# All Parameters Adaptive Fractional Order PI/PD Type Iterative Learning Control

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**Abstract:** In this paper, a novel strategy is applied to the design of all parameters adaptive fractional order PI/PD type iterative learning control scheme. The current result of order adaptive fractional order iterative learning control guarantees the convergence of fractional order PI/PD ones. To our best knowledge, the flat phase technique is first combined with fractional order iterative learning control schemes that improves the convergence speed for both linear fractional order systems with known structure, and black box systems with known input/output data, where the fractional order impulse response invariant discretization method is applied. A number of numerical simulations show that the designed fractional order PI/PD type iterative learning controller guarantees the fact of convergence and the convergence speed simultaneously which implies the efficiency of the above discussed concepts.

**Key Words:** Fractional calculus; Iterative learning control; Flat phase; Impulse response invariant discretization method.

## 1 INTRODUCTION

The formal concept of iterative learning control (ILC) was published in 1978 by Uchiyama. Later in 1984 it gained wide attention when Arimoto et al presented a systematic iterative learning control algorithm in English [1]. ILC belongs to the scheme of intelligent control which is an efficient approach for improving the transient performance of systems that operate repetitively over a fixed time interval [1, 2]. The aim of ILC is to obtain the input sequence from one trial to the next trial properly so that the output can approach the desired trajectory perfectly. In recent years, various ILC schemes have made significant breakthroughs in linear or nonlinear control systems due to its many advantages. A number of illustrated interesting results can be found in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The definition of fractional calculus was presented more than 300 years ago. In 1695, LHospital wrote to Leibniz regarding the idea of non-integer order derivative, and no substantive answers were received. In the following centuries, fractional calculus developed very slowly until the first book that was published in 1974. So far, the fractional calculus has been applied to quite a few fields such as mechanics, geoscience, biology, physics, engineering, aerography, etc [16, 17, 18].

Fractional order iterative learning control (FOILC) is the combination of ILC and fractional calculus, which can date from 2001 [19]. In[19], the author presented $D^{\alpha}$ type iterative learning control algorithm and analyzed the convergence conditions in frequency domain. [20, 21, 22] pay their main attention to the $PD^{\alpha}$ type ILC updating laws. In recent years, various novel ideas of FOILC are proposed, particularly in the application side, the tuning and auto-tuning rules, the robustness and so on [23], it still can't stop the FOILC becoming a popular topic [24, 25, 26, 27, 28, 29].

This paper extends the FOILC scheme to the perfect control of both fractional order and black box systems by using the flat phase technique and impulse response invariant discretization method. The rest of this paper is organized as follows: in Section 2, some preliminaries are introduced; in Section 3, the convergence condition is discussed; in Section 4, a novel tuning method and its main results are presented; in Section 5, the parameters of fractional order $PI^{\lambda}/PD^{\lambda}$ controller are tuned due to the above discussions; in Section 6, some numerical simulations are provided to validate the concepts; in Section 7, conclusions and main innovations are summarized.

## 2 PRILIMINARIES

In this section, some basic definitions and properties are introduced, which will be used in the following part of this paper.

### 2.1 Fractional calculus

Fractional calculus refers to the integral-differential of any order and plays an important role in modern science [19, 30, 31]. A commonly used definition of the fractional operator is Riemann-Liouville operator and its integral is defined as

$$ {}_aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d(\tau), \qquad (1)$$

where $f(t)$ is an arbitrary integrable function, ${}_aD_t^{-\alpha}$ is the fractional integral of order $\alpha \in (0,1)$, $\Gamma(.)$ is the Gamma function. Besides, the Riemann-Liouville fractional order

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derivative is defined as

$$\begin{aligned} {}_a^{RL}D_t^{\beta} f(t) =& \frac{d^n}{dt^n}[{}_aD_t^{-(n-\beta)} f(t)] \\ =& \frac{1}{\Gamma(n-\beta)} \frac{d^n}{dt^n}[\int_a^t \frac{f(\tau)}{(t-\tau)^{\beta-n+1}} d\tau], \end{aligned} \quad (2)$$

where $\beta$ is an arbitrary positive constant, and $n \le \beta < n+1$. Another fractional order differential approach is the Caputo differentiation defined as

$${}_a^{C}D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{y^{m+1}(\tau)}{(t-\tau)^{\alpha}} d\tau \quad (3)$$

where $m$ is an integer and $\alpha = m+\gamma$ , $0<\gamma<1$ .

### 2.2 Laplace transform and Z transform

The Laplace Transform of $f(t)$ is defined as

$$F(s) = \mathscr{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt \quad (4)$$

Where $f(t)$ is continuous or piecewise continuous on every finite interval in $[0,\infty)$ and satisfy $|f(t)| \le Me^{ct}$ $0 \le t < +\infty$ and sufficient large constant $M > 0$. Furthermore, for a $n$ -th fractional derivative, the Laplace Transform of it is denoted by

$$L\{D^n f(t)\} = s^n F(s), \quad (5)$$

where the zero initial conditions are applied.
The inverse Laplace transform of $F(s)$ is defined as

$$f(t) = \mathscr{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma-j\omega}^{\sigma+j\omega} e^{st} F(s) ds, \quad (6)$$

where the path of integration is a vertical line $Re(s) = \sigma$ in the region of convergence of $F(s)$ so that every singularity $s_k$ of $F(s)$ satisfies $\sigma > Re(s_k)$. The Z transform of a discrete-time signal $x(n)$ is defined as

$$X(z) = \mathscr{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n}. \quad (7)$$

The inverse Z transform is given as follows

$$x[n] = \mathscr{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint_{\mathscr{C}} X(z) z^{n-1} dz, \quad (8)$$

where $\mathscr{C}$ is a counterclockwise closed path encircling the origin and entirely in the region of convergence.

### 2.3 Generalized Mittag-Leffler functtion

The Mittag-Leffler function is a very important function in fractional order differential equations [34]. It was difined by the series

$$E_{\rho}(\zeta) = \sum_{k=0}^{\infty} \frac{\zeta^k}{\Gamma(k\rho+1)}, \quad (9)$$

where $\zeta \in \mathbf{C}$ and $\rho$ is an arbitrary positive constant. The Laplace transform of it is

$$\mathscr{L}\{E_{\rho}(-\lambda t^{\rho})\} = \frac{s^{\rho-1}}{s^{\rho}+\lambda}, \quad (10)$$

where $Re\{s\} > |\lambda|^{1/\rho}$. Moreover,the Mittag-Leffler function with two parameters has the following form:

$$E_{\rho,\mu}(\zeta) = \sum_{k=0}^{\infty} \frac{\zeta^k}{\Gamma(k\rho+\mu)}, \quad (11)$$

where $\zeta \in \mathbf{C}$ and $\rho$ and $\mu$ are arbitrary positive constants. When $\mu = 1$, $E_{\rho,1}(\zeta) = E_{\rho}(\zeta)$. The Laplace transform of the Mittag-Leffler function with two parameters is

$$\mathscr{L}\{t^{\mu-1} E_{\rho,\mu}(-\lambda t^{\rho})\} = \frac{s^{\rho-\mu}}{s^{\rho}+\lambda}, \quad (12)$$

where $Re\{s\} > |\lambda|^{1/\rho}$.
Finally, the Generalized Mittag-Leffler function is defined as

$$E_{\rho,\mu}^{\gamma}(\zeta) = \sum_{k=0}^{\infty} \frac{(\gamma)_k}{\Gamma(\rho k+\mu)} \frac{\zeta^k}{k!}, \quad (13)$$

where $\zeta \in \mathbf{C}$ and $\rho$ , $\mu$ and $\gamma$ are arbitrary positive constants, $(\gamma)_k$ is the Pochammers symbol [35]. It can be easily seen that $E_{\rho,\mu}^1(\zeta) = E_{\rho,\mu}(\zeta)$ and $E_{\rho,1}^1(\zeta) = E_{\rho}(\zeta)$.
The Laplace transform of Generalized Mittag-Leffler function is

$$\mathscr{L}\{t^{\mu-1} E_{\rho,\mu}^{\gamma}(-\lambda t^{\rho})\} = \frac{s^{\rho\gamma-\mu}}{(s^{\rho}+\lambda)^{\gamma}}, \quad (14)$$

where $Re\{s\} > |\lambda|^{1/\rho}$.

### 2.4 Impulse response invariant discretization method

The impulse response invariant discretization method converts analog filter transfer functions to digital filter transfer functions in such a way that their impulse responses are the same (invariant) at the sampling instants. Thus, if $g(t)$ denotes the impulse response of an analog (continuous-time) filter, then the digital (discrete-time) filter given by the impulse-invariant method will have impulse response $g(nT_s)$ , where $T_s$ denotes the sampling period in seconds. The basic technique is the so-called Prony method [36].

## 3 CONVERGENCE ANALYSIS

In this paper, the $PI^{\gamma}/PD^{\gamma}$ type updating law is discussed. The input-output relationship at the $k$-th iteration can be described as follows

$$Y_k(s) = G(s) U_k(s),$$

where $G(s)$ is the controlled system, $y_d(t)$ is the desired output and $y_k(0) = y_d(0)$ . Let the updating law be

$$U_{k+1}(s) = U_k(s) + \lambda C(s) E_k(s),$$

where $\lambda$ is a scalar learning gain, and $C(s)$ denotes the controller, it follows that

$$E_{k+1}(s) = [1 - \lambda G(s) C(s)] E_k(s).$$

The convergence condition of the above ILC is [32],[33]

$$\|I - \lambda G(jw) C(jw)\|_{\infty} < 1.$$

Particularly, when the fractional order of the controlled system is unknown, the order-independent FOILC schemes

[19] will be very useful. The first one is order-adaptive FOILC, it was noted that $\gamma \leq \alpha$ is sufficient but not necessary for the convergence of FOILC and the tracking speed is fastest when $\alpha = \gamma$ [24]. The second one is the half-order FOILC [25], it can guarantee higher tracking accuracy and learning speed at high frequency. The last one is the band-stop type FOILC, it combines the advantages of the above mentioned two methods that guarantees the learning speed at both low and high frequencies [29].

## 4 FLAT PHASE METHOD

Let the fractional order controlled plant be

$$G(s) = \frac{1}{Ts^{\alpha}+1}. \tag{15}$$

The open-loop transfer function of the system is shown as

$$L(jw) = C(jw)G(jw), \tag{16}$$

It is obvious that if the loop gain of system changes, then the crossover frequency and the phase margin will also change, the overshoot of the response will vary accordingly. Therefore, in this paper a novel method is applied to prevent the phase margin from the influence of loop gain variations, and the robustness of control system is considered as well.

Firstly, by the definition of the crossover frequency, we can get

$$|L(jw_{cg})| = |C(jw_{cg})G(jw_{cg})| = 1, \tag{17}$$

where $w_{cg}$ is the cut-off frequency.

Moreover, given a desired phase margin, it arrives at

$$\arg[L(jw_{cg})] = \arg[C(jw_{cg})G(jw_{cg})] = -\pi + \phi_m. \tag{18}$$

According to (18), the flat phase condition (19) is added to guarantee the adaptiveness of the control system so that the phase margin does not vary too much according to the slight change of crossover frequency, i.e.

$$\frac{d\arg[L(jw)]}{dw}|_{w=w_{cg}} = 0. \tag{19}$$

So far, three tuning equations (17), (18), (19) are introduced. If the model of controlled plant can be derived, the parameters of fractional order $PI^{\gamma}/PD^{\gamma}$ controller can be achieved graphically by using those three equations.

## 5 TUNING OF $PI^{\gamma}/PD^{\gamma}$ CONTROLLER

It should be noted that the fractional order of the controller should be lower than it in the system [4]. Thus the transfer function of the learning controller in this paper can be defined as

$$C(s) = K_p(1+\frac{K_i}{s^{\lambda}}), \tag{20}$$

where $\lambda$ is a constant that is restricted in (-1,1), and $K_p$ and $K_i$ are both to be tuned positive numbers. According to the characteristic analysis in frequency domain by (16), it arrives at

$$\begin{aligned} L(jw) =& |C(jw)G(jw)| \\ =& \frac{K_p a\sqrt{(1+K_i w^{-\lambda}\cos\frac{\lambda\pi}{2})^2+(K_i w^{-\lambda}\sin\frac{\lambda\pi}{2})^2}}{\sqrt{A_0^2+B_0^2}}, \end{aligned} \tag{21}$$

where

$$A_0 = 1 + Tw^{\alpha}\cos\frac{\alpha\pi}{2},$$

$$B_0 = Tw^{\alpha}\sin\frac{\alpha\pi}{2}.$$

According to (18), the phase angle of system is

$$\begin{aligned} \arg[L(jw_{cg})] =& -\arctan\frac{K_i w_{cg}^{-\lambda}\sin\frac{\lambda\pi}{2}}{1+K_i w_{cg}^{-\lambda}\cos\frac{\lambda\pi}{2}} - \arctan\frac{B}{A} \\ =& -\pi + \phi_m, \end{aligned} \tag{22}$$

where

$$A = 1 + Tw_{cg}^{\alpha}\cos\frac{\alpha\pi}{2},$$

$$B = Tw_{cg}^{\alpha}\sin\frac{\alpha\pi}{2}.$$

Then the relationship between $K_i$ and $w_{cg}$ can be established,

$$K_i = \frac{-\tan(\arctan\frac{B}{A}+\phi_M)}{w_{cg}^{-\lambda}\sin\frac{\lambda\pi}{2} + w_{cg}^{-\lambda}\cos\frac{\lambda\pi}{2}\tan(\arctan\frac{B}{A}+\phi_m)}. \tag{23}$$

Next, substitute (17) into (21), the expression of $K_p$ is shown as

$$K_p = \frac{1}{a}\sqrt{\frac{A^2+B^2}{(1+K_i w_{cg}^{-\lambda}\cos\frac{\lambda\pi}{2})^2+(K_i w_{cg}^{-\lambda}\sin\frac{\lambda\pi}{2})^2}}. \tag{24}$$

Then substitute (22) into (19) yields

$$\begin{aligned} & \frac{d(\arg(L(jw)))}{dw}|_{w=w_{cg}} \\ =& \frac{K_i\lambda w_{cg}^{-\lambda-1}\sin\frac{\lambda\pi}{2}}{K_i^2 w_{cg}^{-2\lambda}+2K_i w_{cg}^{-\lambda}\cos\frac{\lambda\pi}{2}+1} - C = 0, \end{aligned} \tag{25}$$

where

$$C = \frac{\alpha T w_{cg}^{\alpha-1}\sin\frac{\lambda\pi}{2}}{A^2+B^2}.$$

Then, the relationship between $K_i$ and $w_{cg}$ can be established as

$$K_i = \frac{-D \pm \sqrt{D^2-4C^2 w_{cg}^{-2\lambda}}}{2Cw_{cg}^{-2\lambda}}, \tag{26}$$

where

$$D = 2Cw_{cg}^{-\lambda}\cos\frac{\lambda\pi}{2} - \lambda w_{cg}^{-\lambda-1}\sin\frac{\lambda\pi}{2}.$$

Given $\phi_m$ and $\lambda \in (-1,1)$ , then $K_p$ , $K_i$ and $w_{cg}$ can be obtained graphically so that the parameters of the fractional order $PI^{\lambda}/PD^{\gamma}$ type ILC is obtained.

## 6 SIMULATIONS

A number of numerical examples are shown in this section to verify the presented strategy.

**Example 1: Fractional first order case**

In (15), let $a = 2$ , $T = 0.4$ and $\alpha = 0.4$ , $G(s)$ becomes

$$G(s) = \frac{2}{0.4s^{0.4} + 1} \tag{27}$$

Given $\phi_m = 72$ and $\lambda = 0.3$ , the relation of $K_i$ w.r.t $w_{cg}$ is shown in (23) and (26), so that $K_i$ and $w_{cg}$ is the intersection point in the $K_i - w_{cg}$ coordinate (Figure 1), i.e. $K_i = 0.2627$ and $w_{cg} = 0.01956$. Using (24) yields $K_p = 0.2960$. Thus

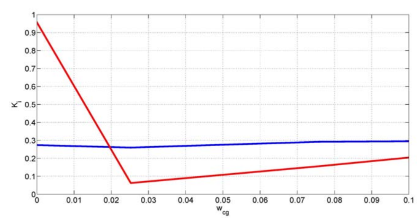

Figure 1: The determination of $K_i$ and $w_{cg}$.

the $PI^{\lambda}$ type FOILC is finally shown as

$$u_{k+1} = u_k + 0.2960e_k + 0.2627e_k^{(-0.3)}. \tag{28}$$

Let the reference be $y_d(t) = 12t^2(1-t)$ and $t \in [0,1]$ , the simulation results are shown in Figure 2. It can be seen

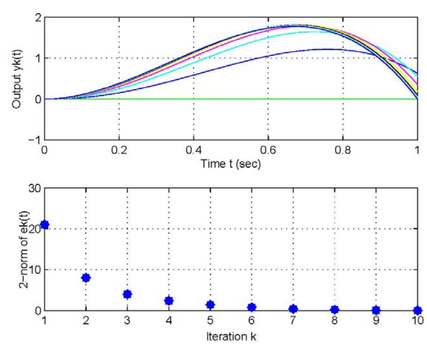


Figure 2: The upper figure shows the first ten iteration results for system (27) and iterative learning control scheme (28). The lower figure shows the 2-norm of the errors in each iterations

from these two figures that the $PI^{\lambda}$ type FOILC guarantees the convergence for the fractional order system. The tracking errors are decreasing and the tracking process is working well when $k \geq 8$ ($\|y_d(t) - y_8(t)\|_2 = 0.2092$).

By using the Bode's ideal loop transfer function $L(s) = (\frac{s}{w_{cg}})^{\alpha}$, the optimal $\mathrm{PI}^{\alpha} - \mathrm{ILC}$ controller is designed to compare with the flat phase based FOILC, the control effect of the optimal $PI^{\alpha} - ILC$ controller is as Figure 3. It can be seen from Figures 2 and 3 that the system's convergence speed using flat phase method is faster than that of the optimal $PI^{\alpha} - ILC$ controller. Its tracking error is

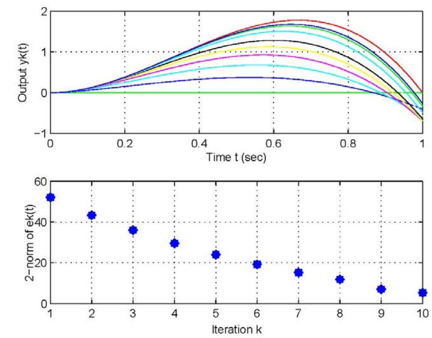


Figure 3: The upper figure shows the first ten iteration results for system (27) with the optimal $PI^{\alpha} - ILC$ controller. The lower figure shows the 2-norm of the errors in each iterations

also a lot smaller than the optimal $PI^{\alpha} - ILC$ controller although their tracking errors are both decreasing. This example illustrates the efficiency of the flat phase based FOILC scheme.

**Example 2: Fractional second order case**

Suppose the fractional second order system is defined as

$$G(s) = \frac{20}{s(0.3s^{0.5} + 1)}. \tag{29}$$

In this case, the tuning method in Section 5 is still working. Nevertheless, according to the next black box case, a manual tuning method is discussed by using the Bode diagram. Based on the flat phase method and according to the order adaptive FOILC scheme, the corresponding Bode diagram is presented in Figure 4. It can be seen from the Figure 4

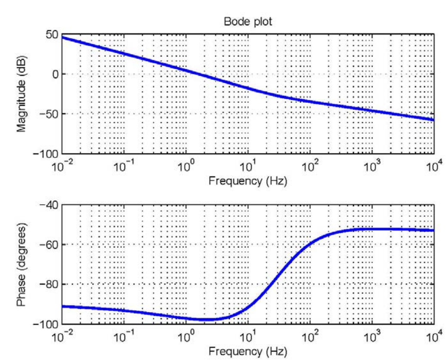


Figure 4: Bode diagram of the controlled model

that when the magnitude reaches zero, the corresponding phase is flat around this region, where $K_p = 0.1$, $K_d = 0.05$ and $\gamma = 0.9$. Thus the learning law is shown as

$$u_{k+1} = u_k + 0.1e_k + 0.05e_k^{(0.9)}. \tag{30}$$

Applying the above learning law to the system (29), where $y_d(t) = 12t^2(1-t)$ and $t \in [0,1]$, the simulation results are shown in Figure 5. It can be seen that the flat phase based manual tuning method can also guarantee the

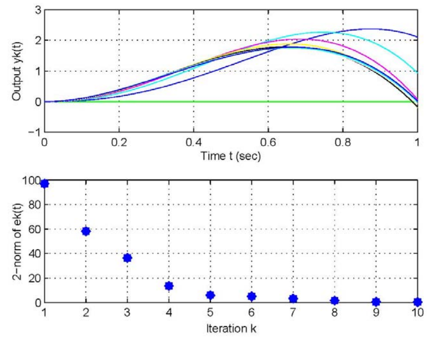


Figure 5: The upper figure shows the first ten iteration results for system (29) with the ILC updating law (30). The lower figure shows the 2-norm of the errors in each iterations

convergence for the fractional second order system and the tracking errors are very small after the ninth iteration ($\|y_d(t) - y_9(t)\|_2 = 0.5993$). This example also implies the efficiency of the flat phase based FOILC scheme.

**Example 3: Black box case**

This example illustrates the application of flat phase based FOILC method to the black box system by using input/output data, where a discrete model is derived by using the impulse response invariant discretization method that is illustrated in Figure 6. In this example, a relaxation process

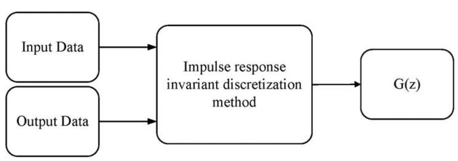


Figure 6: Derivation of $G(z)$.

of a RC circuit is sampled with $Ts = 0.01$ second. Given the input/output data, the black box system is replaced by

$$G(z) = \frac{0.1910 - 0.3754z^{-1} + 0.1845z^{-2}}{1 - 1.9965z^{-1} + 0.9965z^{-2}}, \tag{31}$$

and the Bode diagram of $G(z)$ is shown in Figure 7. Next

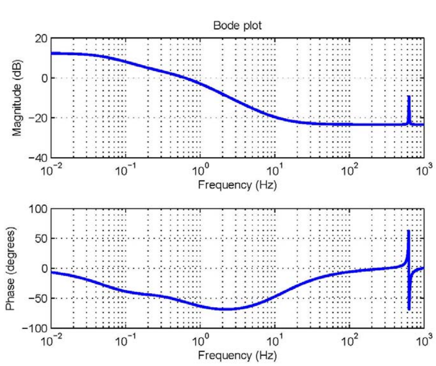


Figure 7: Bode diagram of the discrete-time model

the Bode diagram of the open loop transfer function is shown in Figure 8. Thus the parameters of the learning

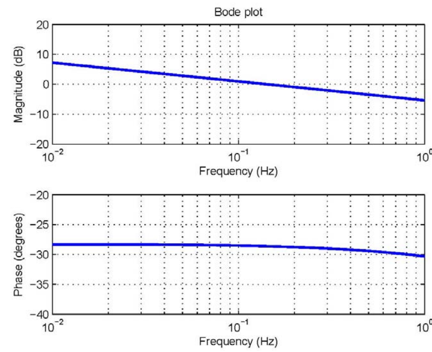


Figure 8: Bode diagram of the system designed by input/output data

law can be shown as $K_p = 0.1, K_i = 0.1, \lambda = 0.1$ so that

$$u_{k+1} = u_k + 0.1e_k + 0.1e_k^{(-0.1)} \tag{32}$$

Lastly, applying the above control law to the RC circuit yields Figure 9. The tracking error decrease to a very smal-

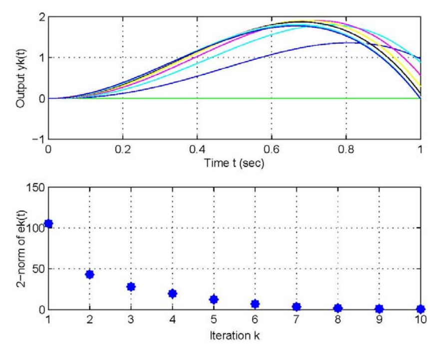


Figure 9: The upper figure shows the first ten iteration results for the unknown model with the ILC updating law (32). The lower figure shows the 2-norm of the errors in each iterations

l values when $k \geq 9$ ($\|y_d(t) - y_9(t)\|_2 = 0.6960$). Therefore, the above mentioned scheme can also be applied to the black box systems.

## 7 CONCLUSIONS AND MAIN INNOVATIONS

In this paper, a FOPI/FOPD controller tuning method for a set of known/unknown systems is presented and a $PI^{\lambda}/PD^{\gamma}$ type FOILC scheme is built up accordingly, where the flat phase and impulse response invariant discretization methods are successfully applied. A number of numerical simulations are provided to validate the efficiency of above concepts. Main innovations are summarized as follows:

(1) Given the system model, the learning gains of $PI^{\lambda}/PD^{\gamma}$ type FOILC can be computed by using the graphical or the manual tuning method.

(2) Given the system structure without exactly the knowledge of parameters, the order adaptive FOILC can be applied.

(3) Given the input/output data of a black box system, an equivalent discrete system is found by using the impulse response invariant discretization method so that the $PI^{\lambda}/PD^{\gamma}$ type FOILC can be chosen manually.

(4)The flat phase method guarantees the adaptiveness of all parameters.

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