Observer-Based Fuzzy Tracking Control for Switched Stochastic Nonlinear Systems

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Abstract: This paper deals with the problem of adaptive fuzzy output feedback tracking control for a class of switched stochastic nonlinear lower-triangular systems without the measurements of the states. First, a reduced-order state observer is introduced to estimate the unmeasurable states of the switched stochastic system. Then fuzzy logic systems (FLSs) are employed to approximate the unknown nonlinearities and the adaptive backstepping technique is used to construct an output feedback controller. The proposed controller guarantees that all the closed-loop signals are the semi-globally uniformly ultimately boundedness (SGUUB), while the tracking error converges to a small neighborhood of the origin.

Key Words: Switched stochastic systems, Adaptive fuzzy control, Reduced-order observer

1 INTRODUCTION

Over the past few decades, the problem of adaptive backstepping-based tracking control of uncertain non-switched nonlinear systems in lower-triangular form has received increasing attention using intelligent control techniques, such as FLSs or Neural Networks (NNs) to parameterize the unknown nonlinearities [1–3]. Correspondingly, many novel adaptive fuzzy control design approaches have been proposed for uncertain non-switched stochastic nonlinear systems in lower-triangular form. It should be noted that the aforementioned control approaches all assume that the states of the systems are available for measurement in the control design. In many practical plants, however, state variables are often rarely fully measured, which motivates observer-based control schemes.

The last decade has also witnessed a considerable growth of interest in switched systems [4–7]. The main reason for the wide interest in the study on such systems is that numerous real world models can be described as switched systems, such as in power systems, computer-controlled systems, transportation systems and control, communication networks and many other fields. As an important class of switched systems, the study on switched stochastic nonlinear systems is of great significant and challenging. Just recently, for a special class of switched stochastic nonlinear systems, the global stabilization problem of switched stochastic nonlinear systems in lower triangular form have been investigated systematically by backstepping in [8].

This paper aims to solve the adaptive fuzzy output feedback tracking control problem for a class of switched stochastic nonlinear systems in lower-triangular form. In the control process, FLSs are first utilized to approximate the unknown nonlinear functions, and a reduced-order state observer is designed and thus via it the immeasurable states are obtained. By adaptive backstepping technique, an adaptive fuzzy output feedback controller is constructed.

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2 PROBLEM FORMULATION

Consider the following switched stochastic nonlinear systems described by

$$\begin{aligned} dx_i &= (x_{i+1} + f_{\sigma(t),i}(\bar{x}_i))dt + g_{\sigma(t),i}(y)d\omega, \\ dx_n &= (u + f_{\sigma(t),n}(x))dt + g_{\sigma(t),n}(y)d\omega, \\ y &= x_1, i = 1, 2, \cdots, n-1, \end{aligned} \tag{1}$$

where $\bar{x}_i = (x_1, x_2, \cdots, x_i)^T \in R^i, i = 1, 2, \cdots, n, x = \bar{x}_n$ is the system state, $u \in R$ is the plant input and $y \in R$ is the system output, $x_2, \cdots, x_n$ are unmeasurable. $\sigma(t) : [0, \infty) \to \mathcal{S} = \{1, 2, \cdots, s\}$ is the switching signal. For $\forall i = 1, 2, \cdots, n$, $k = 1, 2, \cdots, s$, $f_{k,i}(\cdot)$ are unknown nonlinear smooth functions. Assume that $g_{k,i}(y) = y\varphi_{k,i}(y)$, and $\varphi_{k,i}(y)$ are known smooth functions satisfying local Lipschitz condition, $i = 1, 2, \cdots, n$, $k = 1, 2, \cdots, s$. ω is a standard Wiener process satisfying $E\{d\omega(t)\} = 0$.

The main objective of this paper is to design a fuzzy output feedback tracking controller for systems (1) such that the system output $y(t)$ tracks a desired trajectory $y_d(t)$, while ensuring that all closed-loop signals remain bounded in probability.

Assumption 1. The desired trajectory $y_d(t)$ and its time derivatives up to the n-th order are continuous and bounded.

Assumption 2. For $2 \le i \le n$ and $k \in \mathcal{S}$, there exist unknown continuous functions $f_{k,i1}(y)$ such that $|f_{k,i}(\bar{x}_i)| \le f_{k,i1}(y)$.

Lemma 1. For $x \in R, y \in R$, and $p \ge 1$ is a constant, then

$$|x + y|^p \le 2^{p-1}(x^p + y^p). \tag{2}$$

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Remark 1. In this study, FLSs will be introduced to approximate the unknown nonlinear system functions. The related theory of FLSs can be found in [1–3, 9].

3 CONTROL DESIGN

3.1 Reduced-Order Observer Design

We consider the following reduced-order observer

$$\begin{aligned}\dot{\hat{x}}_i &= \hat{x}_{i+1} + k_{i+1}y - k_i(\hat{x}_1 + k_1 y),\\ i &= 1, \cdots, n-2,\\ \dot{\hat{x}}_{n-1} &= u - k_{n-1}(\hat{x}_1 + k_1 y),\end{aligned} \quad (3)$$

where $\hat{x}_i, i = 1, \cdots, n-1$, are the estimates of x_i with $\tilde{x}_i = x_i - \hat{x}_{i-1} - k_{i-1}y, i = 2, \cdots, n, k_i, i = 1, \cdots, n-1$ are constant design parameters to be determined, then combining (1) with (3), we can obtain the observer errors $\tilde{x} = (\tilde{x}_2, \cdots, \tilde{x}_n)$, which satisfy

$$d\tilde{x} = (A\tilde{x} + F)dt + Gd\omega, \quad (4)$$

where

$$A = \begin{bmatrix} -k_1 & & \\ \vdots & I_{n-2} & \\ -k_{n-1} & 0 \quad \cdots & 0 \end{bmatrix},$$

$$F = \begin{bmatrix} f_{\sigma(t),2} - k_1 f_{\sigma(t),1} \\ \vdots \\ f_{\sigma(t),n} - k_{n-1} f_{\sigma(t),1} \end{bmatrix},$$

$$G = \begin{bmatrix} g_{\sigma(t),2} - k_1 g_{\sigma(t),1} \\ \vdots \\ g_{\sigma(t),n} - k_{n-1} g_{\sigma(t),1} \end{bmatrix}.$$

Furthermore, choosing appropriate positive constants $k_1, \cdots, k_{n-1}$ such that A is a Hurwitz matrix, hence there exists a positive definite matrix P satisfying

$$A^T P + PA = -I. \quad (5)$$

Then the switched system can be rewritten as

$$\begin{aligned} d\tilde{x} &= (A\tilde{x} + F)dt + Gd\omega,\\ dy &= (\hat{x}_1 + k_1 y + \tilde{x}_2)dt + f_{\sigma(t),1}(x_1)dt\\ &\quad + g_{\sigma(t),1}(y)d\omega,\\ d\hat{x}_i &= (\hat{x}_{i+1} + k_{i+1}y - k_i(\hat{x}_1 + k_1 y))dt,\\ i &= 1, \cdots, n-2,\\ d\hat{x}_{n-1} &= (u - k_{n-1}(\hat{x}_1 + k_1 y))dt. \end{aligned} \quad (6)$$

3.2 Reduced-Order Observer Design

For system (6), the feasible virtual control signals and the actual control law will be constructed via the backstepping-based adaptive fuzzy output feedback control approach in this subsection.

For system (6), we choose the following coordinate transformation:

$$\begin{aligned} z_1 &= y - y_d,\\ z_2 &= \hat{x}_1 - \alpha_1(y, y_d, \dot{y}_d, \hat{\theta}),\\ z_{i+1} &= \hat{x}_i - \alpha_i(y, \bar{y}_d^{(i)}, \hat{\theta}, \bar{\hat{x}}_{i-1}), \end{aligned} \quad (7)$$

where α_i are the virtual control signals, and $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \cdots, y_d^{(i)}]$, $y_d^{(i)}$ being the ith order time derivative of y_d, $i = 2, \cdots, n-1$.

For any $k \in \mathcal{S}$, it follows from (6) and (7) that

$$\begin{aligned} dz_1 &= (\hat{x}_1 + k_1 y + \tilde{x}_2 + f_{k,1} - \dot{y}_d)dt + g_{k,1}d\omega,\\ dz_2 &= (\hat{x}_2 + k_2 y - k_1(\hat{x}_1 + k_1 y) - \frac{\partial \alpha_1}{\partial y}(\hat{x}_1\\ &\quad + k_1 y + \tilde{x}_2 + f_{k,1}) - \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial y^2} g_{k,1} g_{k,1}^T\\ &\quad - \sum_{j=0}^{1} \frac{\partial \alpha_1}{\partial y_d^{(j)}} y_d^{(j+1)} - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}})dt\\ &\quad - \frac{\partial \alpha_1}{\partial y} g_{k,1} d\omega, \end{aligned}$$

$$\begin{aligned} dz_i &= (\hat{x}_i + k_i y - k_{i-1}(\hat{x}_1 + k_1 y) - \frac{\partial \alpha_{i-1}}{\partial y}(\hat{x}_1\\ &\quad + k_1 y + \tilde{x}_2 + f_{k,1}) - \frac{1}{2}\frac{\partial^2 \alpha_{i-1}}{\partial y^2} g_{k,1} g_{k,1}^T\\ &\quad - \sum_{j=0}^{i-1} \frac{\partial \alpha_1}{\partial y_d^{(j)}} y_d^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\\ &\quad - \sum_{j=1}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j}\dot{\hat{x}}_j)dt - \frac{\partial \alpha_{i-1}}{\partial y} g_{k,1} d\omega,\\ dz_n &= (u - k_{n-1}(\hat{x}_1 + k_1 y)\\ &\quad - \frac{\partial \alpha_{n-1}}{\partial y}(\hat{x}_1 + k_1 y + \tilde{x}_2 + f_{k,1})\\ &\quad - \frac{1}{2}\frac{\partial^2 \alpha_{n-1}}{\partial y^2} g_{k,1} g_{k,1}^T - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)}\\ &\quad - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{j=1}^{n-2} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_j}\dot{\hat{x}}_j)dt\\ &\quad - \frac{\partial \alpha_{n-1}}{\partial y} g_{k,1} d\omega. \end{aligned} \quad (8)$$

We choose the following Lyapunov-Krasovskii functional for system (6)

$$V = V_1 + V_2$$

with

$$V_1 = \frac{b}{2}(\tilde{x}^T P \tilde{x})^2, V_2 = \sum_{i=1}^{n} \frac{1}{4} z_i^4, \quad (9)$$

where the matrix P satisfies inequality (5).

Firstly, we consider the time derivative of the Lyapunov function candidate V_1, and we can have

$$\begin{aligned} \dot{V}_1 &= -b\tilde{x}^T P \tilde{x}|\tilde{x}|^2 + 2b(\tilde{x}^T P \tilde{x})(\tilde{x}^T P F)\\ &\quad + bTr\{G^T(2P\tilde{x}\tilde{x}^T P + \tilde{x}^T P \tilde{x} P)G\}. \end{aligned} \quad (10)$$

Then, according to Assumption 2, Lemma 1 and Young's inequality, we have

$$\begin{aligned} &2b(\tilde{x}^T P \tilde{x})(\tilde{x}^T P F)\\ &\le 2b\lambda_{\max}(P)\|P\|_N |\tilde{x}|^3 |F|\\ &\le \frac{3}{2} b\lambda_{\max}(P)\|P\|_N c_1^{\frac{4}{3}} |\tilde{x}|^4 \end{aligned}$$

$$
+\frac{4b(n-1)\lambda_{\max}(P)\|P\|_N}{c_1^4}
\cdot\sum_{i=2}^{n}(|f_{k,i}(\bar{x}_i)|^4+k_{i-1}^4|f_{k,1}(x_1)|^4)
\leq \frac{3}{2}b\lambda_{\max}(P)\|P\|_N c_1^{\frac{4}{3}}|\tilde{x}|^4
+\frac{4b(n-1)\lambda_{\max}(P)\|P\|_N}{c_1^4}
\cdot\sum_{i=2}^{n}(f_{k,i1}^4(y)+k_{i-1}^4 f_{k,1}^4(y))
= A_1 c_1^{\frac{4}{3}}|\tilde{x}|^4+\frac{8(n-1)A_1}{3c_1^4}
\cdot\sum_{i=2}^{n}(f_{k,i1}^4(y)+k_{i-1}^4 f_{k,1}^4(y)),
$$

$$
bTr\{G^T(2P\tilde{x}\tilde{x}^TP+\tilde{x}^TP\tilde{x}P)G\}
\leq 2b\|G^TP\tilde{x}\|_F^2+b\lambda_{\max}^2(P)|\tilde{x}|^2\|G\|_F^2
\leq (2b\|P\|_N^2+b\lambda_{\max}^2(P))|\tilde{x}|^2\|G\|_F^2
\leq (2b\|P\|_N^2+b\lambda_{\max}^2(P))(|\tilde{x}|^4+\frac{1}{4}\|G\|_F^4)
\leq (2b\|P\|_N^2+b\lambda_{\max}^2(P))|\tilde{x}|^4
+\frac{(2b\|P\|_N^2+2b\lambda_{\max}^2(P))(n-1)}{4}
\cdot\sum_{i=2}^{n}(|g_{k,i}|^4+k_{i-1}^4|g_{k,1}|^4)
\leq (2b\|P\|_N^2+b\lambda_{\max}^2(P))|\tilde{x}|^4
+\frac{(2b\|P\|_N^2+2b\lambda_{\max}^2(P))(n-1)}{4}
\cdot\sum_{i=2}^{n}y^4(\varphi_{k,i}^4+k_{i-1}^4\varphi_{k,1}^4)
= A_2|\tilde{x}|^4+\frac{(n-1)A_2}{4}
\cdot\sum_{i=2}^{n}y^4(\varphi_{k,i}^4+k_{i-1}^4\varphi_{k,1}^4), \quad (11)
$$

where $A_1 = \frac{3}{2}b\lambda_{\max}(P)\|P\|_N, A_2 = 2b\|P\|_N^2 + b\lambda_{\max}^2(P), c_1$ is a positive parameter.

Taking (11) into account, (10) can be written as

$$
\dot{V}_1 \leq -(b\lambda_{\min}(P)-A_1c_1^{\frac{4}{3}}-A_2)|\tilde{x}|^4
+\frac{8(n-1)A_1}{3c_1^4}\sum_{i=2}^{n}(f_{k,i1}^4+k_{i-1}^4 f_{k,1}^4)
+\frac{(n-1)A_2}{4}\sum_{i=2}^{n}y^4(\varphi_{k,i}^4+k_{i-1}^4\varphi_{k,1}^4). \quad (12)
$$

Next, we will consider the Lyapunov function V_2 step by step. Make $V_2=\sum_{i=1}^{n}V_{2i}, V_{2i}=\frac{1}{4}z_i^4$.

Under tha action of Itô differentiation rule and (8), we obtain

$$
\mathcal{L}V_{21} = z_1^3(z_2+\alpha_1+k_1y+\tilde{x}_2+f_{k,1}-\dot{y}_d)
+\frac{3}{2}z_1^2g_{k,1}g_{k,1}^T. \quad (13)
$$

From Young's inequality and the definition of $\tilde{x}$, we have

$$
z_1^3z_2 \leq \frac{3}{4}z_1^4+\frac{1}{4}z_2^4,
$$
$$
z_1^3\tilde{x}_2 \leq \frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1^4+\frac{1}{4\epsilon_1^4}\tilde{x}_2^4 \leq \frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1^4+\frac{1}{4\epsilon_1^4}|\tilde{x}|^4,
$$
$$
z_1^3f_{k,1}(x_1) \leq \frac{3}{4}l_1^{\frac{4}{3}}z_1^4+\frac{1}{4l_1^4}f_{k,1}^4(y),
$$
$$
\frac{3}{2}z_1^2g_{k,1}g_{k,1}^T \leq \frac{3}{4c_2}z_1^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4. \quad (14)
$$

Then substituting (14) into (13) yield that

$$
\mathcal{L}V_{21} \leq z_1^3(\alpha_1+\frac{3}{4}z_1+k_1y+\frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1+\frac{3}{4}l_1^{\frac{4}{3}}z_1
+\frac{3}{4c_2}z_1-\dot{y}_d)+\frac{1}{4}z_2^4+\frac{1}{4\epsilon_1^4}|\tilde{x}|^4
+\frac{1}{4l_1^4}f_{k,1}^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4. \quad (15)
$$

Furthermore, according to (8) we can get

$$
\mathcal{L}V_{22} = z_2^3(z_3+\alpha_2+k_2y-k_1(\hat{x}_1+k_1y)
-\frac{\partial\alpha_1}{\partial y}(\hat{x}_1+k_1y+\tilde{x}_2+f_{k,1})
-\frac{1}{2}\frac{\partial^2\alpha_1}{\partial y^2}g_{k,1}g_{k,1}^T-\sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)}
-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}})+\frac{3}{2}z_2^2(\frac{\partial\alpha_1}{\partial y})^2g_{k,1}g_{k,1}^T. \quad (16)
$$

Then, similar to (14), one obtains

$$
z_2^3z_3 \leq \frac{3}{4}z_2^4+\frac{1}{4}z_3^4,
$$
$$
-z_2^3\frac{\partial\alpha_1}{\partial y}\tilde{x}_2 \leq \frac{3}{4}\epsilon_2^{\frac{4}{3}}(1+(\frac{\partial\alpha_1}{\partial y})^2)^{\frac{2}{3}}z_2^4+\frac{1}{4\epsilon_2^4}|\tilde{x}|^4,
$$
$$
-z_2^3\frac{\partial\alpha_1}{\partial y}f_{k,1}(x_1) \leq \frac{3}{4}l_2^{\frac{4}{3}}(1+(\frac{\partial\alpha_1}{\partial y})^2)^{\frac{2}{3}}z_2^4
+\frac{1}{4l_2^4}f_{k,1}^4(y),
$$
$$
-\frac{1}{2}z_2^3\frac{\partial^2\alpha_1}{\partial y^2}g_{k,1}g_{k,1}^T \leq \frac{1}{4\kappa_2}(\frac{\partial^2\alpha_1}{\partial y^2})^2z_2^6
+\frac{\kappa_2}{4}y^4\varphi_{k,1}^4,
$$
$$
\frac{3}{2}z_2^2(\frac{\partial\alpha_1}{\partial y})^2g_{k,1}g_{k,1}^T \leq \frac{3}{4c_2}(\frac{\partial\alpha_1}{\partial y})^4z_2^4
+\frac{3c_2}{4}y^4\varphi_{k,1}^4. \quad (17)
$$

From (16) and (17), we get

$$
\mathcal{L}V_{22} \leq z_2^3(\alpha_2+\frac{3}{4}z_2+k_2y-k_1(\hat{x}_1+k_1y)
-\frac{\partial\alpha_1}{\partial y}(\hat{x}_1+k_1y)+(1+(\frac{\partial\alpha_1}{\partial y})^2)^{\frac{2}{3}}
\cdot z_2(\frac{3}{4}\epsilon_2^{\frac{4}{3}}+\frac{3}{4}l_2^{\frac{4}{3}})+\frac{1}{4\kappa_2}(\frac{\partial^2\alpha_1}{\partial y^2})^2z_2^3
-\sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)}-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}
+\frac{3}{4c_2}(\frac{\partial\alpha_1}{\partial y})^4z_2)+\frac{1}{4}z_3^4+\frac{1}{4\epsilon_2^4}|\tilde{x}|^4
$$

$$+\frac{1}{4l_2^4}f_{k,1}^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4$$
$$=z_2^3(\alpha_2+\Delta_2)+\frac{1}{4}z_3^4+\frac{1}{4\epsilon_2^4}|\tilde{x}|^4$$
$$+\frac{1}{4l_2^4}f_{k,1}^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4, \tag{18}$$

where

$$\Delta_2=\frac{3}{4}z_2+k_2y-k_1(\hat{x}_1+k_1y)-\frac{\partial\alpha_1}{\partial y}(\hat{x}_1+k_1y)+(1+(\frac{\partial\alpha_1}{\partial y})^2)^{\frac{2}{3}}z_2(\frac{3}{4}\epsilon_2^{\frac{4}{3}}+\frac{3}{4}l_2^{\frac{4}{3}})+\frac{1}{4\kappa_2}(\frac{\partial^2\alpha_1}{\partial y^2})^2z_2^3-\sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)}-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}+\frac{3}{4c_2}(\frac{\partial\alpha_1}{\partial y})^4z_2. \tag{19}$$

Similarly, for $i=3,\cdots,n-1, V_{2i}=\frac{1}{4}z_i^4$, we can infer

$$\mathcal{L}V_{2i}=z_i^3(z_{i+1}+\alpha_i+k_iy-k_{i-1}(\hat{x}_1+k_1y)-\frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_1+k_1y+\tilde{x}_2+f_{k,1})-\frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_{k,1}g_{k,1}^T-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j)+\frac{3}{2}z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2g_{k,1}g_{k,1}^T, \tag{20}$$

then, using a similar way in (17) yields that

$$z_i^3z_{i+1}\le\frac{3}{4}z_i^4+\frac{1}{4}z_{i+1}^4,$$
$$-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}\tilde{x}_2\le\frac{3}{4}\epsilon_i^{\frac{4}{3}}(1+(\frac{\partial\alpha_{i-1}}{\partial y})^2)^{\frac{2}{3}}z_i^4+\frac{1}{4\epsilon_i^4}|\tilde{x}|^4,$$
$$-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}f_{k,1}(x_1)\le\frac{3}{4}l_i^{\frac{4}{3}}(1+(\frac{\partial\alpha_{i-1}}{\partial y})^2)^{\frac{2}{3}}z_i^4+\frac{1}{4l_i^4}f_{k,1}^4(y),$$
$$-\frac{1}{2}z_i^3\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_{k,1}g_{k,1}^T\le\frac{1}{4\kappa_i}(\frac{\partial^2\alpha_{i-1}}{\partial y^2})^2z_i^6+\frac{\kappa_i}{4}y^4\varphi_{k,1}^4,$$
$$\frac{3}{2}z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2)g_{k,1}g_{k,1}^T\le\frac{3}{4c_2}(\frac{\partial\alpha_{i-1}}{\partial y})^4z_i^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4. \tag{21}$$

Further, we have

$$\mathcal{L}V_{2i}\le z_i^3(\alpha_i+\Delta_i)+\frac{1}{4}z_{i+1}^4+\frac{1}{4\epsilon_i^4}|\tilde{x}|^4+\frac{1}{4l_i^4}f_{k,1}^4+\frac{\kappa_i}{4}y^4\varphi_{k,1}^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4, \tag{22}$$

where

$$\Delta_i=\frac{3}{4}z_i+k_iy-k_{i-1}(\hat{x}_1+k_1y)-\frac{\partial\alpha_{i-1}}{\partial y}\cdot(\hat{x}_1+k_1y)+(1+(\frac{\partial\alpha_{i-1}}{\partial y})^2)^{\frac{2}{3}}z_i(\frac{3}{4}\epsilon_i^{\frac{4}{3}}+\frac{3}{4}l_i^{\frac{4}{3}})+\frac{1}{4\kappa_i}(\frac{\partial^2\alpha_{i-1}}{\partial y^2})^2z_i^3-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j+\frac{3}{4c_2}(\frac{\partial\alpha_{i-1}}{\partial y})^4z_i. \tag{23}$$

When $i=n$, differentiating V_{2n} produces that

$$\mathcal{L}V_{2n}=z_n^3(g_{v_\tau}v+d(v)-k_{n-1}(\hat{x}_1+k_1y)-\frac{\partial\alpha_{n-1}}{\partial y}(\hat{x}_1+k_1y+\tilde{x}_2+f_{k,1})-\frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial y^2}g_{k,1}g_{k,1}^T-\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}-\sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j-\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}})+\frac{3}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial y})^2g_{k,1}g_{k,1}^T, \tag{24}$$

similar to the procedures as the former steps, we have

$$-z_n^3\frac{\partial\alpha_{n-1}}{\partial y}\tilde{x}_2\le\frac{3}{4}\epsilon_n^{\frac{4}{3}}(1+(\frac{\partial\alpha_{n-1}}{\partial y})^2)^{\frac{2}{3}}z_n^4+\frac{1}{4\epsilon_n^4}|\tilde{x}|^4,$$
$$-z_n^3\frac{\partial\alpha_{n-1}}{\partial y}f_{k,1}(x_1)\le\frac{3}{4}l_n^{\frac{4}{3}}(1+(\frac{\partial\alpha_{n-1}}{\partial y})^2)^{\frac{2}{3}}z_n^4+\frac{1}{4l_n^4}f_{k,1}^4(y),$$
$$-\frac{1}{2}z_n^3\frac{\partial^2\alpha_{n-1}}{\partial y^2}g_{k,1}g_{k,1}^T\le\frac{1}{4\kappa_n}(\frac{\partial^2\alpha_{n-1}}{\partial y^2})^2z_n^6+\frac{\kappa_n}{4}y^4\varphi_{k,1}^4,$$
$$\frac{3}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial y})^2g_{k,1}g_{k,1}^T\le\frac{3}{4c_2}(\frac{\partial\alpha_{n-1}}{\partial y})^4z_n^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4. \tag{25}$$

Combining (24) with (25), we obtain

$$\mathcal{L}V_{2n}\le z_n^3(u+\Delta_n)+\frac{1}{4\epsilon_n^4}|\tilde{x}|^4+\frac{1}{4l_n^4}f_{k,1}^4+\frac{\kappa_n}{4}y^4\varphi_{k,1}^4+\frac{3c_2}{4}y^4\varphi_{k,1}^4, \tag{26}$$

where

$$\Delta_n=-k_{n-1}(\hat{x}_1+k_1y)-\frac{\partial\alpha_{n-1}}{\partial y}(\hat{x}_1+k_1y)+(1+(\frac{\partial\alpha_{n-1}}{\partial y})^2)^{\frac{2}{3}}z_n(\frac{3}{4}\epsilon_n^{\frac{4}{3}}+\frac{3}{4}l_n^{\frac{4}{3}})$$

$$+\frac{1}{4\kappa_n}(\frac{\partial^2\alpha_{n-1}}{\partial y^2})^2 z_n^3 - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}$$
$$-\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j$$
$$+\frac{3}{4c_2}(\frac{\partial\alpha_{n-1}}{\partial y})^4 z_n. \quad (27)$$

Therefore, we obtain

$$\begin{aligned}\mathcal{L}V_2 \le\ & z_1^3(\alpha_1 + \frac{3}{4}z_1 + k_1 y + \frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1 + \frac{3}{4}l_1^{\frac{4}{3}}z_1 \\ & + \frac{3}{4c_2}z_1 - \dot{y}_d) + z_2^3(\alpha_2 + \frac{1}{4}z_2 + \Delta_2) \\ & + \sum_{i=3}^{n-1} z_i^3(\alpha_i + \frac{1}{4}z_i + \Delta_i) + z_n^3(u \\ & + \frac{1}{4}z_n + \Delta_n) + \sum_{i=1}^{n}\frac{1}{4\epsilon_i^4}|\tilde{x}|^4 \\ & + \sum_{i=1}^{n}\frac{1}{4l_i^4}f_{k,1}^4 + \frac{3nc_2}{4}y^4\varphi_{k,1}^4 \\ & + \sum_{i=2}^{n}\frac{\kappa_i}{4}y^4\varphi_{k,1}^4. \end{aligned} \quad (28)$$

According to (12) and (28), we can get that

$$\begin{aligned}\mathcal{L}V \le\ & z_1^3(\alpha_1 + \frac{3}{4}z_1 + k_1 y + \frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1 + \frac{3}{4}l_1^{\frac{4}{3}}z_1 \\ & + \frac{3}{4c_2}z_1 - \dot{y}_d) + z_2^3(\alpha_2 + \frac{1}{4}z_2 + \Delta_2) \\ & + \sum_{i=3}^{n-1} z_i^3(\alpha_i + \frac{1}{4}z_i + \Delta_i) + z_n^3(u \\ & + \frac{1}{4}z_n + \Delta_n) - c_0|\tilde{x}|^4 + H(y), \end{aligned} \quad (29)$$

where

$$c_0 = b\lambda_{\min}(P) - \sum_{i=1}^{n}\frac{1}{4\epsilon_i^4} - A_1 c_1^{\frac{4}{3}} - A_2, \quad (30)$$

and

$$\begin{aligned}H(y) =\ & \frac{8(n-1)A_1}{3c_1^4}\sum_{i=2}^{n}(f_{k,i1}^4 + k_{i-1}^4 f_{k,1}^4) \\ & + \frac{(n-1)A_2}{4}\sum_{i=2}^{n} y^4(\varphi_{k,i}^4 + k_{i-1}^4\varphi_{k,1}^4) \\ & + \sum_{i=1}^{n}\frac{1}{4l_i^4}f_{k,1}^4 + \frac{3nc_2}{4}y^4\varphi_{k,1}^4 \\ & + \sum_{i=2}^{n}\frac{\kappa_i}{4}y^4\varphi_{k,1}^4. \end{aligned} \quad (31)$$

To complete the design process, we define

$$\begin{aligned}\bar{f}_k(Z) =\ & \frac{3}{4}z_1 + k_1 y + \frac{3}{4}\epsilon_1^{\frac{4}{3}}z_1 + \frac{3}{4}l_1^{\frac{4}{3}}z_1 + \frac{3}{4c_2}z_1 \\ & - \dot{y}_d + \frac{16}{z_1^3}\tanh^2(\frac{z_1}{\nu})H(y) \end{aligned} \quad (32)$$

with $\nu > 0$ is a design parameter, $Z = [y, y_d, \dot{y}_d] \in \Sigma_Z \subset R^3$, and Σ_Z being some known compact set in R^3. The reason is the term $\frac{H(y)}{z_1^3}$ is discontinuous at $z_1 = 0$, which means it cannot be approximated by the FLSs directly. To compensate this term, a hyperbolic tangent function $\tanh(\frac{z_1}{\nu})$ is introduced, since that $\lim_{z_1\to 0}\frac{16}{z_1^3}\tanh^2(\frac{z_1}{\nu})H(y)$ exists.
Substituting (32) into (29), we have

$$\begin{aligned}\mathcal{L}V \le\ & z_1^3(\alpha_1 + \bar{f}_k(Z)) + z_2^3(\alpha_2 + \frac{1}{4}z_2 + \Delta_2) \\ & + \sum_{i=3}^{n-1} z_i^3(\alpha_i + \frac{1}{4}z_i + \Delta_i) + z_n^3(u \\ & + \frac{1}{4}z_n + \Delta_n) - c_0|\tilde{x}|^4 + (1 - \\ & 16\tanh^2(\frac{z_1}{\nu}))H(y). \end{aligned} \quad (33)$$

By virtue of Lemma 2, FLSs $W_k^T S_k$ are utilized to approximate $\bar{f}_k(Z)$ such that $\bar{f}_k(Z)$ can be expressed as

$$\bar{f}_k(Z) = W_k^T S_k(Z) + \delta(Z), |\delta(Z)| \le \varepsilon, \quad (34)$$

where $\delta(Z)$ is the approximation error and ε is a given positive design parameter, $k \in \mathcal{S}$.
According to the triangular inequality, we can get

$$\begin{aligned} z_1^3(W_k^T S_k(Z) + \delta(Z)) \le\ & \frac{\vartheta\theta}{2\lambda^2}S_k^T(Z)S_k(Z)z_1^6 \\ & + \frac{\lambda^2}{2\vartheta} + \frac{3}{4}z_1^4 + \frac{\varepsilon^4}{4}, \end{aligned} \quad (35)$$

where $\theta = \|W_{\max}\|^2$ is an unknown constant, ϑ and λ are positive design parameters.
Taking (34) and (35) into account, (33) can be written as

$$\begin{aligned}\mathcal{L}V \le\ & z_1^3(\alpha_1 + \frac{3}{4}z_1 + \frac{\vartheta\theta}{2\lambda^2}S_k^T(Z)S_k(Z)z_1^3) \\ & + z_2^3(\alpha_2 + \frac{1}{4}z_2 + \Delta_2) + \sum_{i=3}^{n-1} z_i^3(\alpha_i \\ & + \frac{1}{4}z_i + \Delta_i) + z_n^3(u + \frac{1}{4}z_n + \Delta_n) \\ & - c_0|\tilde{x}|^4 + \frac{\lambda^2}{2\vartheta} + \frac{\varepsilon^4}{4} \\ & + (1 - 16\tanh^2(\frac{z_1}{\nu}))H(y). \end{aligned} \quad (36)$$

According to (36), we construct the adaptive fuzzy output feedback tracking control laws and the parameter adaptive law as follows

$$\alpha_1 = -\xi_1 z_1 - \frac{3}{4}z_1 - \frac{\vartheta\hat{\theta}}{2\lambda^2}S_k^T(Z)S_k(Z)z_1^3, \quad (37)$$
$$\alpha_2 = -\xi_2 z_2 - \frac{1}{4}z_2 - \Delta_2, \quad (38)$$
$$\alpha_i = -\xi_i z_i - \frac{1}{4}z_i - \Delta_i, i = 3, \cdots, n-1, \quad (39)$$
$$u = -(\xi_n z_n + z_n + \Delta_n), \quad (40)$$
$$\dot{\hat{\theta}} = \vartheta S_k^T(Z)S_k(Z)z_1^6 - \rho\hat{\theta}, \quad (41)$$

where ξ_j, ϑ and ρ are positive design parameters. $\hat{\theta}$ is the estimation of $\theta = \|W_{\max}\|^2$ with $\|W_{\max}\|^2 \geq \|W_k\|^2 (k = 1, 2, \cdots, s)$.
Based on (54), we can get the following inequalities

$$z_n^3 g_{v_\tau} v \leq -\xi_n z_n^4 - z_n^4 - \Delta_n z_n^3, \quad (42)$$

$$z_n^3 d(v) \leq \frac{3}{4} z_n^4 + \frac{1}{4}\mathfrak{D}^4. \quad (43)$$

Finally, taking (37)-(53) and the above inequalities into account, we can get

$$\begin{aligned}\mathcal{L}V \leq & -\sum_{j=1}^{n} \xi_j z_j^4 + \frac{\vartheta\tilde{\theta}}{2\lambda^2} S_k^T(Z) S_k(Z) z_1^6 \\ & -c_0|\tilde{x}|^4 + \frac{\lambda^2}{2\vartheta} + \frac{\varepsilon^4}{4} + \frac{1}{4}\mathfrak{D}^4 \\ & +(1 - 16\tanh^2(\frac{z_1}{\nu}))H(y). \end{aligned} \quad (44)$$

Consider a stochastic Lyapunov function candidate as

$$V^* = V + \frac{1}{4\lambda^2}\tilde{\theta}^2, \quad (45)$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error.
Then, the following inequality can be obtained

$$\begin{aligned}\mathcal{L}V^* \leq & -\sum_{j=1}^{n} \xi_j z_j^4 + \frac{\rho}{2\lambda^2}\tilde{\theta}\hat{\theta} - c_0|\tilde{x}|^4 \\ & +\frac{\lambda^2}{2\vartheta} + \frac{\varepsilon^4}{4} + \frac{1}{4}\mathfrak{D}^4 \\ & +(1 - 16\tanh^2(\frac{z_1}{\nu}))H(y), \end{aligned} \quad (46)$$

where parameters b, c_1 and $\epsilon_i, i = 1, \cdots, n$, are chosen appropriately such that $b\lambda_{\min}(P) - \sum_{i=1}^{n} \frac{1}{4\epsilon_i^4} - A_1 c_1^{\frac{4}{3}} - A_2 \geq c_0 > 0$.
For the term $\tilde{\theta}\hat{\theta}$, we have

$$\tilde{\theta}\hat{\theta} \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2. \quad (47)$$

Substituting (47) into (46), we can obtain

$$\begin{aligned}\mathcal{L}V^* \leq & -\sum_{j=1}^{n} \xi_j z_j^4 - \frac{\rho}{4\lambda^2}\tilde{\theta}^2 - c_0|\tilde{x}|^4 + b_0 \\ & +(1 - 16\tanh^2(\frac{z_1}{\nu}))H(y), \end{aligned} \quad (48)$$

where $b_0 = \frac{\lambda^2}{2\vartheta} + \frac{\varepsilon^4}{4} + \frac{1}{4}\mathfrak{D}^4 + \frac{\rho}{4\lambda^2}\theta^2$.
Theorem 1. Under Assumptions 1-3, consider the closed-loop system consisting of the plant (1), the reduced-order state observer (3), the controllers (37)-(40) and the adaptive law (41). Suppose that for $1 \leq i \leq n, k \in \mathcal{S}$, all the unknown nonlinear functions can be approximated by FLS in the sense that the approximation error ε is bounded in probability. Then, with the bounded initial condition $\hat{\theta}(0) \geq 0$, the following properties are guaranteed under arbitrary switchings.
(i) All the signals of the closed-loop system are bounded in probability.
(ii) The error signal $\varphi(t)$, in the mean square sense, belong to the following compact set Ω defined by

$$\Omega := \{\varphi \in R^{2n} | \Phi \leq \tilde{\Phi}\} \quad (49)$$

and eventually converge to the neighborhood of the origin Ω_1 defined by

$$\Omega_1 := \{\varphi \in R^{2n} | \Phi \leq \tilde{\Phi}_1\}, \quad (50)$$

where $\varphi = (z_1, \cdots, z_n, \tilde{x}_2, \cdots, \tilde{x}_n)^T, \Phi = \frac{1}{t}E\{\int_0^t |\varphi(\tau)|^4 d\tau\}$.
Proof: The proof is omitted here since it is very similar to the one in [9].

4 CONCLUSIONS

In this paper, the problem of adaptive fuzzy output feedback tracking control is addressed for a class of switched stochastic nonlinear lower-triangular systems in the presence of input saturation under arbitrary switchings. Based on a well-designed reduced-order state observer and an appropriate stochastic Lyapunov function, we employed FLSs combined with the backstepping technique to construct an adaptive fuzzy output feedback controller.

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