

Design of PID and ADRC Based Quadrotor Helicopter Control System

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Abstract: In this paper, an active disturbance rejection control (ADRC) and PID based control method is proposed to perform position and attitude control of a small unmanned quadrotor helicopter model. For the double closed-loop control system, ADRC is applied in the inner loop control, and an extended state observer (ESO) is designed to observe and compensate the internal uncertainties and the external disturbances in real time. PID is used in the outer loop control. Simulation experiments are performed to evaluate the efficiency of proposed control system.

Key Words: Quadrotor, Active disturbance rejection control, PID, Double closed-loop control

1 INTRODUCTION

In recent years, unmanned quadrotor helicopter has shown an extensive application prospect in both military and civil fields. Now, quadrotor helicopter can accomplish several kinds of dangerous and difficult missions such as search and rescue missions, surveillance, inspection, mapping and aerial cinematography [1].

Quadrotor helicopter is simple in structure but complicated in control. The quadrotor model we consider is an under-actuated system with four inputs and six outputs, which is also highly coupled and nonlinear. Besides, due to the light weight of quadrotor, it can be easily influenced by external disturbance such as wind speed. To perform the position and attitude control of considered quadrotor model, several control methods have been applied to quadrotor system. Sliding mode control is efficient for systems with large uncertainties, time varying properties and bounded external disturbances [2]. But the switching function creates a chattering phenomenon which is a high frequency oscillation that may cause control system unstable. Robust adaptive control is applied to quadrotor in article [3] to perform control of systems with unknown external disturbances and internal uncertainties. The control performance is good but the design of controller is complicated.

Active disturbance rejection control (ADRC) is proposed by Han [4] to deal with nonlinear systems with mixed uncertainty and external disturbance. ADRC estimates the states and the disturbances through the extended state observer (ESO), and compensates for the total

disturbances in real time [6]. The controller is proved to be efficient and of strong robustness. Thus ADRC is fit in the control of quadrotor helicopter because of its superiority in solving control problems of nonlinear models with uncertainties and strong disturbances.

The remaining parts of the paper are organized as follows. In section 2, a typical quadrotor model is introduced. In section 3, an ADRC based control system is designed for a typical quadrotor model. In section 4, simulation results are provided. Finally, concluding remarks are given in section 5.

2 QUADROTOR HELICOPTER MODEL

The considered quadrotor helicopter dynamic model [7] is a six DOF system with four control inputs, which is a nonlinear, under-actuated system shown in Eq. (1),

$$\begin{cases} \ddot{x} = U_1(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) - K_1\dot{x} / m \\ \ddot{y} = U_1(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) - K_2\dot{y} / m \\ \ddot{z} = U_1(\cos\phi \cos\psi) - g - K_3\dot{z} / m \\ \ddot{\phi} = U_2 - IK_5\dot{\phi} / I_2 \\ \ddot{\theta} = U_3 - IK_4\dot{\theta} / I_1 \\ \ddot{\psi} = U_4 - K_6\dot{\psi} / I_3 \end{cases} \quad (1)$$

where the vector $[x, y, z]$ denotes the position of quadrotor, and the states ϕ , θ , ψ represent the roll, the pitch and the yaw respectively. $I_i (i = 1, 2, 3)$ represent the moments of inertia and $K_i (i = 1, \dots, 6)$ represent the drag coefficients. g denotes gravity.

This model is inaccurate in a way because of the neglect of the gyroscopic effect and the frictional resistance [8]. Meanwhile, the quadrotor helicopter can be influenced by

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the wind easily, so there are both internal uncertainties and external disturbances.

3 PID/ADRC CONTROL SYSTEM DESIGN

Divide the quadrotor model Eq. (1) into two parts:

$$\begin{cases} \ddot{x} = U_1(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) - K_1\dot{x} / m \\ \ddot{y} = U_1(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) - K_2\dot{y} / m \\ \ddot{z} = U_1(\cos\phi \cos\psi) - g - K_3\dot{z} / m \end{cases} \quad (2)$$

$$\begin{cases} \ddot{\phi} = U_2 - IK_5\dot{\phi} / I_2 \\ \ddot{\theta} = U_3 - IK_4\dot{\theta} / I_1 \\ \ddot{\psi} = U_4 - K_6\dot{\psi} / I_3 \end{cases} \quad (3)$$

Eq. (2) denotes the position loop of the system while Eq. (3) denotes the attitude loop of the system. Thus a double closed loop control system based on PID and ADRC is proposed for the quadrotor control problem. The inner loop is the attitude loop using ADRC control scheme, and the outer loop is the position loop using PID control algorithm. The structure of proposed control system is shown in Fig. 1.

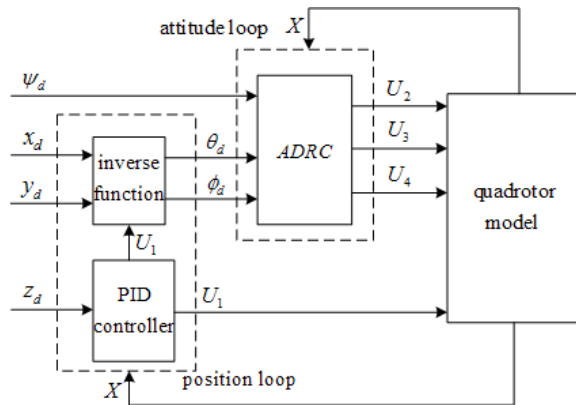


Fig.1 Structure of PID/ADRC control system

3.1 ADRC Design

ADRC consists of tracking differentiator (TD), extended state observer (ESO) and nonlinear state error feedback (NLSEF). The structure of ADRC is shown in Fig. 2.

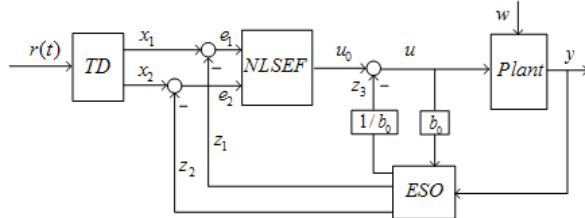


Fig.2 Structure of ADRC

Based on the feedback error, ADRC scheme does not rely on a specific mathematical model [9]. As shown in Fig. 2, the input signal is $r(t)$, which is the desired value of the state. The tracking differentiator is employed to arrange the transition process where x_1 traces $r(t)$ smoothly and x_2 achieves an approximate differential of $r(t)$. The

extended state observer is designed to observe the states. As shown in Fig. 2, z_1, z_2 are the observations of x_1, x_2 , while z_3 is the estimation of total disturbances and used for the compensation. The control input u is achieved by the nonlinear state error feedback control law which is a combination of several specific nonlinear error functions. Extended state observer is the core of ADRC and ensures the control system strong robustness and disturbance rejection ability.

Taking the row loop model as an example, let $\phi = x_1$, so the second-order system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = U_2 - IK_5x_2 / I_2 \end{cases} \quad (4)$$

3.1.1 Tracking Differentiator

Consider ϕ_d as the input signal, so the tracking error is defined as $e = x_1 - \phi_d$. According to article [6], the tracking differentiator is designed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -l \cdot \text{sign}(e + \frac{x_2|x_2|}{2l}) \end{cases} \quad (5)$$

l is the acceleration limit. Eq. (5) provides x_1 a fast tracking of input signal ϕ_d and its approximate differential under the limitation of l . This ensures the ADRC system a smooth input signal and its differential, and avoids the overshoot of output signal caused by the drastic changes in reference signal, enhancing the robustness of the control system.

3.1.2 Extended State Observer

Extended state observer is the core of ADRC and enables quadrotor control system the ability of disturbances observation and rejection [10].

A third-order ESO is designed for the second-order roll loop model as follows:

$$\begin{cases} e = z_1 - \phi \\ \dot{z}_1 = z_2 - \beta_1 \cdot e \\ \dot{z}_2 = z_3 - \beta_2 \cdot \text{fal}(e, \alpha_1, \delta) + b_0 u \\ \dot{z}_3 = -\beta_3 \cdot \text{fal}(e, \alpha_2, \delta) \end{cases} \quad (6)$$

$$\text{fal}(e, \alpha, \delta) = \begin{cases} e / \delta^{1-\alpha}, & |e| \leq \delta \\ |e|^\alpha \cdot \text{sign}(e), & |e| > \delta \end{cases} \quad (7)$$

where z_1, z_2 denote the observed value of x_1, x_2 ; z_3 is the extended state, and provides the observed value of total disturbances; e is the observation error. $\beta_1, \beta_2, \beta_3$ are observer gains and $\alpha_1, \alpha_2, \delta$ are the parameters of the controller.

3.1.3 Nonlinear State Error Feedback

ADRC employs a nonlinear combination of tracking errors, which performs more effective in control than the linear combination form, to obtain the control variable. As shown in the formula.

$$\begin{cases} e_1 = x_1 - z_1 \\ e_2 = x_2 - z_2 \\ u_0 = k_1 \cdot \text{fal}(e_1, \alpha_3, \delta) + k_2 \cdot \text{fal}(e_2, \alpha_4, \delta) \\ u = u_0 - z_3 / b_0 \end{cases}, \quad (8)$$

where e_1, e_2 are observation errors between x_1, x_2 given by tracking differentiator and z_1, z_2 given by extended state observer. The control variable u_0 is given by a combination of nonlinear functions $\text{fal}(\cdot)$ shown in Eq. (7). k_1, k_2 are adjustable parameters.

z_3 is the estimated value of total disturbances. Taking z_3 / b_0 as the compensation value of total disturbances, the final control variable u can be achieved as shown in Eq. (8). So ADRC accomplishes the observation and compensation of disturbances in real time.

3.2 PID Control Design

Position loop consists of altitude loop shown in Eq. (9) and lateral position loop shown in Eq. (10).

$$\ddot{z} = U_1 (\cos \phi \cos \psi) - g - K_3 \dot{z} / m \quad (9)$$

$$\begin{cases} \ddot{x} = U_1 (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) - K_1 \dot{x} / m \\ \ddot{y} = U_1 (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) - K_2 \dot{y} / m \end{cases} \quad (10)$$

It is clear that altitude loop is independent of lateral control loop, which means the control variable U_1 plays a role in the altitude control exclusively. The lateral loop, however, is coupled with the inner attitude loop through the attitude angles. The attitude loop control variables U_2, U_3 take charge of lateral position control instead of U_1 , and the reference values of roll angle and pitch angle are calculated by the inverse function shown in Fig. 1, illustrating the quadrotor system highly coupled and under-actuated.

PID control algorithm is adopted in the outer loop control system, which is shown in the formula.

$$\ddot{z} = K_{pz} (z_d - z) + K_{iz} \int (z_d - z) dt + K_{dz} (\dot{z}_d - \dot{z}) \quad (11)$$

$$\begin{cases} \ddot{x} = K_{px} (x_d - x) + K_{ix} \int (x_d - x) dt + K_{dx} (\dot{x}_d - \dot{x}) \\ \ddot{y} = K_{py} (y_d - y) + K_{iy} \int (y_d - y) dt + K_{dy} (\dot{y}_d - \dot{y}) \end{cases} \quad (12)$$

Combining Eq. (9) and Eq. (11) gives rise to:

$$U_1 = \frac{K_{pz} (z_d - z) + K_{dz} (\dot{z}_d - \dot{z}) + g + K_3 \dot{z} / m}{\cos \phi \cos \psi} \quad (13)$$

The inverse function combines two equations in Eq. (10), giving rise to the reference value as follows:

$$\theta_d = \arcsin \left(\frac{\left(\ddot{x} + \frac{K_1 \dot{x}}{m} - \sin \phi \sin \psi \right) U_1}{\cos \phi \cos \psi} \right) \quad (14)$$

$$\phi_d = \arcsin \left(\frac{\left(\left(\ddot{x} + \frac{K_1 \dot{x}}{m} \right) \sin \psi - \left(\ddot{y} + \frac{K_2 \dot{y}}{m} \right) \cos \psi \right) U_1}{U_1} \right) \quad (15)$$

4 SIMULATION RESULTS

Simulation researches of quadrotor PID/ADRC control system are carried on using MATLAB. The parameters of the adopted quadrotor model are shown as follows:

$$\begin{aligned} K_1 = K_2 = K_3 = 0.010, \quad K_4 = K_5 = K_6 = 0.012, \\ I_1 = I_2 = 1.25, \quad I_3 = 2.5, \quad m = 2\text{kg}, \quad l = 0.2\text{m}, \\ g = 9.8\text{m/s}^2. \end{aligned}$$

4.1 Simulation without Disturbances and Uncertainties

Sliding mode control is widely used in quadrotor control, and is also an efficient control method of strong robustness. To illustrate the efficiency of proposed control method, comparative studies are conducted between sliding mode control system and PID/ADRC control system.

Assume the initial position coordinates of the quadrotor to be $[0 \ 0 \ 0]$, and the desired hovering position to be $[4 \ 3 \ 6]$. Simulation experiment demands that the roll and the pitch converge to 0 and the reference value of yaw be 45° . The external disturbances and internal uncertainties are not considered in this experiment.

The main ADRC controller parameters are shown as follows:

$$r = 1.5, \quad \beta_1 = 30, \quad \beta_2 = 300, \quad \beta_3 = 1000, \quad \alpha_1 = 0.5, \\ \alpha_2 = 0.25, \quad b_0 = 2, \quad k_1 = 6, \quad k_2 = 8.$$

A sliding mode controller shown in Eq. (16) is proposed to quadrotor model to conduct comparative studies.

$$U_1 = \frac{[\varepsilon_1 \text{sgn}(S_z) + \alpha_1 S_z + k_1 \dot{e}_z + \ddot{z}_d + g + K_3 \dot{z} / m + e_z]}{\cos \phi \cos \psi}$$

$$U_4 = \varepsilon_4 \text{sgn}(S_\psi) + \alpha_4 S_\psi + k_4 \dot{e}_\psi + \ddot{\psi}_d + K_6 \dot{\psi} / I_3 + e_\psi \quad (16)$$

$$\begin{bmatrix} U_2 \\ U_3 \end{bmatrix} = - \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}_3} \right)^{-1} (\mathbf{R} + M \text{sgn}(\mathbf{S}) + \lambda \mathbf{S})$$

where, S_z, S_ψ and \mathbf{S} are linear sliding surface. \mathbf{R}, \mathbf{f} and \mathbf{x} are shown as follows:

$$\mathbf{R} = c_1 \mathbf{x}_2 + c_2 \mathbf{f}_1 + c_3 \left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} \mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_3} \mathbf{x}_4 \right) + \frac{d}{dt} \left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} \right) \mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} \dot{\mathbf{f}}_1 + \frac{d}{dt} \left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_3} \right) \mathbf{x}_4 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_3} \mathbf{f}_2 - c_1 \dot{\mathbf{x}}_{1d} - c_2 \dot{\mathbf{x}}_{2d}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix}, \quad \mathbf{x}_1 = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} \theta \\ \phi \end{bmatrix},$$

$$\mathbf{f}_1 = U_1 \begin{bmatrix} \cos \psi_d & \sin \psi_d \\ \sin \psi_d & -\cos \psi_d \end{bmatrix} \begin{bmatrix} \sin \theta \cos \phi \\ \sin \phi \end{bmatrix} + \begin{bmatrix} -K_1 \dot{x} / m \\ -K_2 \dot{y} / m \end{bmatrix},$$

$$\mathbf{f}_2 = \begin{bmatrix} -IK_4 \dot{\theta} / I_1 \\ -IK_5 \dot{\phi} / I_2 \end{bmatrix},$$

$$\mathbf{S} = c_1 \mathbf{e}_1 + c_2 \mathbf{e}_2 + c_3 \mathbf{e}_3 + \mathbf{e}_4.$$

The main sliding mode controller parameters are given as follows:

$$\alpha_1 = 0.5, \varepsilon_1 = 0.5, k_1 = 5, \quad \alpha_4 = 1, \varepsilon_4 = 2, k_4 = 5, \quad c_1 = 40, \quad c_2 = 60, \quad c_3 = 25, \quad M = 10.$$

Fig. 3 and Fig. 4 show the results of position coordinates and attitude angles using different control methods. As seen in Fig. 3, the position coordinates x , y , z converge rapidly to their corresponding desired value in both methods. As shown in Fig. 4, The pitch and roll angles converge to 0 smoothly after a short period of time and the range is within $\pm 20^\circ$. Thus, both control methods ensure that the quadrotor hovers at desired point rapidly with steady attitude. It is clear that the convergence rate of PID/ADRC control is faster than sliding mode control, though the fluctuation ranges of pitch and roll angles are larger than sliding mode control. But on the whole, PID/ADRC performs more excellent although sliding mode control is effective enough for the quadrotor control.

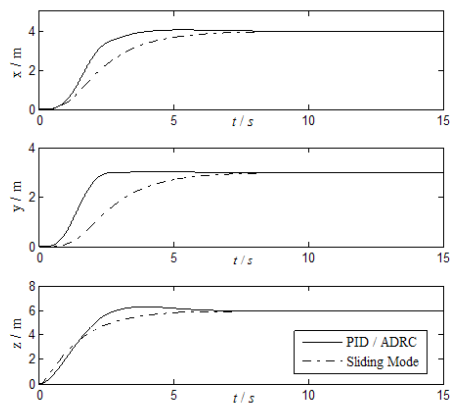


Fig. 3. Stabilization of position.

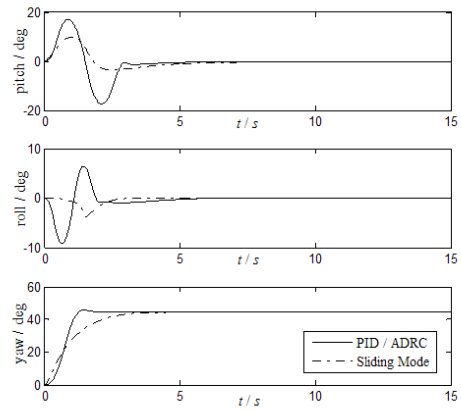


Fig. 4. Stabilization of attitude angles.

4.2 Simulation with Disturbances and Uncertainties

In order to illustrate the robustness and disturbance rejection performance of PID/ADRC control system, external disturbances and internal uncertainties are added in the simulation research conducted in 4.1. The inertia parameters have 5% uncertainties, considered as $\Delta I_i = 0.05 I_i (i=1, \dots, 6)$. The external disturbances added to the attitude angles consist of a white gaussian noise signal from the initial time and a unit step disturbance signal added at the time $t = 10s$.

The simulation results are shown in Fig. 5 and Fig.6. According to Fig. 5, the position coordinates can still converge to reference value rapidly and smoothly. According to Fig. 6, there are slight concussions in attitude angles at the time $t = 10s$ due to the unit step disturbance signal, but the attitude angles converge to desired value after a short period of time adjusting. So the ADRC controller can ensure satisfactory performance and smooth trajectory in quadrotor system with external disturbances and internal uncertainties.

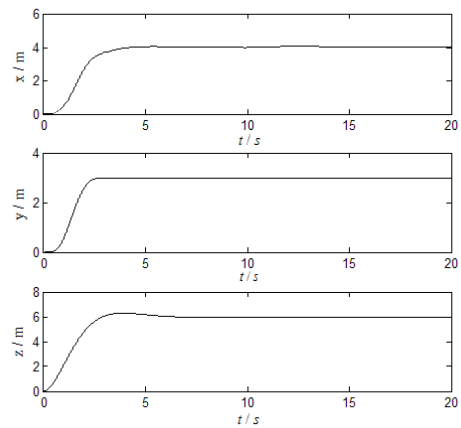


Fig. 5. Stabilization of position with disturbances and uncertainties.

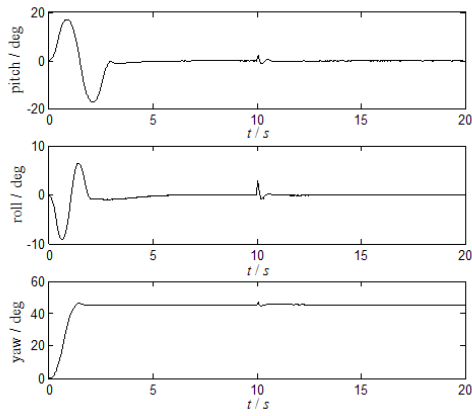


Fig. 6. Stabilization of attitude angles with disturbances and uncertainties.

ADRC is the core of the proposed control system. Added simulations are performed to analyze the performance of ADRC, and illustrate its ability of disturbance observation and rejection. The applied disturbances and uncertainties are consistent with that applied in the previous simulation experiments.

Fig. 7 shows the input and output signals of tracking differentiator in the yaw loop. As seen in the figure, x_1 smoothes and tracks the yaw reference signal which is a fixed value and x_2 is the differential of x_1 . Fig. 8 shows the smooth tracking and the differential of the roll trajectory generated by the outer loop. Fig. 9 shows the states observation performance of extended state observer. Fig. 10 shows the disturbance compensation conducted by ESO, which illustrates that the white gaussian noise signal added at the initial time, the unit step disturbance signal added at the time $t=10s$ and the model uncertainties have been compensated effectively.

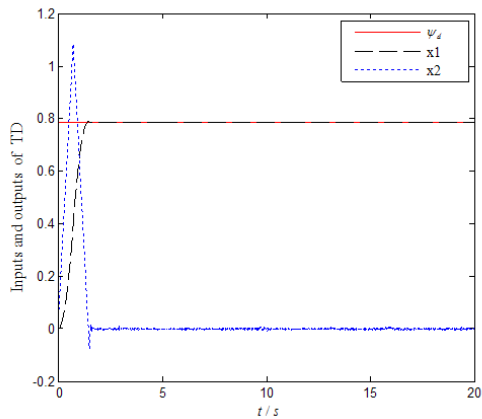


Fig. 7. Input and output signals of tracking differentiator in the yaw loop.

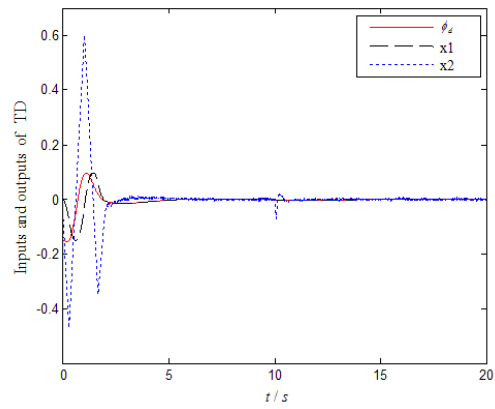


Fig. 8. Input and output signals of tracking differentiator in the roll loop.

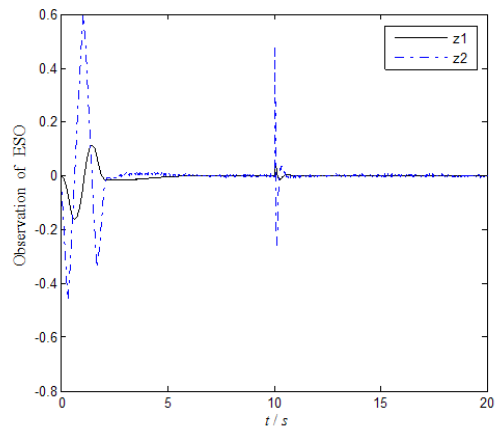


Fig. 9. States observation of ESO.

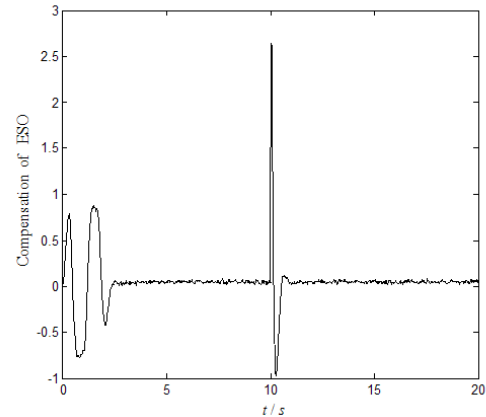


Fig. 10. Disturbance compensation of ESO.

5 CONCLUSION

In this paper, in order to solve the control problem of quadrotor system with disturbances and model uncertainties, a PID/ADRC based double closed-loop control system is proposed. An extended state observer is designed to observe the states and total disturbances and compensate the disturbances as well. Therefore, ADRC ensures the system strong robustness and disturbance

rejection ability. Firstly a comparative simulation study is conducted to show the effectiveness of ADRC. Then simulation researches with disturbances and uncertainties are conducted to illustrate the robustness and excellent performance in disturbance rejection of proposed ADRC based control system, proving that ADRC is effective in quadrotor control.

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