hat{d}(k)\hat{d}^{T}(k) \parallel \theta^{T}(k)\hat{d}(k)e_{y^{c}}^{T}(k)Y \\ &+ \beta^{2} \parallel I - \alpha\hat{d}(k)\hat{d}^{T}(k) \parallel \theta^{T}(k) \parallel I - \alpha\hat{d}(k)\hat{d}^{T}(k) \\ &\times \theta(k) - \tilde{\theta}^{T}(k)\tilde{\theta}(k) \end{aligned}$$
(19)

Let $\tilde{y}(k)$ stand for the real measurement output of process during monitoring. Then apply Cauchy-Schwarz inequality, equivalent relation $\theta(k) = -(\tilde{d}(k) - \theta(k)) + \tilde{d}(k)$ and definition $e_{y^c}(k) = \tilde{y}(k) - \theta(k) - (y(k) - \hat{d}(k))$ to the above inequality, we have

$$\Delta \mathfrak{L}_{\theta}(k) \leq \frac{1}{\alpha} Tr\{2\tilde{\theta}^{T}(k)\tilde{\theta}(k) + 3\alpha^{2}(\hat{d}(k)\tilde{\theta}^{T}(k)Y)^{T}\hat{d}(k) \\ \times \tilde{\theta}^{T}(k)Y - 3\beta^{2} \parallel I - \alpha \hat{d}(k)\hat{d}^{T}(k) \parallel^{2} \tilde{\theta}^{T}(k)\tilde{\theta}(k) \\ + \pi(\tilde{d}(k))\},$$
(20)

where $\pi(d(k)) > 0$ represents the unknown functional that is obtained by adding the unmeasured values in above inequality. According to the results in [21] and Frobenius norm property, from (20) there is the following basic constraint for real time $\hat{d}(k)$

$$2 + 3\alpha^2 \lambda^2 \parallel \hat{d}(k) \parallel^2 < 3\beta^2 \parallel I - \alpha \hat{d}(k) \hat{d}^T(k) \parallel^2$$
. (21)

Finally, (21) can directly deduce the condition in (17).

Remark 3 Actually, the constraint (17) is not sufficient tight because there is not sufficient information about the sensor fault in our assumption. But in many practical plant, the sensor basis fault types or the possible sensor fault distributions are often (partial) known. The unknown element of fault is usually its magnitude and occurrence time (i.e. time profile). In these cases, the disturbance learning law (16) together with parameter determination (17) even including the identification accuracy by SMI can be more significantly improved.

To the end, the integrally operating procedures for fault compensation are summarized as below:

	Algorithm	I Off-line	design	of	disturbance	estimator
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- 1: Collect the input and output data sets of studied process and plant over window horizon N, and name them as (U, Y).
- 2: Identify the appropriate state-space model (1) and obtain system (2) a by using SMI method on the basis of (U, Y).
- 3: Design disturbance estimator (5) by solving optimization problem (9).

Algorithm II On-line sensor fault compensation

- 1: Use the designed estimator to calculate the disturbance estimation by $\hat{d}(k) = \begin{bmatrix} 0 & I \end{bmatrix} q(k)$.
- 2: Initiate the learning law (16) by choosing suitable parameters that satisfy constraint (17).
- 3: Perform output compensation by transforming $y^{c}(k)$ to the controller.

4 NUMERICAL EXAMPLE

In this subsection, to demonstrate the effectiveness of the proposed SMI-based fault estimation and fault compensation framework, the following numerical example of an 3order discrete-time state-space model ([22]) is used here:

$$x(k+1) = Ax(k) + Bu(k) + W\omega(k),$$

$$y(k) = \hat{C}x(k) + \omega(k),$$
(22)

with

$$\hat{A} = \begin{bmatrix} 0.8 & -0.4 & 0.2 \\ 0 & 0.3 & -0.5 \\ 0 & 0 & 0.5 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 & 0 \\ 0 & -0.6 \\ 0.5 & 0 \end{bmatrix},$$
$$\hat{W} = \begin{bmatrix} 0.055 \\ 0.04 \\ 0.45 \end{bmatrix}, \hat{C} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For simplification, we assume that $\omega(k)$ and u(k) are random white noise sequences of variance 0.1 and 1, respectively. Then, a total of 2000 data samples were generated in the above closed loop system. The first 1500 samples of data were used to establish the identified state space model on the basis of SMI method (here N4SID) and the subsequent 500 samples of data were used to verify approximation effect of such model. Figure 1 (200 samples) was given to show identification results of SMI-based method for considered system (22), where the discrete-time identified state-space model fits to the estimation data by 42.19% - 50.95%. The corresponding obtained state-space model without feedthrough matrix is provided in the form of (1)

$$A = \begin{bmatrix} 0.8747 & 0.355 & -0.4603 \\ -0.05532 & 0.5016 & 0.2835 \\ -0.06554 & -0.1889 & 0.2361 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.00748 & 0.008184 \\ -0.01789 & -0.01441 \\ -0.0162 & 0.02116 \end{bmatrix},$$

$$W = \begin{bmatrix} -0.01817 & 0.006317 \\ -3.634e - 05 & -0.01336 \\ -0.009942 & -0.01133 \end{bmatrix},$$

$$C = \begin{bmatrix} -9.541 & 10 & -3.966 \\ -4.535 & -15.43 & -9.041 \end{bmatrix}.$$

(23)

Now, to design the estimator, we assume that system (23) is subjected to sensor fault $Ff_s(k)$ as (2) with unknown fault distribution matrix. Then, by solving Theorem 1, we obtain the parameters of disturbance estimator are

$$L_q = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, L_p = \begin{bmatrix} 0.0121 & -0.0063 \\ 0.0114 & -0.0256 \\ 0.0035 & 0.0083 \\ -0.3360 & -0.1757 \\ -0.2874 & -0.1423 \end{bmatrix}.$$
 (24)

Then, we verify the effectiveness of above estimator (24) for the actual system (22). Let $\omega(k)$ be the band-limited white-noise signal with noise power 0.1, static output feedback gain matrix K be $diag\{0.58, 0.2\}$, and the fault type in (22) be the second sensor offset by 5 at k = 100. In what follows, the outputs of (22) with/without compensation are illustrated in Figure 2. Note that, the functional signal $\chi(\hat{d}(k))$ is just taken in the form of $\hat{d}(k)$ due to that such simple compensation is sufficient for the sensor fault case in the system (22). From Figure 2, it can be seen that the fault output can be timely corrected via the disturbance compensation, which further implies the sensor fault tolerant capacity of the proposed data-driven SMI aided fault compensation strategy in this paper.



Figure 1: The Measured and Simulated Output Data



Figure 2: The Real and Compensated System Output Signals Under Sensor Fault

5 CONCLUSION

In this paper, a effective sensor fault compensation strategy is proposed by combining the data-driven and modelbased techniques. The specific SMI, MDE and ILA methods are integrated in the compensation framework. The fault tolerant control capability of such methodology has also been proved by a simple numerical simulation. As can be seen, there is still a large room to improve the related work in this direction. For example, to monitor and control the more complicated industrial process or the key performance system, the Bayesian network and adaptive dynamic programming methods can be suitably applied in the proposed framework to enhance the fault tolerant effects.

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