Gradient-based step response identification of low-order model for time delay systems

Rui Yan, Fengwei Chen, Shijian Dong, Tao Liu*

Institute of Advanced Control Technology, Dalian University of Technology, Dalian, 116024, P. R. China
* Corresponding author. Tel: 86-411-84706465 email: liurouter@ieee.org (T. Liu)

Abstract: In this paper, a step response identification method is proposed to obtain a low-order model for industrial processes with time delay. Based on a classification of the model pole distribution, the corresponding time domain expressions of the output response to a step change are derived and therefore, a gradient searching algorithm is developed to simultaneously identify the rational model parameters together with the delay parameter. The computation effort can be significantly reduced compared to the existing time domain multiple-integral methods for model fitting. The convergence of the proposed algorithm is analyzed with a strict proof. Two illustrative examples from the literature are shown to demonstrate the effectiveness of the proposed method.

Key Words: Step response identification, time delay, second-order model, gradient searching, convergence

1 INTRODUCTION

For control-oriented model identification in industrial engineering applications, step response tests have been widely used owing to its implemental simplicity and economy. In the past decades, the identification of process models with time delay has received increasing attention in view of the fact that time delay is usually involved with industrial processes and system operations [1-3]. Early references (e.g. [4]) proposed the approximation of time delay by using Padé approximation and Laguerre expansion, resulting in a higher order rational transfer function model that contains more parameters to be estimated and may cause unacceptable fitting error when the process response has a long time delay. By fitting a few representative points in the transient output response to a step change, the reference [5] presented alternative identification methods for obtaining a first-order-plus-dead-time (FOPDT) or second-order-plus-dead-time (SOPDT) model. Based on numerical integral to the time domain expression of step response, the papers [6, 7] introduced n -fold multiple integrals to identify a n -th order process model. Note that these multiple-integral based identification methods can give good accuracy and robustness in comparison with the previous methods, but the computation load is relatively high and the use of multiple time integrals for parameter estimation may be sensitive to the test data length. In the recent years, a frequency domain step identification method was proposed [8] to reduce the computation effort by introducing a damping factor to the step response for the computation of the Laplace transform.

A generalized expectation-maximization algorithm was suggested for robust identification of linear parameter varying (LPV) systems with fixed input delay [9]. For identifying a canonical state space model with state delay, a recursive least-squares parameter identification algorithm was presented in [10] based on using a state filter. To handle the nonlinear estimation problem, a few numerical optimization methods were developed for estimating both the linear model parameters and the delay parameter, e.g. hierachical identification strategies [11], Gradient-based searching algorithm [12,13], Newton-like approaches [14, 15]. Note that these numerical optimization methods were developed based on using persistent excitation signals such as the pseudo random binary sequence (PRBS), which may not be executable in many industrial applications.

In this paper, a step response identification method is proposed to obtain a low-order model for industrial processes with time delay, to further extend our previous work [16] that was limited to overdamped processes. By introducing a new time domain expression of the process transfer function model, a modified gradient-based identification method is proposed which can simultaneously identify linear model parameters together with the delay parameter from the step response data. Moreover, no time integral is required for computation, significantly reducing the computation effort compared to the recently developed multiple-integral based identification methods. Owing to the use of a step test, it is verified that the cost function of prediction error becomes convex with respect to all the model parameters in a wide region, such that good convergence can be guaranteed. For clarity, the paper is organized as follows: Section 2 presents the low-order process models to be identified. The corresponding identification method is detailed in Section 3, together with some guidelines for model structure selection. The convergence is analyzed in Section 4. Two numerical examples are shown in Section 5. Conclusions are drawn in Section 6.

2 PROBLEM FORMULATION

For industrial processes, e.g. multi-component blending reactors and fermentation tanks, low-order models, e.g. FOPDT and SOPDT, are widely used for control system design and tuning [17]. Without loss of generality, the process models are divided into three types according to the distribution of poles,

the first one has a single or repetitive poles,

$$G_{\mathrm{m1}}(s)=\frac{k_{\mathrm{p}}}{(\tau_{\mathrm{p}}s+1)^{m}}e^{-Ls} \tag{1}$$

the second one has two distinct real poles,

$$G_{m2}(s)=\frac{k_p}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ls} \tag{2}$$

and the last one has two conjugate poles,

$$G_{m3}(s)=\frac{k_p\left(\tau_1^2+\tau_2^2\right)}{(s+\tau_1)^2+\tau_2^2}e^{-Ls} \tag{3}$$

where k_p denotes the process static gain, L is the process time delay, τ_p, τ_1, τ_2 are time constants, and m is the number of repetitive poles or the model order.

It should be noted that higher-order processes can be effectively described by the above models [17], [18].

Under a step test, the input excitation with a magnitude of h can be described by

$$u(t)=h,\ t\ge 0. \tag{4}$$

Correspondingly, the time domain step response of the process models shown in (1), (2), (3) can be expressed by

$$y(t)=\bar{y}(\infty)\left[1-e^{-(t-L)/\tau_p}\right]+v(t),\ t\ge L \tag{5}$$

$$y(t)=\bar{y}(\infty)\left[1+\frac{\tau_1}{\tau_2-\tau_1}e^{-(t-L)/\tau_1}+\frac{\tau_2}{\tau_1-\tau_2}e^{-(t-L)/\tau_2}\right]+v(t),t>L \tag{6}$$

$$y(t)=\bar{y}(\infty)\left[1-\cos(\tau_2(t-L))-\frac{\tau_1}{\tau_2}\sin(\tau_2(t-L))\right]\cdot e^{-\tau_1(t-L)}+\frac{b_1}{\tau_2}e^{-\tau_1(t-L)}\sin(\tau_2(t-L))+v(t),t>L \tag{7}$$

where $v(t)$ is assumed to be a white noise with zero mean.

Generally, the process static gain, k_p, can be directly computed from

$$k_p=\frac{\bar{y}(\infty)}{h} \tag{8}$$

where $\bar{y}(\infty)$ is obtained by averaging 20-50 measured output values after the process response moves into a steady state.

3 PROPOSED IDENTIFICATION ALGORIEHM

To give a general identification algorithm for the models shown in (1), (2), and (3), the identification algorithm is detailed for a second-order model with a zero as expressed by

$$G_{m4}(s)=\frac{b_1 s+b_0}{(s+\tau_1)^2+\tau_2^2}e^{-Ls} \tag{9}$$

Correspondingly, the time domain step response is shown into

$$y(t)=\bar{y}(\infty)\left[1-\cos(\tau_2(t-L))-\frac{\tau_1}{\tau_2}\sin(\tau_2(t-L))\right]\cdot e^{-\tau_1(t-L)}+\frac{b_1}{\tau_2}e^{-\tau_1(t-L)}\sin(\tau_2(t-L))+v(t),t>L \tag{10}$$

To estimate all the unknown model parameters, we define a parameter vector for estimation,

$$\theta=[\tau_1,\tau_2,b_1,L]^T \tag{11}$$

The output prediction error can be computed by

$$\delta(t,\theta)=y(t)-\left[1-\cos(\tau_2(t-L))-\frac{\tau_1}{\tau_2}\sin(\tau_2(t-L))\right]\cdot \bar{y}(\infty)e^{-\tau_1(t-L)}-\frac{b_1}{\tau_2}e^{-\tau_1(t-L)}\sin(\tau_2(t-L)) \tag{12}$$

It is obvious that the unknown time delay impede the computation of prediction error in (12). To resolve the problem, the following cost function is adopted herein,

$$J(\hat{\theta})=\frac{1}{2}\varepsilon^T\varepsilon \tag{13}$$

where $\varepsilon=[\delta(t_1,\hat{\theta}),\cdots,\delta(t_N,\hat{\theta})]^T$, $\hat{\theta}$ denotes an estimate of the unknown parameters, $\hat{y}(t)$ is the predicted output, N denotes the collected data length, and t_i ($i=1,2,\cdots,N.$) are the sampling instants which should be chosen such as $L\le t_1<t_2<\cdots<t_N$.

To minimize the cost function in (13), we compute the first-order derivative for (12) with respect to τ_1, τ_2, b_1 and L, respectively, obtaining

$$\frac{\partial\delta(t_i,\theta)}{\partial\tau_1}=\left[\bar{y}(\infty)\frac{1-\tau_1(t-L)}{\tau_2}+\frac{b_1(t-L)}{\tau_2}\right]e^{-\tau_1(t-L)}\sin[\tau_2(t-L)]-(t-L)\bar{y}(\infty)e^{-\tau_1(t-L)}\cos[\tau_2(t-L)] \tag{14}$$

$$\frac{\partial\delta(t_i,\theta)}{\partial\tau_2}=(t-L)e^{-\tau_1(t-L)}\cos[\tau_2(t-L)]\left[\bar{y}(\infty)\frac{\tau_1}{\tau_2}-\frac{b_1}{\tau_2}\right]-e^{-\tau_1(t-L)}\sin[\tau_2(t-L)]\left[\bar{y}(\infty)(t-L+\frac{\tau_1}{\tau_2^2})-\frac{b_1}{\tau_2^2}\right] \tag{15}$$

$$\frac{\partial\delta(t_i,\theta)}{\partial b_1}=-\frac{1}{\tau_2}e^{-\tau_1(t-L)}\sin[\tau_2(t-L)] \tag{16}$$

$$\frac{\partial\delta(t_i,\theta)}{\partial L}=e^{-\tau_1(t-L)}\sin[\tau_2(t-L)]\left[\bar{y}(\infty)(\tau_2+\frac{\tau_1^2}{\tau_2})-\frac{b_1\tau_1}{\tau_2}\right]+b_1e^{-\tau_1(t-L)}\cos[\tau_2(t-L)] \tag{17}$$

The Jacobian matrix of (12) is defined by

$$\Xi(t_i,\theta)=\frac{\partial\delta(t_i,\theta)}{\partial\theta}=\left[\frac{\partial\delta(t_i,\theta)}{\partial\tau_1},\frac{\partial\delta(t_i,\theta)}{\partial\tau_2},\frac{\partial\delta(t_i,\theta)}{\partial b_1},\frac{\partial\delta(t_i,\theta)}{\partial L}\right]^T \tag{18}$$

Denote

$$\Psi(\theta)=\left[\Xi(t_1,\theta),\Xi(t_2,\theta),\cdots,\Xi(t_N,\theta)\right]^T \tag{19}$$

The gradient matrix of $J(\theta)$ with respect to θ is therefore defined by

$$g(\theta)=\Psi^T(\theta)\varepsilon \tag{20}$$

The corresponding Hessian matrix is formulated by

$$H(\theta)=\Psi^T(\theta)\Psi(\theta) \tag{21}$$

Hence, the parameter vector θ can be estimated using a Guass-Newton iteration approach, i.e.

$$\hat{\theta}_k = \hat{\theta}_{k-1} - H_k^{-1} g_k \tag{22}$$

where H_k and g_k can be computed from (20) and (21) using $\hat{\theta}_{k-1}$ estimated from the previous iteration.

It is well known that the standard Guass-Newton iteration method may give a slow convergence rate in the initial iterations, causing a considerable computation effort. To overcome the deficiency, we propose the use of an adaptive searching step for iteration based on the Armijo line searching strategy (see e.g. in [19]), i.e.

$$J(\hat{\theta}_{k-1} + \rho^m d_k) \le J(\hat{\theta}_{k-1}) + \sigma \rho^m g_k^T d_k \tag{23}$$

where $d_k = -H_k^{-1} g_k$ denotes the searching step size, m is the minimal nonnegative integer satisfying (23). For the convenience of implementation, we take $\sigma = \rho$ and $\rho = \min\{\rho_0, 0.8\}$ where $\rho_0 = 1 - 1/(J_k/J_{k-1} + J_{k-1}/J_k)$, following a similar choice given in our previous work [16] to facilitate the convergence.

A suitable initial value is required for the Guass-Newton iteration to avoid convergence to an improper local minimum. However, it is well known that the convexity of the cost function is related to the type of input excitation for identification [4]. To address this issue, a graphical study on the relationship between $J(\theta)$ and the SOPDT model parameters is explored based on using different input excitations. Consider a second-order process with time delay described by $G(s) = e^{-Ls}/(s^2 + \alpha_1 s + 1)$, where $\alpha_1 = 2$ and $L = 5$. The input excitation is taken as a unity step change or a PRBS sequence with the variance of $\sigma^2 = 1$, respectively. The number of measured output data is $N = 3000$ and the sampling period is $T_s = 0.01$ (s). The plots of $J(\theta)$ with respect to α_1 and L are shown in the Fig.1 and Fig.2. It is seen from Fig.1 that there is a unique minimum, while Fig.2 has multiple minima. The reason lies with that a step signal contains only low frequency components that provoke the output response mainly in the low frequency range. This implies that the cost function is convex in a wide region using a step test for model identification, and the initial choice of model parameters for iteration is not very limited. A small positive real number is therefore suggested to initialize all the model parameters for iteration.

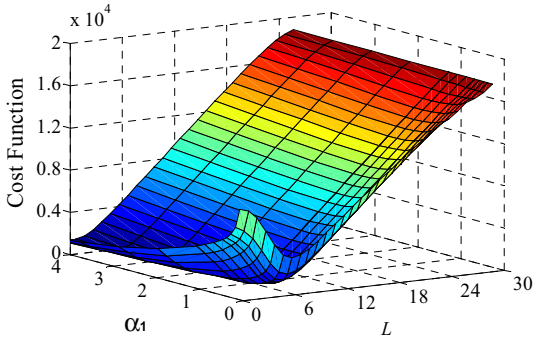

Figure 1 Plot of the cost function under a step test

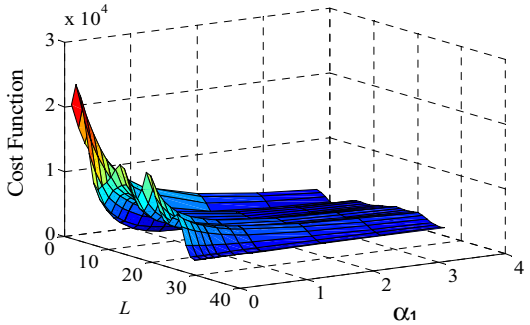

Figure 2 Plot of the cost function under a PRBS excitation

Hence, the proposed identification algorithm is summarized as below:

(i) Collect the input and output observation data, $\{u(t_i), y(t_i)\}$ ($i = 1, 2, \cdots, N.$), from a step test.

(ii) Take an initial estimation of θ denoted by θ_0 with small positive real numbers.

(iii) Compute $\delta(t, \hat{\theta}_{k-1})$ and $J(\hat{\theta}_{k-1})$ by (12) and (13).

(iv) Determine the searching step size $d_k = -H_k^{-1} g_k$ by computing the gradient matrix g_k in (20) and the Hessian matrix H_k in (21).

(v) Update $\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_k d_k$, where $\mu_k = \rho^{m_k}$, and m is the minimal nonnegative integer that satisfies (23).

(vi) End the algorithm if the fitting condition, $1/N \sum_{i=1}^{N} [y(t_i) - \hat{y}(t_i)]^2 < err$, is satisfied, where err is a user specified fitting threshold. Otherwise, let $k = k + 1$ and go to step (iii).

With the above algorithm, b_0 is obtained by

$$b_0 = \frac{\bar{y}(\infty)(\tau_1^2 + \tau_2^2)}{h} \tag{24}$$

4 CONVERGENCE ANALYSIS

To analyze the convergence property of the proposed algorithm, the following proposition is given accordingly.

Proposition 1: Assuming that $\delta(t, \theta)$ is Lipschitz continuously differentiable in a neighborhood Ω of the bounded set $S = \{\theta | J(\theta) \le J(\theta_0)\}$, and the Jacobian matrix $\Psi(\theta)$ in (19) is full row rank together with the positive definite Hessian matrix $H(\theta)$ in (21), the proposed algorithm algorithm guarantee the uniform convergence satisfying

$$\lim_{k \to \infty} g_k(\theta) = \lim_{k \to \infty} \Psi^{\mathrm{T}}(\theta_k) \varepsilon(\theta_k) = 0 \tag{25}$$

where $\varepsilon(\theta_k) = [\delta(t_1, \theta_k), \cdots, \delta(t_N, \theta_k)]^T$.

Proof: Since $\delta(t, \theta)$ is Lipschitz continuously differentiable in the bounded set S, there follows for some positive constants β_1 and β_2 that

$$\|\delta(t, \theta)\| \le \beta_1, \ \forall \theta \in S \ \text{and} \ \forall t > L \tag{26}$$

$$\left\|\Xi(t,\theta)\right\| \le \beta_2,\ \forall \theta \in S \text{ and } \forall t > L \tag{27}$$

where $\|\bullet\|$ denotes the matrix 2-norm.

It can be verified that there exists a constant $q > 0$ such that

$$\left\|\Psi(\theta)\right\| = \left\|\Psi^{\mathrm{T}}(\theta)\right\| \le q \tag{28}$$

Owing to that $\Psi(\theta)$ is full column rank, there stands for a sufficient large constant, $p > 0$, such that

$$\left\|\Psi(\theta)z\right\| \ge p\left\|z\right\| \tag{29}$$

Denote by ω_k the intersection angle between the negative gradient direction, $-g_k$, and the Guass-Newton searching direction of d_k^{GN}. It can be derived using (28) and (29) that

$$\begin{aligned}\cos(\omega_k) &= -\frac{g_k^T d_k^{GN}}{\left\|g_k\right\|\left\|d_k^{GN}\right\|} = -\frac{(d_k^{GN})^T \Psi^T(\theta_k)\Psi(\theta_k)d_k^{GN}}{\left\|\Psi^T(\theta_k)\Psi(\theta_k)d_k^{GN}\right\|\left\|d_k^{GN}\right\|} \\ &= \frac{\left\|\Psi(\theta_k)d_k^{GN}\right\|^2}{\left\|\Psi^T(\theta_k)\Psi(\theta_k)d_k^{GN}\right\|\left\|d_k^{GN}\right\|} \ge \frac{p^2\left\|d_k^{GN}\right\|^2}{q^2\left\|d_k^{GN}\right\|^2} \\ &= \frac{p^2}{q^2} > 0\end{aligned} \tag{30}$$

It follows from (30) that

$$-g_k^T p_k = g_k^T d_k^{GN} \Big/ \left\|d_k^{GN}\right\| = \left\|g_k\right\|\cos(\alpha_k) \tag{31}$$

where $p_k = s_k \big/ \left\|s_k\right\|$, and $s_k = \rho^m d_k^{GN}$.

Take the limit to Eq.(31), we get

$$\lim_{k\to\infty} -g_k^T d_k^{GN} \Big/ \left\|d_k^{GN}\right\| = \lim_{k\to\infty} \left\|g_k\right\|\cos(\alpha_k) > 0 \tag{32}$$

Hence, there is

$$\lim_{k\to\infty} g_k^T p_k < 0 \tag{33}$$

Assume θ^* is a cluster point, but $\Delta J(\theta^*) = \mathrm{g}(\theta^*) \ne 0$

From the assumption and the Armijo condition in (23), we have

$$J(\hat{\theta}_{k-1} + \rho^m d_k^{GN}) - J(\hat{\theta}_{k-1}) \to 0 \tag{34}$$

$$g_k^T s_k \to 0 \tag{35}$$

$$\left\|s_k\right\| \to 0 \tag{36}$$

Note that in the Armijo condition, m_k is the minimum nonnegative integer satisfying the inequality (23). So the inequality (23) can be rearranged as

$$J(\hat{\theta}_{k-1} + \rho^{m_k-1} d_k^{GN}) - J(\hat{\theta}_{k-1}) > \sigma g_k^T \rho^{m_k-1} d_k^{GN} \tag{37}$$

From the assumption in (36), we know $Q_k = s_k / \rho \to 0$.

Then (37) can be rearranged as

$$\frac{J(\hat{\theta}_{k-1} + Q_k p_k) - J(\hat{\theta}_{k-1})}{Q_k} > \sigma g_k^T p_k \tag{38}$$

Take the limit on both sides of Eq.(38), we obtain

$$\lim_{k\to\infty} g_k^T p_k \ge \sigma \lim_{k\to\infty} g_k^T p_k \tag{39}$$

It follows that

$$\lim_{k\to\infty} g_k^T p_k \ge 0 \tag{40}$$

which is contradictory to (33). Hence, there stands

$$\lim_{k\to\infty} g_k^T p_k = 0 \tag{41}$$

Since $\left\|p_k\right\| = 1$ and $\{\left\|p_k\right\|\}$ is bounded, there exists a sub-sequence of $\{\left\|p_k\right\|\}$ that converges to $P^* \ne 0$ which satisfies (41). According to the properties of a bounded series [20], there follows for $\{\left\|p_k\right\|\}$ with $k \to \infty$ that

$$\lim_{k\to\infty} g_k(\theta) = 0 \tag{42}$$

This completes the proof.

Note that the convergence result shown in (25) indicates that the cost function $J(\theta)$ converges to a steady value when $k \to \infty$, that is to say, the estimated model parameters converge to the true values in case perfect model match with the plant, leading to

$$\lim_{k\to\infty} \tilde{\theta}_k = \lim_{k\to\infty} (\theta_k - \theta_{k-1}) = 0 \tag{43}$$

Owing to the use of an instrumental model in the proposed algorithm, the iteration is isolated from the measurement noise, guaranteeing good robustness for convergence.

Note that the true model structure of the process to be identified is difficult to be known in practical applications. It is desirable to determine the optimal model structure for representing the process dynamic response characteristics from a step test. The following hypothesis testing condition [8], can be taken for choosing the optimal model order,

$$\frac{\sum_{i=1}^{N}[\hat{y}(t_i) - y(t_i)]^2\Big|_{n_1}}{\sum_{i=1}^{N}[\hat{y}(t_i) - y(t_i)]^2\Big|_{n_2}} \le 0.1 \tag{44}$$

where $\hat{y}(t)$ and $y(t)$ denotes, respectively, the measured process output and the model output to the step change, n_1 denotes the current model order and n_2 a higher order to be verified. It can be seen from (44) that a higher model order n_2 should be accepted only if the output prediction error is larger than one-tenth of that of the current model with n_1. When the model order is determined, it is easy to distinguish the type of the model poles from the step response test. Note that a SOPDT model with repetitive poles has an obviously larger curvature in the transient step response compared to an SOPDT model with two distinct poles, and a SOPDT model with two conjugate poles has obvious oscillation in the transient step response.

5 ILLUSTRATION

Two examples from the literature are studied to illustrate the performance of the proposed method. The following time domain fitting criterion is used to assess estimate accuracy,

$$err = \frac{1}{N}\sum_{i=1}^{N}[y(t_i) - \hat{y}(t_i)]^2 \tag{45}$$

The parameter estimation error is assessed by the following criterion,

$$ERR=\sqrt{\frac{\left\|\hat{\theta}_k-\theta\right\|_2}{\|\theta\|_2}}\times 100\% \tag{46}$$

where $\hat{\theta}_k$ is an estimation of the true parameter vector θ.

Example 1. Consider a second-order process with a positive zero studied in the references [6, 8],

$$G(s)=\frac{(-4\mathrm{s}+1)e^{-s}}{9s^2+2.4s+1}$$

Based on a unity step response test, Liu et al [8] gave a SOPDT model,

$$G(s)=\left(-3.9989s+0.9998\right)e^{-1.00s}\Big/\left(9.0183s^2+2.3951s+1\right)$$

and Wang et al [6] obtained

$$G(s)=\left(-0.4444s+0.1111\right)e^{-1.00s}\Big/\left(s^2+0.2667s+0.1111\right)\ .$$

By performing the same step test, the proposed algorithm gives the process model,

$$G(s)=\left(-4.0000s+1.0000\right)e^{-1.00s}\Big/\left(9.0000s^2+2.4000s+1\right)$$

in terms of the sampling interval $T_s=0.01\,(\mathrm{s})$, the response time $T=200$ (s), and the initial values for iteration $\theta_0=[0.1,0.2,0.2,0.2]^T$.

To demonstrate the robustness under different noise to signal ratio (NSR) levels, i.e. NSR=10%, 20%, 100 Monte Carlo tests are conducted. Table 1 shows the identification result of the proposed algorithm, indicating rapid convergence under these noise levels. Table 2 lists the averaged iteration numbers where the internal loop indicates the sum of iteration number for determining m in (23) in each test, and the external loop indicates the iteration number of searching step for converging to the optimal estimation. It can be seen from Table 1 and Table 2 that the proposed algorithm guarantees a fast convergent speed and maintain good robustness against measurement noise. Table 3 shows the estimation results of the proposed method with respect to the sampling frequency $f=1/T_s$ (corresponding to different data lengths for identification) under $NSR=10\%$. The parameters shown here are the mean of 100 Monte Carlo estimation. The numbers in the parentheses are the standard deviations of 100 estimates. It can be seen that good identification accuracy and consistent estimation are obtained by the proposed algorithm, while it is not sensitive to the data length collected for model identification.

Table 1. Identification results under different noise levels

NSR	k	$\hat{\tau}_1$	$\hat{\tau}_2$	$\hat{b}_1$	$\hat{L}$	*ERR(%)*
10%	1	20.0000	4.0000	4.0000	0.20	134.03
	3	10.1868	1.6461	-2.7708	1.60	19.17
	5	9.5332	2.1633	-3.6060	1.11	6.96
	7	9.0538	2.3927	-3.9601	1.03	0.74
	9	9.0466	2.4009	-3.9724	1.03	0.63
15%	1	20.0000	4.0000	4.0000	0.20	134.03
	3	10.1817	1.5910	-2.7133	1.69	19.97
	5	9.5607	2.1312	-3.5568	1.15	7.6035
	7	9.0688	2.3898	-3.9451	1.04	0.9881
	9	9.0637	2.1010	-3.9617	1.04	0.8702
True		9	2.4	-4	1	0

Table 2. Averaged iteration numbers under 100 Monte-Carlo tests

NSR	External loop	Internal loop	Success ratio
10%	13	15	100
20%	14	15	100

In addition, the computation load for the proposed method is listed in Table 4 in comparison with a multiple integral method given by Wang et al. [6]. In Table 4, N denotes the number of sampling data and K is the iteration number. It can be seen from Table 4 that the proposed algorithm uses obviously less computational effort compared to the multiple integral method since N is much larger than K.

Example 2. Consider the fifth-order system with time delay studied by Wang et al [6],

$$G(s)=\frac{1.08}{(s+1)^2(2s+1)^3}e^{-10s}$$

Based on a step test, Wang et al [6] derived an SOPDT model,

Table 3. Parameter estimation under different sampling frequencies

Frequency	$\hat{a}_2$	$\hat{a}_1$	$\hat{b}_1$	$\hat{b}_0$	$\hat{L}$
5Hz	8.99(±0.3719)	2.40(±0.0944)	-4.02(±0.1942)	0.99(±0.0056)	1.00(±0.1703)
25Hz	9.02(±0.1846)	2.40(±0.0441)	-4.01(±0.0980)	0.99(±0.0026)	0.99(±0.1015)
50Hz	8.99(±0.1252)	2.40(±0.0280)	-4.01(±0.0685)	1.00(±0.0017)	1.00(±0.0638)
75Hz	8.98(±0.1078)	2.40(±0.0234)	-3.98(±0.0556)	1.00(±0.0015)	1.01(±0.0499)
100Hz	9.00(±0.0870)	2.40(±0.0233)	-4.00(±0.0543)	1.00(±0.0014)	1.00(±0.0493)
True	9	2.4	-4	1	1

Table 4. Comparison of the computation load

Methods	Addition	Multiplication
Proposed	$K(58N+114)$	$K(90N+297)$
Ref. [8]	$\frac{1}{6}N(N+1)(2N+1)+N^2+50N+690$	$\frac{1}{6}N(N+1)(2N+1)+\frac{1}{2}N(N+1)+N^2+62N+1747$

$G(s) = 1.0800e^{-12.71s} / 7.8910s^2 + 5.3231s + 1$

corresponding to the mean-squared output fitting error, $err = 5.71 \times 10^{-4}$. For comparison, the same step test is performed for the sampling time $T = 150$ (s) with the sampling interval $T_s = 0.01$ (s). An SOPDT model is obtained by using the proposed algorithm,

$G(s) = 1.0800e^{-12.59s} / 7.7510s^2 + 5.5679s + 1$,

corresponding to $err = 4.88 \times 10^{-4}$.

Then assume that the output measurement noise level is NSR=10%, the proposed method gives

$G(s) = 1.0791e^{-12.49s} / 7.8539s^2 + 5.6051s + 1$, corresponding to $err = 3.17 \times 10^{-5}$, compared with that of Wang et al [6], $G(s) = 1.0789e^{-14.20s} / 0.1111s^2 + 3.8731s + 1$, corresponding to $err = 2.34 \times 10^{-4}$. The step responses of these models are shown in Fig.3, well demonstrating good fitting of the proposed method.

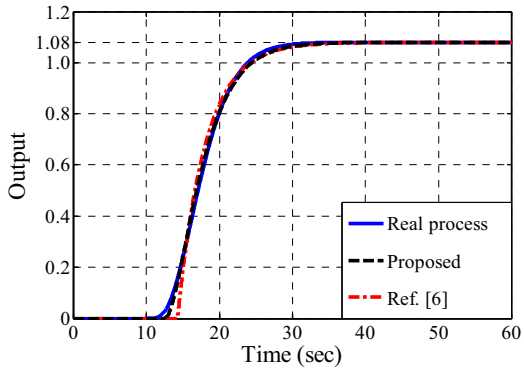

Figure 3. Step response fitting for Example 2

6 CONCLUSIONS

Based on a classification of the model pole distribution, a step response identification method has been proposed for industrial processes with time delay, along with a proof on the convergence. By developing a gradient-based searching approach to minimize the output prediction error, the linear model parameters together with the delay parameter can be simultaneously identified from the time domain expression of a low-order model response to a step change. The computation effort can be significantly reduced compared to recently developed step response identification methods (e.g. [6, 7]) based on using multiple integrals to step response data. Moreover, the proposed algorithm is not sensitive to the data length collected for model identification, therefore facilitating practical applications.

REFERENCES

[1] H. Garnier and L. Wang (Ed). Identification of continuous-time models from sampled data. Springer-Verlag, London, 2008.

[2] P. C. Young, Recursive Estimation and Time-series Analysis: An Introduction for the Student and Practitioner. Springer-Verlag, Berlin, 2011.

[3] T. Liu, Q. G. Wang, H. P. Huang, A tutorial review on process identification from step or relay feedback test, Journal of Process Control, Vol. 23, No. 10, 1597-1623, 2013.

[4] P. J. Gawthrop, M. T. Nihtilä, Identification of time delays using a polynomial identification method, Systems & Control Letters, Vol. 5, No. 4, 267-271, 1985.

[5] H. P. Huang, M. W. Lee, C. L. Chen, A system of procedures for identification of simple models using transient step response, Industrial & Engineering Chemistry Research, Vol. 40, No. 8, 1903-1915, 2001.

[6] Q. G. Wang, X. Guo, Y. Zhang, Direct identification of continuous time delay systems from step responses, Journal of Process Control, Vol. 11, No. 5, 531-542, 2001.

[7] T. Liu, F. Zhou, Y. Yang, F. Gao, Step response identification under inherent-type load disturbance with application to injection molding, Industrial & Engineering Chemistry Research, Vol. 49, No. 22, 11572-11581, 2010.

[8] T. Liu, F. Gao, A frequency domain step response identification method for continuous-time processes with time delay, Journal of Process Control, Vol. 20, No. 7, 800-809, 2010.

[9] X. Yang, Y. Lu, Z. Yan, Robust global identification of linear parameter varying systems with generalised expectation–maximisation algorithm, IET Control Theory & Applications, Vol. 9, No. 7, 1103-1110, 2015.

[10] Y. Gu, F. Ding, J. Li, State filtering and parameter estimation for linear systems with d-step state-delay, IET Signal Processing, Vol. 8, No.6, 639-646, 2014.

[11] F. Ding, X. Liu, J. Chu, Gradient-based and least-squares-based iterative algorithms for Hammerstein systems using the hierarchical identification principle, IET Control Theory & Applications, Vol. 7, No. 2, 176-184, 2013.

[12] J. Na, X. M. Ren, Y. Xia, Adaptive parameter identification of linear SISO systems with unknown time-delay, Systems & Control Letters, Vol. 66, 43-50, 2014.

[13] F. Chen, H. Garnier, M. Gilson, Robust identification of continuous-time models with arbitrary time-delay from irregularly sampled data, Journal of Process Control, Vol. 25, 19-27, 2015.

[14] K. Ji, L. Xu, W. Xiong, L. Chen, Newton iterative algorithm based modeling and proportional derivative controller design for second-order systems, Journal of Applied Mathematics and Computing, 1-16, 2014.

[15] S. W. Sung, I. B. Lee, Prediction error identification method for continuous-time processes with time delay, Industrial & Engineering Chemistry Research, Vol. 40, No.24, 5743-5751, 2001.

[16] R. Yan, T. Liu, F. Chen, S. Dong, Gradient-based step response identification of overdamped processes with time delay, Systems Science & Control Engineering, Vol. 3, No.1, 504-513, 2015.

[17] D. E. Seborg, D. A. Mellichamp, T. F. Edgar, F. J. Doyle III, Process Dynamics and Control, 3nd Edition, Hoboken, NJ: John Wiley & Sons, 2010.

[18] T. Liu, F. Gao, Industrial Process Identification and Control Design: Step-test and Relay-Experiment-Based Methods. Springer, London UK, 2012.

[19] S. J. Wright, J. Nocedal, Numerical Optimization. New York: Springer, 1999.

[20] B. P. Rynne, M. A. Youngson, Linear functional analysis. Springer, London UK, 2000.