Output Tracking Combing Output Redefinition and Non-causal Stable Inversion for Non-minimum Systems

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Abstract: A method to achieve precise output tracking for non-minimum systems by combining output redefinition with non-causal stable inversion is presented. Firstly, output redefinition approach can make the system reach Stable Initial State after a fixed time period. An optimal solution by a search algorithm to minimize the magnitude of the redefined trajectory designed with exponential form is obtained. Then a precise output trajectory tracking is achieved via stable inversion. In contrast with conventional stable inversion, the new method can reduce the pre-actuation time. The simulation of the proposed method for a tip trajectory tracking of a one-link flexible manipulator shows that the system state can be transferred accurately to Stable Initial State with no vibrating output over the transition process.

Key Words: Output tracking, non-minimum systems, output redefinition, non-causal stable inversion, search algorithm, Stable Initial State

1 INTRODUCTION

Output tracking for non-minimum systems draws many scholars's attentions all the time [1]. Chen and Devasia applied stable inversion method to this problem first [2][3]. Bayo solved the inversed dynamical equations in frequency domain [4], however it was time consuming. A stable inversion approach was put forward in time domain in [5]. All mentioned stable inversion situations require the information of the whole trajectory, which means the calculation is off-line. Zou came up with a preview-based stable inversion to track the output trajectory, making online calculation possible [6]. Yet stable inversion requires pre- and post- actuation process and the system is limited to have no imaginary eigenvalues [7].

A causal inversion via output redefinition can be seen in [8]. It can cancel the effect of the unstable zeros of the system, dealing equally with hyperbolic systems and non-hyperbolic systems and leading to causal control. Thus, the systems can start from an arbitrary initial state. An approximate causal inversion controller is designed in [9]. The causal inversion controller is a feed-forward controller, combining with a optimal feedback controller to achieve precise tracking. Compared with the stable inversion methods, causal inversion doesn't need pre-actuation process. It is further studied to achieve rest-rest output tracking in [10]. To get rid of the effect of unstable zeros, the whole output trajectory is redefined with exponential form. It is applicable in both hyperbolic and non-hyperbolic systems, with no pre- or post-actuation process being required.

From [11], we know the non-causality of stable inversion lies at the Stable Initial State. The pre-actuation process can make the system run to the Stable Initial State from time negative infinity [12]. To have the state moved to an expected value in a finite time period, an optimal state transition method is proposed [13]. It can be used to get the Stable Initial State [14]. In this paper, a new method of output redefinition together with non-causal stable inversion to achieve precise output tracking for non-minimum systems is proposed. We can obtain the desired Stable Initial State after a fixed time period control via output redefinition. Using a search algorithm, we get the most optimal solution of the redefined output in a family tracks. Then a precise output tracking is achieved from the Stable Initial State through non-causal stable inversion. Consequently, the magnitude of the redefined trajectory is minimized. There are several different ways to achieve the ideal Stable Initial State for the inversion system. In contrast with the available techniques, the new method can move the state to the desired value precisely at a limited time. It is well designed with exponential form accompanying quite small amplitude as we expected. No oscillation circumstance can be found here. To verify the effectiveness of this new approach, simulation results are presented for a tip trajectory tracking of a one-link flexible manipulator.

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2 BASIC SYSTEM FRAMEWORK

For a linear non-minimum SISO system,

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t)\end{cases} \tag{1}$$

Where $x \in \Re^{n\times 1}$, $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $C \in \Re^{1\times n}$, y is a scalar. The relative degree of the system is r, in the trivial case, it is assumed that $r < n$ strictly [14].

$$y^{(r)} = CA^r x + CA^{(r-1)}Bu \tag{2}$$

From equation (2), we have

$$u = (CA^{(r-1)}B)^{-1}(y^{(r)} - CA^r x) \tag{3}$$

Substituting equation (3) into equation (1), consequently

$$\dot{x} = \overline{A}x + \overline{B}y^{(r)}(t) \tag{4}$$

Where $\bar{A} = A - B(CA^{(r-1)}B)^{-1}CA^{(r)}$, $\bar{B} = B(CA^{(r-1)}B)^{-1}$.

Thus we have the equivalent system

$$\begin{cases} \dot{x} = \bar{A}x + \bar{B}y^{(r)}(t) \\ y = Cx \end{cases} \tag{5}$$

There exists a matrix T_1 , making the coordinate transformation,

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = T_1 x \text{ , } x = T_1^{-1} \begin{bmatrix} \xi \\ \eta \end{bmatrix} \tag{6}$$

Substituting equation (6) into (3),

$$\begin{aligned} u &= (CA^{(r-1)}B)^{-1}(y^{(r)} - CA^{r}x) \\ &= (CA^{(r-1)}B)^{-1}(y^{(r)} - CA^{r}T_1^{-1}\begin{bmatrix} \xi \\ \eta \end{bmatrix}) \\ &= (CA^{(r-1)}B)^{-1}(y^{(r)} - \begin{bmatrix} Q_\xi & Q_\eta \end{bmatrix}\begin{bmatrix} \xi \\ \eta \end{bmatrix}) \\ &= C_y y^{(r)} + C_\xi \xi + C_\eta \eta \end{aligned} \tag{7}$$

Where

$C_y = (CA^{(r-1)}B)^{-1}$, $C_\xi = -(CA^{(r-1)}B)^{-1}Q_\xi$,

$C_\eta = -(CA^{(r-1)}B)^{-1}Q_\eta$.

Defining the external state vector

$$\xi = \begin{bmatrix} y, & \dot{y}, \cdots & y^{(r-1)} \end{bmatrix}^T \tag{8}$$

The original system can be written in the new coordinates as

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = T_1 \bar{A} T_1^{-1} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + T_1 \bar{B} y^{(r)} \tag{9}$$

Further we get the internal states equation

$$\dot{\eta} = A_\eta \eta + B_\eta y^{(r)} \tag{10}$$

We name η as internal state vector,

$\eta \in \Re^{(n-r)\times 1}$, $A_\eta \in \Re^{(n-r)\times(n-r)}$, $B_\eta \in \Re^{(n-r)\times 1}$, $y^{(r)} \in \Re^{(n-r)\times 1}$.

And (10) can be further decomposed. For matrix A_η , there exist invertible matrix V , we make the linear transformation

$$\eta = V \begin{bmatrix} \eta_s \\ \eta_u \end{bmatrix} \tag{11}$$

Then we get

$$\begin{bmatrix} \dot{\eta}_s \\ \dot{\eta}_u \end{bmatrix} = \bar{J}_A \begin{bmatrix} \eta_s \\ \eta_u \end{bmatrix} + \bar{J}_B y^{(r)} \tag{12}$$

$\bar{J}_A$ is a real Jordan form of A_η , matrix $\bar{J}_A$, $\bar{J}_B$ can be divided into blocks as

$$\bar{J}_A = V^{-1} A_\eta V = \begin{bmatrix} A_S & 0 \\ 0 & A_u \end{bmatrix}, \ \bar{J}_B = V^{-1} B_\eta = \begin{bmatrix} B_s \\ B_u \end{bmatrix}$$

where A_S is a diagonal matrix with diagonal elements that are the eigenvalues of A_S with negative real parts, and A_u is a diagonal matrix with diagonal elements that are the eigenvalues of A_u with positive real parts. As a result, the internal stable and unstable state differential equations can be written as

$$\dot{\eta}_s(t) = A_s \eta_s(t) + B_s y^{(r)}(t) \tag{13}$$

$$\dot{\eta}_u(t) = A_u \eta_u(t) + B_u y^{(r)}(t) \tag{14}$$

Definition 1: Inverse system

Equation (10), together with equation (7) compose the inverse system,

$$\begin{cases} \dot{\eta}(t) = A_\eta \eta(t) + B_\eta y^{(r)}(t) \\ u(t) = C_y y^{(r)}(t) + C_\xi \xi(t) + C_\eta \eta(t) \end{cases} \tag{15}$$

In which the reference trajectory becomes the new input and the original input $u(t)$ will become the new output.

Definition 2: The reference output trajectory

The reference output trajectory $y_d(t)$ is defined on time interval $[t_0, t_f]$. Meanwhile the reference output trajectory is required to be sufficiently smooth which means $y_d^{(i)}(t) \in L_1 \cap L_\infty$, $i = 0, 1, \ldots, r$.

3 NON-CAUSAL STABLE INVERSION

3.1 Stable Inversion Process

To solve the inverse system (15), we let $y = y_d$, $\xi = \xi_d$, and $\xi_d = [y_d, ..., y_d^{(r-1)}]^T$. We have the internal state differential equations of the system, as equation (13) and (14) shows, the following bounded dynamic can be calculated.

$$\eta_{sd}(t) = \int_{-\infty}^{t} e^{A_s(t-\tau)} B_s y_d^{(r)}(\tau) d\tau \tag{16}$$

$$\eta_{ud}(t) = -\int_{t}^{\infty} e^{-A_u(\tau-t)} B_u y_d^{(r)}(\tau) d\tau \tag{17}$$

It can be easily integrated of the stable part in forward time and the unstable part in backward time. Then we get η_d ,

$$\eta_d = V \begin{bmatrix} \eta_{sd} \\ \eta_{ud} \end{bmatrix} \tag{18}$$

Because ξ_d is already known, from equation (7), we have

(19)

Remark 1 The non-causal stable inversion can be regarded as the only precise technique for output tracking of non-minimum systems. Non-causality means the system doesn't start from the actual initial state, but a desired initial value existing in the unstable manifolds. We call the ideal initial state of the system Stable Initial State [11].

3.2 Stable Initial State

The desired Stable Initial State can be obtained as

$$x_d(t_0) = T^{-1} \begin{bmatrix} \mathbf{0}_{r\times 1} \\ \eta_d(t_0) \end{bmatrix}_{n\times 1}, \ \eta_d(t_0) = V \begin{bmatrix} \eta_{sd}(t_0) \\ \eta_{ud}(t_0) \end{bmatrix} \tag{20}$$

With equation (17), we get the desired initial value of unstable state

$$\eta_{ud}(t_0) = -\int_{t_0}^{\infty} e^{-A_u(\tau-t_0)} B_u y_d^{(r)}(\tau) d\tau \tag{21}$$

To assure the tracking accuracy, $x(t)$ must equal to the desired state $x_d(t)$ for all $t(t \ge t_0)$, then $\eta_u(t) = \eta_{ud}(t)$ must be fulfilled. It can be proved that the initial condition $\eta_u(t_0) = \eta_{ud}(t_0)$ in stable inversion cases is only need to be considered. The proof is presented as follows.

From (15),

$$\begin{aligned}\eta_u(t) &= \eta_u(t_0)e^{A_u(t-t_0)} + \int_{t_0}^{t} e^{A_u(t-\tau)} B_u y_d^{(r)}(\tau)d\tau \\ &= e^{A_u t}(e^{-A_u t_0}\eta_u(t_0) + \int_{t_0}^{t} e^{-A_u\tau} B_u y_d^{(r)}(\tau)d\tau)\end{aligned} \tag{22}$$

Supposing the internal unstable state reach its desired initial value,

$$\eta_u(t_0) = -\int_{t_0}^{+\infty} e^{-A_u(\tau-t_0)} B_u y_d^{(r)}(\tau)d\tau \tag{23}$$

Substituting equation (23) into (22), we get

$$\begin{aligned}\eta_u(t) &= e^{A_u t}(-e^{-A_u t_0}\int_{t_0}^{+\infty} e^{-A_u(\tau-t_0)} B_u y_d^{(r)}(\tau))d\tau \\ &\quad + \int_{t_0}^{t} e^{-A_u\tau} B_u y_d^{(r)}(\tau)d\tau) \\ &= e^{A_u t}(\int_{+\infty}^{t} e^{-A_u\tau} B_u y_d^{(r)}(\tau)d\tau) \\ &= -\int_{t}^{\infty} e^{-A_u(\tau-t)} B_u y_d^{(r)}(\tau)d\tau \\ &= \eta_{ud}(t)\end{aligned} \tag{24}$$

As to stable inner state

$$\eta_{sd}(t_0) = \int_{-\infty}^{t_0} e^{A_s(t_0-\tau)} B_s y_d^{(r)}(\tau)d\tau \tag{25}$$

According to definition 3, we know $y_d = 0$, resulting $y_d^{(r)} = 0$ for $t \notin [t_0, t_f]$, thus $\eta_{sd}(t_0) = 0$. Then

$$\begin{aligned}\eta_s(t) &= \eta_s(t_0)e^{A_s t} + \int_{t_0}^{t} e^{A_s(t-\tau)} B_u y_d^{(r)}(\tau)d\tau \\ &= \int_{t_0}^{t} e^{A_s(t-\tau)} B_u y_d^{(r)}(\tau)d\tau \\ &= \int_{-\infty}^{t} e^{A_s(t-\tau)} B_s y_d^{(r)}(\tau)d\tau \\ &= \eta_{sd}(t)\end{aligned} \tag{26}$$

Remark 2 For any time $t > t_0$, once the internal unstable initial state value is precisely achieved, the Stable Initial State can be reached. Consequently the precise output tracking can be realized. The output tracking error depends on the error between the actual state value and Stable Initial State at t_0.

4 METHODS TO OBTAIN SATBLE INITIAL STATE

4.1 Output redefinition method

The main idea of output redefinition method is based on redefining a new trajectory with functions so that a bounded continuous input signal can be obtained, while canceling the effect of unstable zeros at the same time. As a result, the initial state of the system can be converted to Stable Initial State. We re-define the output functions over $t \in [t_i, t_0]$,

$$\tilde{y}_d = \sum_{j=1}^{h} c_j e^{m_j t}, \quad \ddot{\tilde{y}}_d = \sum_{j=1}^{h} m_j^2 c_j e^{m_j t} \tag{27}$$

$$\tilde{\xi}_d = [\tilde{y}_d, ..., \tilde{y}_d^{(r-1)}]^T \tag{28}$$

Substituting $\tilde{y}_d$, $\tilde{\xi}_d$ into the system. Suppose the system $r = 2$, $n = 4$. The equation (13), (14) can be rewritten as

$$\dot{\tilde{\eta}}_{sd} = a_s\tilde{\eta}_{sd} + b_s\ddot{\tilde{y}}_d \ (a_s < 0) \tag{29}$$

$$\dot{\tilde{\eta}}_{ud} = a_u\tilde{\eta}_{ud} + b_u\ddot{\tilde{y}}_d \ (a_u > 0) \tag{30}$$

where h is the number of the exponential function term determined by the restriction conditions set in desired states and output trajectory. Constant m_j is chosen in advance with a search algorithm to find the most optimistic value. At last, c_j is only needed to be considered.

Firstly, we introduce the search algorithm to find the optimal constant m_j. Regarding $m_{\max}$ and $m_{\min}$ as the variables, according to equation (31), all the constants m_j, $(j = 1...r)$ are variables relative to $m_{\max}$ and $m_{\min}$, and $m_{\min} \le m_{\max} \le 0$.

$$m_j = m_{\max} + \frac{(m_{\min} - m_{\max})}{(r-1)}(j-1), (j = 1...r) \tag{31}$$

The optimum principle is to have the smallest tracking error

$$y_{err} = \sqrt{\int_{t_i}^{t_0} (y_d - \tilde{y}_d)^2 dt} \tag{32}$$

And we set the reference output to get the Stable Initial State $y_d = 0$, $t \in [t_i, t_0]$, then

$$y_{err} = \sqrt{\int_{t_i}^{t_0} (\tilde{y}_d)^2 dt} \tag{33}$$

Then y_{err} is a cost function merely related to variable $m_{\max}$ and $m_{\min}$. The plane $m_{\max}$ and $m_{\min}$ is meshed at first. Then at each node the value of y_{err} is calculated. Finally $m_{\max}$ and $m_{\min}$, which give the minimal value for y_{err} are selected as the best pair. Then the corresponding m_j is obtained.

Secondly, the process to solve the inverse system employing the new redefined trajectory is listed as follows. The solution of equation (29) is composed of the general solution $(\tilde{\eta}_s)_g$ of the homogeneous differential equation $\dot{\tilde{\eta}}_{sd} = a_s\tilde{\eta}_{sd}$, and a particular solution $(\tilde{\eta}_s)_p$ of (29). Then the general solution is $\tilde{\eta}_{sd} = (\tilde{\eta}_s)_g + (\tilde{\eta}_s)_p$, where

$$(\tilde{\eta}_{sd})_g = d_s e^{a_s t}, \quad (\tilde{\eta}_{sd})_p = \sum_{j=1}^{h} \frac{b_s m_j^2 c_j}{m_j - a_s} e^{m_j t} \tag{34}$$

At last we have

$$\tilde{\eta}_{sd} = d_s e^{a_s t} + \sum_{j=1}^{h} \frac{b_s m_j^2 c_j}{m_j - a_s} e^{m_j t} \tag{35}$$

Similarly, to solve equation (30), $\tilde{\eta}_{ud} = (\tilde{\eta}_u)_g + (\tilde{\eta}_u)_p$,

$$(\tilde{\eta}_u)_g = d_u e^{a_u t}, \quad (\tilde{\eta}_u)_p = \sum_{j=1}^{h} \frac{b_u m_j^2 c_j}{m_j - a_u} e^{m_j t} \tag{36}$$

because $a_u > 0$, to make sure $\tilde{\eta}_u$ is stable, condition $d_u = 0$ must be satisfied, then

$$\tilde{\eta}_{ud} = (\tilde{\eta}_u)_p = \sum_{j=1}^{h} \frac{b_u m_j^2 c_j}{m_j - a_u} e^{m_j t} \tag{37}$$

The new trajectory has to satisfy the restriction conditions originated from the system states and output trajectory .That means $\tilde{\eta}_{ud}$, $\tilde{\eta}_{sd}$, $\tilde{y}_d$, $\dot{\tilde{y}}_d$, $\ddot{\tilde{y}}_d$ should satisfy the initial and final conditions at time t_i, t_0. To be mentioned particularly, the bound conditions at time t_0 must be the Stable Initial State, meaning

$$\tilde{\eta}_u(t_0) = \eta_{ud}(t_0), \quad \tilde{\eta}_s(t_0) = \eta_{sd}(t_0) \tag{38}$$

The number of bound conditions is $h = 2n + 2$.

To simplify the matrix equation as

$$ZC = B_c \tag{39}$$

where Z is a constant matrix, C is the vector to be solved, B_c is the bound value. Once we get C, from (27) we can get $\tilde{y}_d$, $\dddot{\tilde{y}}_d$, further from (35) and (37), we have $\tilde{\eta}_{sd}$, $\tilde{\eta}_{ud}$, then $\tilde{\eta}_d$, $\tilde{x}_d$. At last, the output of the inverse system

$$\begin{aligned}\tilde{u}_d &= (CA^{(r-1)}B)^{-1}(\tilde{y}_d^{(r)} - CA^r \tilde{x}_d) \\ &= C_y \tilde{y}_d^{(r)} + C_\xi \tilde{\xi}_d + C_\eta \tilde{\eta}_d\end{aligned} \tag{40}$$

4.2 The pre-actuation method

If the pre-actuation process works from time negative infinity, the Stable Initial state can be obtained precisely. Apparently this is not practical in actual situation [12].

The inner unstable state error $\delta\eta_u(t_0)$ converges to zero as time decrease to negative infinity

$$\lim_{t \to -\infty} e^{-A_u(t_0 - t)} \delta\eta_u(t_0) = 0 \tag{41}$$

In practical situation, the system starts from a finite moment t_p, $t_p \in [-\infty, t_0]$, resulting the state error

$$\delta\eta_u(t_p) = e^{-A_u(t_0 - t_p)} \delta\eta_u(t_0) \tag{42}$$

Remark 3 Pre-actuation method is an approximate method to reach Stable Initial State during a finite time interval.

4.3 The optimal state to state transition (OST)

The optimal state to state transition method can transit an original state to the desired one in a finite time. But the output amplitude can't be optimal. In this way, the Stable Initial State can be achieved from the actual starting state. At a bounded time interval, we can find the input u_{ost} and system state x_{ost} to transfer the system state from one state $x(t_1)$ at time t_1 to another state $x(t_2)$ at time $t_2 (t_2 \geq t_1)$, assuring the control input energy is the smallest. The process is named as the optimal state to state transition. The input design formula is presented as

$$u(t) = R^{-1} B^{\mathrm{T}} e^{A^{\mathrm{T}}(t_2 - t)} G_{(t_1,t_2)}^{-1} [x(t_2) - e^{A(t_2 - t_1)} x(t_1)] \tag{43}$$

R is an arbitrary positive definite symmetric real matrix. A and B are given system parameters. $G_{(t_1,t_2)}$ is controllable and invertible and named as grammian matrix.

$$G_{(t_1,t_2)} = \int_{t_1}^{t_2} e^{A(t_2 - \tau)} B R^{-1} B^{\mathrm{T}} e^{A^{\mathrm{T}}(t_2 - \tau)} d\tau \tag{44}$$

We use OST method to achieve precise state transition from $\eta_u(t_i)$ to $\eta_u(t_0)$.

5 SIMULATION RESULTS

A linearized one-link flexible manipulator model is used for our simulation [15].The end-effector angular position is taken as the output for the system. The detailed systems parameters are shown as follows.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.0857 & -0.6857 & 0.0171 \\ 0 & -0.2743 & 1.7143 & -0.0549 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 68.5714 \\ -171.4286 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0.5 & 0 & 0 \end{bmatrix}, \; D = 0.$$

The desired output trajectory is defined as

$$y = \begin{cases} 0 & t \in [0,5] \\ 1 - e^{(-0.004(t-5)^5)} & t \in (5,13] \\ 1 - e^{(-0.004(21-t)^5)} & t \in (13,21] \\ 0 & t \in (21,26] \end{cases}$$

where $t_i = 0$, $t_0 = 5$, $t_f = 26$.

The relative degree $r = 2$, the transition matrix

$$T = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

The system can be transformed to have the stable and unstable differential equations as (29), (30). Then we can get the constants as

$a_s = -0.4665$, $b_s = 11.2480$, $a_u = 0.5146$, $b_u = -11.4639$.

5.1 Output redefinition method to get the Stable Initial State

The simulation result of the one-order search algorithm is shown as Fig. 1. With different values of $m_{\max}$ and $m_{\min}$, Fig.1 shows the error between the redefined output and the reference output. Then, we can get the minimum error under constant value $m_{\max} = 0$, $m_{\min} = -1$.

The output is redefined on time region $t \in [0,5]$. After the redefinition process, the system achieves the Stable Initial

State, $X_i = [0.3088 \quad -0.6177 \quad 0.1589 \quad -0.3178]^T$. With the Stable Initial State, the stable inversion can precisely track the desired output during time $t \in [5,26]$. The simulation result can be shown as Fig. 2, Fig. 3.

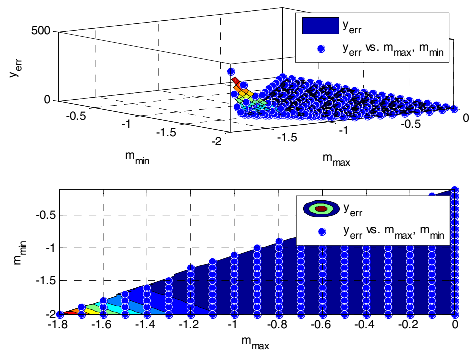

Fig. 1. Error between redefined the output and the reference output versus $m_{\max}$ and $m_{\min}$

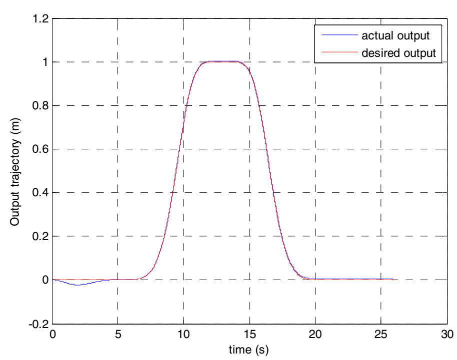

Fig. 2. Output tracking combines with output redefinition and stable inversion

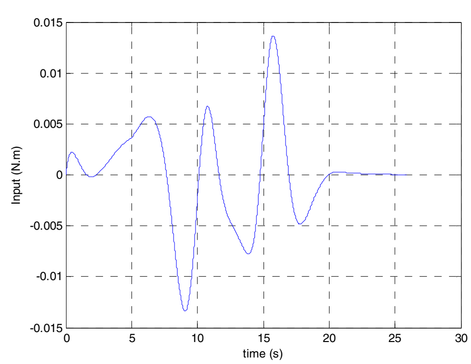

Fig. 3. The input signal of output redefinition and stable inversion process

5.2 Comparison with the pre-actuation technique

We have verified that the tracking accuracy is merely related to the error between the actual initial state and the Stable Initial State. The simulation result demonstrates a big tracking error when the pre-actuation process time is only $5s$. Theoretically, the pre-actuation time should be infinite. A tracking result contrast between the pre-actuation method and output redefinition method can be seen as Fig 4. Both the two technique work during time interval $t \in [0,5]$. It is clear that the redefinition method has a better tracking effect. Fig. 5 shows the tracking effect when pre-actuation starts from time $t = -15s$, a much better tracking effect which further confirms the pre-actuation process time should be as long as possible.

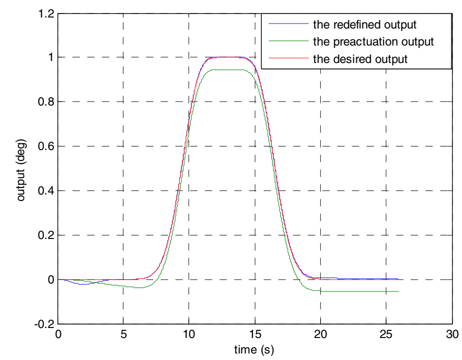

Fig. 4. The output contrast between output redefinition and pre-actuation method at whole time interval

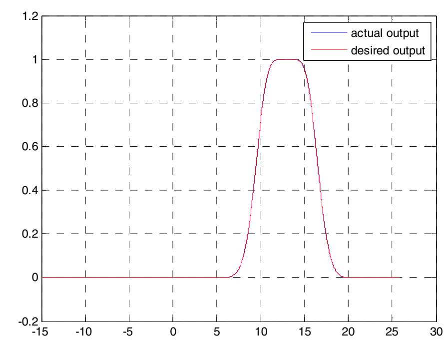

Fig. 5. Pre-actuation tracking effect with time interval $20s$

5.3 Comparison with the optimal state transition technique

The optimal state to state transition works during time period $t \in [0,5]$, and a comparison between the corresponding output and the redefined output can be seen as Fig 6. While the whole output tracking effect contrast is shown in Fig 7.

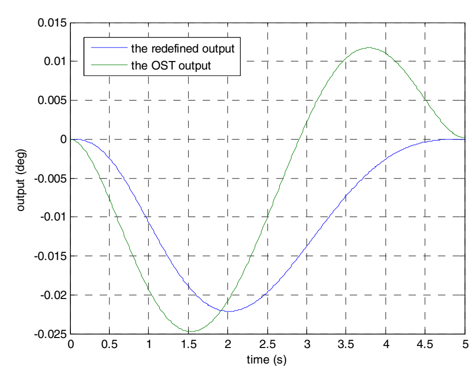

Fig. 6. The output contrast between output redefinition and OST process at time interval $[0,5]$

As the simulation results demonstrate, during the same time interval to obtain the Stable Initial State, the amplitude of redefined output is always smaller than the one from OST. Furthermore, there is no oscillated phenomenon found in the redefined trajectory method.

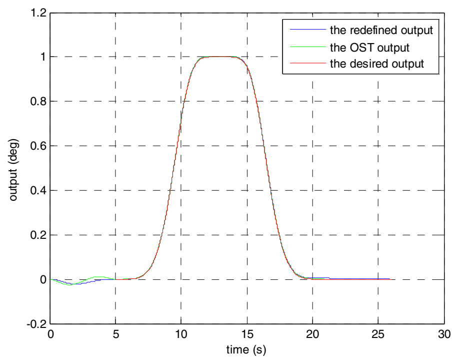

Fig 7. The output contrast between output redefinition and OST method at whole time interval

We can draw the conclusion from the simulation results that during the same time period, the output redefinition method has the most desired output amplitude as we hope. After the redefinition process, the Stable Initial State can be precisely obtained, resulting an accurate output tracking at the following time.

6 CONCLUSIONS

A new method to achieve precise output tracking for non-minimum systems by combining output redefinition with non-causal stable inversion is presented. The main idea amounts to redefine the output trajectory to Stable Initial State. Then stable inversion method can realize accurate output tracking. The output redefinition process can transit the system to the desired state values from the actual states at a limited time period. Besides, a search algorithm to find the most optimal track with the least amplitude is introduced here. Compared with the available techniques, our approach can reduce the pre-actuation time with quite minimal output amplitude. The simulation results support our theory here.

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