Multiple-mode switched observer-based unknown input estimation for a class of switched systems

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Abstract: This paper considers the issue of unknown input estimation for a class of uncertain switched linear systems by designing multiple-mode switched observer. Under average dwell time (ADT), the multiple-mode switched observer is proposed to estimate the states of switched system even if it subjects to unknown inputs, where the observer gain matrices are also obtained by solving a linear matrix inequality (LMI). A switching signal is designed to guarantee the exponential stability of error dynamics. Next, an approach of differential numerical solution is provided to obtain the approximate value of system output derivative. Based on the state estimation and the numerical solution of system output derivative, a kind of algebraic reconstruction method of unknown information is developed such that the unknown input of switched system can be estimated. Finally, a numerical simulation is given to illustrate the effectiveness of the proposed methods.

Key Words: Multiple-mode switching observer, Unknown input estimation, Average dwell time

1 Introduction

The state estimation issue for switched system is of great importance in automatic control and its application. The idea behind unknown input observer design is to decouple the effect of the unknown inputs from the state estimation error dynamics and to estimate asymptotically the states of the switched system even in the presence of unknown inputs. The problems of stability and control issues have been extensively focused and some satisfactory results are also obtained [1-10]. In the meantime, the state estimation problems have also been widely focused for switched linear or nonlinear systems [11-21]. For instance, based on linear matrix inequality (LMI) and common Lyapunov function techniques, Alessandri and Coletta propose a Luenberger switched observer to estimate the states of switched system in [11]. The estimation issue of the continuous states for a class of switched system is also tackled in [12]. Bejarano and Pisano discuss the state estimation problem for the switched linear system with unknown inputs in [13]. For the switched systems with unknown current mode, the state observation for a switched linear system with unknown inputs and slow switching signal is addressed in [16]. The problem of robust state estimation is also tackled for a class of switched linear systems with unknown inputs in [17]. Besides, many works on state estimation do not consider the unknown inputs [11-12,19-21]. The idea of accumulating the information from individual subsystem is employed to design observer for switched linear systems in [19]. The problems of state estimation are also investigated for switched linear systems with average dwell time switching in [20]. Under minimum dwell time switching, the state observability for a class of nonlinear switched systems is discussed in [21]. However, most of the works consider the issue of the state estimation for switched system without disturbance inputs [11-12,19-21]. In this paper, we not only consider the unknown inputs but also can simultaneously estimate both states and unknown inputs of uncertain switched system, and the methods proposed here can easily apply to the fields of fault diagnosis and fault tolerance control, especially in chemical process [22,23].

This paper investigates the designing issues of multiple-mode switched observer and unknown input estimation for a class of uncertain switched systems. First, by an algebraic decoupling technique, a multiple-mode switched observer is constructed such that the states of switched system can be estimated even if it subjects to unknown inputs, the observer gain matrices are also obtained by solving a linear matrix inequality (LMI). Meanwhile, a switching signal with average dwell time (ADT) properties is designed to guarantee the exponential stability of error dynamics. Second, in order to estimate the unknown inputs of switched system, a differential numerical solution approach is considered to approximately obtain the derivative of system output. Finally, a kind of unknown information reconstruction method is proposed to recover the unknown inputs of switched system.

The remainder of the paper is organized as follows. Section 2 presents general model and preliminaries. In section 3, a

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multiple-mode switched observer is proposed to estimate the states of switched system, and an observer-based unknown input reconstruction method is developed in section 4. In section 5, a simulation example is given to illustrate the performance of the proposed methods. Some conclusions are summarized in section 6.

2 General model and preliminaries

Consider a class of uncertain linear time-invariant switched systems with unknown inputs as follows

$$\left\{ \begin{array}{l} \dot{x} = A_{\sigma(t)}x + B_{\sigma(t)}u + D_{\sigma(t)}\eta, \\ y = Cx, \end{array} \right. \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$ are the state vector, output vector and known input vector, respectively. $\eta \in R^k$ stands for the unknown input vector. The switching signal, $\sigma(\cdot) : \mathbb{R}^+ \to \Lambda$, is assumed to be a piecewise and right-continuous constant function that determines its value in an index set $\Lambda = \{1, 2, \cdots, N\}$. Matrix quadruple $\{A_k, B_k, C, D_k\}$, for all $k \in \Lambda$, is a family of constant matrices with appropriate dimensions. For a switching sequence $t_0 < t_1 < \cdots < t_i < t_{i+1} < \cdots$, the kth subsystem is active if $\sigma(t) = k \in \Lambda$ and $t \in [t_i, t_{i+1})$, where t_0 is the initial time of switched system. Let t_i^+ and t_{i+1}^- denote the starting and ending times of the kth mode, respectively, i.e., $\sigma(t_i^+) = \sigma(t_{i+1}^-) = k$ for the ith switching. We also assume that $\mathrm{rank}C = p$ and $\mathrm{rank}(CD_k) = m$ for any $k \in \Lambda$.

Definition 1 *[3] Let $\mathrm{N}_{\sigma(t)}(\tau_1, \tau_2)$ be the number of discontinuities of the switching signal $\sigma(t)$ on the interval (τ_2, τ_1). We say $\sigma(t)$ has an average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that*

$$\mathrm{N}_{\sigma(t)}(\tau_1, \tau_2) \le \mathrm{N}_0 + \frac{\tau_1 - \tau_2}{\tau_a} \tag{2}$$

holds for all $\tau_1 \ge \tau_2 \ge 0$.

3 Multiple-mode switched observer

In order to estimate the states of switched system (1) with disturbance input, we consider the following dynamics

$$\left\{ \begin{array}{l} \dot{z} = L_k z + (I - V_k C) B_k u + U_k y, \\ \hat{x} = z + V_k y, \end{array} \right. \tag{3}$$

where V_k, L_k and U_k are the gain matrices which will be determined later and $\sigma(t) = k \in \Lambda$.

Theorem 1 *If for a scalar $\mu_2 > 1$, $(k, k') \in \Lambda \times \Lambda (k \ne k')$ and a positive definite symmetric matrix $Q_k \in \mathbb{R}^{n\times n}$, there exist matrix $W_k \in \mathbb{R}^{n\times p}$ such that*

$$\left\{ \begin{array}{l} (M_k A_k - W_k C)^T P_k + P_k (M_k A_k - W_k C) = -Q_k \\ P_k < \mu_2 P_{k'} \end{array} \right. \tag{4}$$

has a positive definite symmetric matrix solution $P_k \in \mathbb{R}^{n\times n}$, then the observer system (3) can asymptotically estimate the states of switched system (1) for any switching signal $\sigma(t)$ with a sufficiently large ADT τ_a according to

$$\tau_a > \frac{\ln \mu_2}{\mu_1}, \tag{5}$$

where $\mu_1 = \min_{k\in\Lambda}\left(\frac{\lambda_{\min}(Q_k)}{\lambda_{\max}(P_k)}\right)$, $M_k = I - V_k C$.

Proof. Define the estimation error as $\tilde{x} = x - \hat{x} = x - z - V_k y = (I - V_k C)x - z$, then we have

$$\begin{aligned} \dot{\tilde{x}} &= (I - V_k C)\dot{x} - \dot{z} \\ &= (I - V_k C)(A_k x + B_k u + D_k \eta) - L_k z \\ &\quad - (I - VC) B_k u - U_k y \\ &= (I - V_k C)(A_k x + D_k \eta) - L_k (\hat{x} - V_k y) - U_k y \\ &= [(I - V_k C) A_k - U_k C + L_k V C] x - L_k \hat{x} \\ &\quad + (I - V_k C) D_k \eta. \end{aligned} \tag{6}$$

Due to the rank assumption of $\mathrm{rank}(CD_k) = m$, we can choose gain matrix

$$V_k = D_k \left[(CD_k)^T CD_k\right]^{-1} (CD_k)^T \tag{7}$$

such that the following matrix equation

$$V_k C D_k = D_k \tag{8}$$

has a least square solution matrix V_k. We can obtain from (6) and (8) that

$$\dot{\tilde{x}} = [(I - V_k C) A_k - U_k C + L_k V_k C] x - L_k \hat{x}. \tag{9}$$

If we set matrices $M_k = I - V_k C$, $U_k = W_k + L_k V_k$ and $L_k = M_k A_k - W_k C$, then we can derive from (9) that

$$\begin{aligned} \dot{\tilde{x}} &= [M_k A_k - (L_k V_k + W_k) C + L_k V_k C] x \\ &\quad - (M_k A_k - W_k C)\hat{x} \\ &= (M_k A_k - W_k C)\tilde{x}. \end{aligned} \tag{10}$$

Consider switching Lyapunov function candidate $\Theta_k(\tilde{x}(t)) = \tilde{x}^T P_k \tilde{x}$, the derivative of Θ_k along with the error dynamic system (10) is

$$\begin{aligned} &\dot{\Theta}_k(\tilde{x}(t)) \\ &= \tilde{x}^T \left[(MA_k - W_k C)^T P_k + P_k (MA_k - W_k C)\right] \tilde{x}. \end{aligned} \tag{11}$$

Combining inequality (11) and the first equation of (4), we can obtain

$$\begin{aligned} &\dot{\Theta}_k(\tilde{x}(t)) \\ &\le \tilde{x}^T \left[(MA_k - W_k C)^T P_k + P_k (MA_k - W_k C)\right] \tilde{x} \\ &= -\tilde{x}^T Q_k \tilde{x} \le -\lambda_{\min}(Q_k)\tilde{x}^T\tilde{x} \le -\tfrac{\lambda_{\min}(Q_k)}{\lambda_{\max}(P_k)}\tilde{x}^T P_k \tilde{x} \\ &= -\tfrac{\lambda_{\min}(Q_k)}{\lambda_{\max}(P_k)}\Theta_k(\tilde{x}(t)) \le -\mu_1 \Theta_k(\tilde{x}(t)), \end{aligned}$$

where $\mu_1 = \min_{k\in\Lambda}\left(\frac{\lambda_{\min}(Q_k)}{\lambda_{\max}(P_k)}\right)$. For any $t \in [t_i, t_{i+1})$ and $\sigma(t_i) = k$, we can obtain by integrating the above differential inequality that

$$\Theta_{\sigma(t_i)}(\tilde{x}(t)) < e^{-\mu_1(t - t_i)}\Theta_{\sigma(t_i)}(\tilde{x}(t_i)). \tag{12}$$

Besides, it follows from the second inequality of (4) that $\forall\left(\sigma(t_i) = k, \sigma(t_i^-) = k'\right) \in \Lambda \times \Lambda$ and $k \ne k'$, there is

$$\begin{aligned} \Theta_k(\tilde{x}(t_i)) &= \tilde{x}^T(t_i) P_k \tilde{x}(t_i) \le \mu_2 \tilde{x}^T(t_i^-) P_{k'} \tilde{x}(t_i^-) \\ &= \mu_2 \Theta_{k'}\left((t_i^-)\right). \end{aligned} \tag{13}$$

Thus, by iterating (12) and (13) for $i = 0$ to $i = \mathrm{N}_{\sigma(t)}(t, 0)$ we can obtain that for any $t \in [t_i, t_{i+1})$

$$\begin{aligned}
\Theta_{\sigma(t)}(\tilde{x}(t)) &< \tilde{x}^{-\mu_1(t-t_i)}\Theta_{\sigma(t_i)}(\tilde{x}(t_i)) \\
&\le \mu_2 e^{-\mu_1(t-t_i)}\Theta_{\sigma(t_i^-)}\left(\tilde{x}\left(t_i^-\right)\right) \\
&= \mu_2 e^{-\mu_1(t-t_i)}\Theta_{\sigma(t_{i-1})}\left(\tilde{x}\left(t_i^-\right)\right) \\
&< \mu_2 e^{-\mu_1(t-t_i)}e^{-\mu_1(t_i-t_{i-1})}\Theta_{\sigma(t_{i-1})}(\tilde{x}(t_{i-1})) \\
&= \mu_2 e^{-\mu_1(t-t_{i-1})}\Theta_{\sigma(t_{i-1})}(\tilde{x}(t_{i-1})) \\
&\le \mu_2^2 e^{-\mu_1(t-t_{i-1})}\Theta_{\sigma(t_{i-1}^-)}\left(\tilde{x}\left(t_{i-1}^-\right)\right) \\
&= \mu_2^2 e^{-\mu_1(t-t_{i-1})}\Theta_{\sigma(t_{i-2})}\left(\tilde{x}\left(t_{i-1}^-\right)\right) \\
&< \mu_2^2 e^{-\mu_1(t-t_{i-1})}e^{-\mu_1(t_{i-1}-t_{i-2})}\Theta_{\sigma(t_{i-2})}(\tilde{x}(t_{i-2})) \\
&= \mu_2^2 e^{-\mu_1(t-t_{i-2})}\Theta_{\sigma(t_{i-2})}(\tilde{x}(t_{i-2})) \\
&\vdots \\
&< \mu_2^{\mathrm{N}_{\sigma(t)}(t,0)}e^{-\mu_1(t-t_1)}e^{-\mu_1(t_1-t_0)}\Theta_{\sigma(t_0)}(\tilde{x}(t_0)) \\
&= \mu_2^{\mathrm{N}_{\sigma(t)}(t,0)}e^{-\mu_1(t-t_0)}\Theta_{\sigma(t_0)}(\tilde{x}(t_0)).
\end{aligned} \tag{14}$$

Definition 1 means that the following inequality

$$\mathrm{N}_{\sigma(t)}(t, 0) \le \mathrm{N}_0 + \frac{t-0}{\tau_a} = \mathrm{N}_0 + \frac{t}{\tau_a} \tag{15}$$

holds. We can further derive from (14) and (15) that

$$\begin{aligned}
\Theta_{\sigma(t)}(\tilde{x}(t)) &< \mu_2^{\mathrm{N}_0+\frac{t}{\tau_a}}e^{-\mu_1 t}\Theta_{\sigma(t_0)}(\tilde{x}(t_0)) \\
&= \mu_2^{\mathrm{N}_0}e^{-\left(\mu_1-\frac{\ln\mu_2}{\tau_a}\right)t}\Theta_{\sigma(t_0)}(\tilde{x}(t_0))
\end{aligned} \tag{16}$$

since $t_0 = 0$ is the initial time of switched system. If we denote $\alpha = \mu_2^{\mathrm{N}_0}$ and $\beta = \mu_1 - \frac{\ln\mu_2}{\tau_a}$, one can obtain that $\alpha > 0$ and $\beta > 0$ since $\mathrm{N}_0 > 0$, $\mu_2 > 1$ and (5). So, the inequality (16) implies that $\Theta_{\sigma(t)}(\tilde{x}(t)) < \alpha e^{-\beta t}\Theta_{\sigma(t_0)}(\tilde{x}(t_0))$, $\forall t \ge 0$. . Therefore, we conclude that if (4) and (5) hold, the error dynamics (6) is globally uniformly exponentially stable. That is to say, the observer system (3) with ADT property (5) can asymptotically estimate the states of switched system (1). □

4 Switched observer-based fault unknown input estimation

In the previous section, a multiple-mode switched observer is proposed to estimate the states of switched system (1), where the influence of the disturbance input η on switching system is eliminated by choosing matrix V_k with the form of (7). Next, we will develop a kind of algebraic method to estimate the unknown input η.

Based on the switched system (1), we can obtain the derivative of measurement output y as follows

$$\dot{y} = C\dot{x} = CA_k x + CB_k u + CD_k\eta \tag{17}$$

for any $\sigma(t) = k \in \Lambda$ and $t \in [t_i, t_{i+1})$. Furthermore, the matrix (CD_k) is full column rank since we have assumed $\mathrm{rank}(CD_k) = m$. So, the left-invertible matrix can be denoted as

$$(CD_k)^{\dagger} = \left((CD_k)^T(CD_k)\right)(CD_k)^T. \tag{18}$$

Considering Eqs. (17) and (18) together yields

$$\eta = (CD_k)^{\dagger}\dot{y} - (CD_k)^{\dagger}(CA_k x + CB_k u). \tag{19}$$

If the derivative estimation of measurement output y is known in (19), and the switched observer (3) does not contain any information of unknown input, it is natural to get that a reconstruction method can be developed as

$$\hat{\eta} = (CD_k)^{\dagger}\hat{\dot{y}} - (CD_k)^{\dagger}(CA_k\hat{x} + CB_k u) \tag{20}$$

where $\hat{\eta}$ is the estimation of η, $\hat{\dot{y}}$ is the estimation of $\dot{y}$, and the state estimation $\hat{x}$ is provided by the switched observer (3).

Theorem 2 *Under Eqs. (4) and (5) hold, the information reconstruction for unknown input η given by (20) is an asymptotical estimation.*

Proof. Subtracting Eq. (19) from Eq. (20) leads to

$$\tilde{\eta} = (CD_k)^{\dagger}\tilde{\dot{y}} - (CD_k)^{\dagger}CA_k\tilde{x}$$

where $\tilde{\eta} = \eta - \hat{\eta}$, $\tilde{\dot{y}} = \dot{y} - \hat{\dot{y}}$ and $\tilde{x} = x - \hat{x}$. Considering the facts that the state asymptotical estimation has been implemented via switched observer (3) and that $\hat{\dot{y}}$ is the estimation of $\dot{y}$, we have $\lim\limits_{t\to\infty}\tilde{\eta}(t) = 0$. □

It should be noticed that the derivative information of measurement output y is assumed to be known. In fact, there needs to obtain the derivative estimation , i.e., $\hat{\dot{y}}$ in (20), such that the estimation of the unknown input η can be finished by (20). Next, based on the differential numerical solution methods, we provide a kind of numerical algorithm to get $\hat{\dot{y}}$. Let y_κ $(\kappa = 1, 2, \cdots, p)$ denote the κth component of measurement output y, and suppose the time interval we concern with is $[0, t)$. Then, we can obtain $N+1$ time points $t'_r = t'_0 + rh$ by dividing the time interval into N parts which is equivalent each other, where $r = 0, 1, 2, \cdots, N$ and the step size $h = \frac{t-0}{N}$. For the κth component of measurement output y, its approximate value of derivative, $\left.\frac{dy_\kappa(t)}{dt}\right|_{t={t'}_r}$, is denoted as $\xi_{\kappa,r}$, then by backward difference quotient and forward difference quotient, respectively, the approximate values of derivative of t'_0 and t'_N can be calculated as

$$\begin{cases}
\xi_{\kappa,0} = \frac{y_\kappa({t'}_1)-y_\kappa({t'}_0)}{h}, \\
\xi_{\kappa,N} = \frac{y_\kappa({t'}_N)-y_\kappa({t'}_{N-1})}{h}.
\end{cases} \tag{21}$$

In addition, for $r = 1, 2, \cdots, N-1$, the approximate derivative values of according time points can be computed by central difference quotient as follows

$$\xi_{\kappa,r} = \frac{y_\kappa({t'}_{r+1}) - y_\kappa({t'}_{r-1})}{2h}. \tag{22}$$

Thus, we can derive from (21) and (22) that $\xi_\kappa = \begin{bmatrix} \xi_{\kappa,0} & \xi_{\kappa,1} & \cdots & \xi_{\kappa,N} \end{bmatrix}^T$ is the differential numerical solution of $\dot{y}_\kappa$ $(\kappa = 1, 2, \cdots, p)$ on time interval $[0, t)$. So, we can further obtain that

$$\xi = \begin{bmatrix} \xi_1^T & \xi_2^T & \cdots & \xi_p^T \end{bmatrix}^T \tag{23}$$

is the differential numerical solution of $\dot{y} = \begin{bmatrix} \dot{y}_1^T & \dot{y}_2^T & \cdots & \dot{y}_p^T \end{bmatrix}^T$. The Eq. (23) also means that ξ is the approximate value of $\dot{y}$, that is to say, we can directly replace $\hat{\dot{y}}$ with ξ in (20). So, we can derive the following corollary from (20), (23) and Theorem 2.

Corollary 1 *Under Eqs. (4) and (5) hold, if the differential numerical solution of $\dot{y}$ is obtained as (23), the information reconstruction given by*

$$\hat{\eta} = (CD_k)^{\dagger}\xi - (CD_k)^{\dagger}\left(CA_k\hat{x} + CB_k u\right) \qquad (24)$$

is the estimation of the unknown input η.

Remark 1 *For any $(k, k') \in \Lambda \times \Lambda\,(k \neq k')$ and $\mu_2 > 1$, the solution matrices P_k and W_k in (4) can be obtained by solving the following linear matrix inequality*

$$\begin{cases} (P_k M_k A_k + \Pi_k C)^T + P_k M_k A_k + \Pi_k C < 0 \\ P_k - \mu_2 P_{k'} < 0 \end{cases} \qquad (25)$$

and $Q_k = -(P_k M_k A_k + \Pi_k C)^T - P_k M_k A_k - \Pi_k C$, $W_k = -P_k^{-1}\Pi_k$.

5 Simulation

In this section, a numerical example is given to show the effectiveness of the proposed methods. We assume that the switched system is with two subsystems, and the matrices of switched system (1) are defined as

$$A_1 = \begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & -3 & 1 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & -3 & 2 & -5 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 2 & -3 & 0 & 5 \\ 0 & -2 & -5 & 1 \\ -6 & 0 & 1 & -2 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. The unknown input and known input vectors are set as $\eta = 3.2\sin(3.8t) + 1.2\cos(3.6t)$ and $u = 2.2\cos(2.5t)$, respectively. Based on (7), we can obtain matrix

$$V_1 = V_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

which makes equation (8) be satisfied. One can further obtain from $M_k = I - V_k C$ that

$$M_1 = M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

If the parameter is selected as $\mu_2 = 1.2$, we can then derive from (25) by LMI toolbox that

$$Q_1 = \begin{bmatrix} 34.7074 & 0 & 0 & 0 \\ 0 & 34.7074 & 0 & 0 \\ 0 & 0 & 36.3868 & 0.5777 \\ 0 & 0 & 0.5777 & 62.968 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 34.7074 & 0 & 0 & 0 \\ 0 & 34.7074 & 0 & 0 \\ 0 & 0 & 103.3879 & -3.7718 \\ 0 & 0 & -3.7718 & 82.9302 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 55.9772 & 0 & 0.6699 & 0.1011 \\ 0 & 55.9933 & 0 & 0 \\ 0.6699 & 0 & 10.2721 & 1.0109 \\ 0.1011 & 0 & 1.0109 & 6.2968 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 55.978 & 0 & 0.6125 & 0.0969 \\ 0 & 55.9933 & 0 & 0 \\ 0.6125 & 0 & 10.5531 & 1.0715 \\ 0.0969 & 0 & 1.0715 & 6.0767 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} -0.8177 & -0.5619 \\ 0.5613 & 0.3099 \\ 11.9476 & 0.0363 \\ -1.8408 & 0.0032 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} -4.6867 & 2.0585 \\ -1.0576 & 0.3099 \\ -0.303 & -2.0608 \\ -7.9624 & 2.9938 \end{bmatrix}.$$

We can further derived from $U_k = W_k + L_k V_k$ and $L_k = M_k A_k - W_k C$ that

$$L_1 = \begin{bmatrix} -0.1823 & 0.5619 & 2 & 0 \\ -0.5613 & -0.3099 & 0 & 0 \\ -10.9476 & -0.0363 & -2 & 0 \\ 1.8408 & -0.0032 & 1 & -5 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.3133 & -1.0585 & 0 & 0 \\ 1.0576 & -0.3099 & 0 & 0 \\ 0.303 & 0.0608 & -5 & 1 \\ -0.0376 & 0.0062 & 1 & -7 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} -0.8177 & 0 \\ 0.5613 & 0 \\ 11.9476 & 0 \\ -1.8408 & -5 \end{bmatrix}, U_2 = \begin{bmatrix} -4.6867 & 1 \\ -1.0576 & 0 \\ -0.303 & -1 \\ -7.9624 & -4 \end{bmatrix}.$$

Moreover, it is easy to calculate that $\mu_1 = \min_{k\in\{1,2\}}\left(\frac{\lambda_{\min}(Q_k)}{\lambda_{\max}(P_k)}\right) = 0.6198$ and $\frac{\ln\mu_2}{\mu_1} = 0.2941$. Thus, let us generate a possible switching signal with ADT time $\tau_a = 1$ according to (6), as shown in Fig.1. In the simulation, the initial values are set as $x(0) = \begin{bmatrix} 2.3 & 1.5 & -6.1 & 4.8 \end{bmatrix}^T$ and $z(0) = \begin{bmatrix} -3.6 & -2.5 & 1.8 & -2.1 \end{bmatrix}^T$, respectively. Then, the state response error curves between switched system (1) and switched observer (3) are depicted in Fig. 2, from which we can see that the estimation performance is perfect even if the switched system is subject to disturbance inputs.

In order to reconstruct the unknown input η, we first derive the differential numerical solution, $\xi = \begin{bmatrix} \xi_1^T & \xi_2^T & \cdots & \xi_p^T \end{bmatrix}^T$, of $\dot{y} = \begin{bmatrix} \dot{y}_1^T & \dot{y}_2^T & \cdots & \dot{y}_p^T \end{bmatrix}^T$ from (21) and (22). Next, the estimation of the unknown input can directly be obtained by (24) in Corollary 1 since the state estimation $\hat{x}$ has been provided from the switched observer (3), and the reconstruction results are illustrated in Fig. 3.

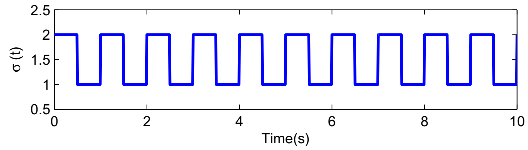

Figure 1: Switching signal $\sigma(t)$ with $\tau_a = 1$

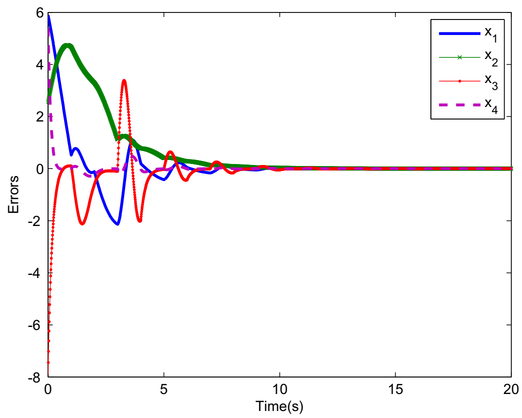

Figure 2: State estimation error curves of x

6 Conclusion

This paper considers the unknown input estimation issues of switched system by designing multiple-mode switched observer. A robust multiple-mode switched observer is constructed to estimate the states of switched system under the ADT switching even if the switched system subjects to unknown inputs. The switching signal with ADT is also designed, which together with the LMI conditions given in this paper guarantees the exponential convergence of state estimation error dynamics. Next, based on the estimated states from the designed switched observer, a kind of unknown input estimation method is developed by providing the differential numerical solution of the system output derivative.

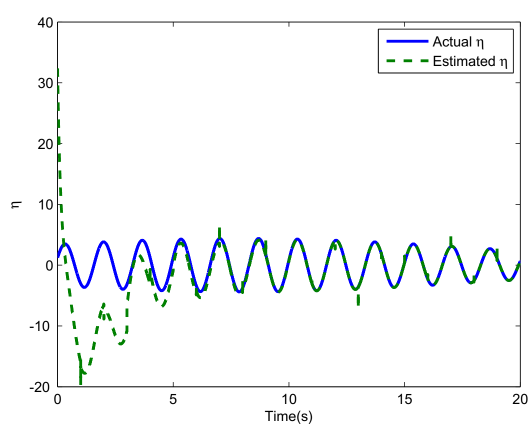

Figure 3: Unknown input estimation

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