An integrated Model Predictive Control Strategy for Batch Processes

Li Jia[1], Chao Han[1], Min-sen Chiu[2]

1.Shanghai Key Laboratory of Power Station Automation Technology, Department of Automation, College of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200072, China
E-mail: jiali@staff.shu.edu.cn

2.Department of Chemical and Biomolecular Engineering, National University of Singapore, Singapore 117576
E-mail: checmc@nus.edu.sg

Abstract: A novel integrated model predictive control (MPC) strategy with model identification for batch processes is proposed in this paper. It systematically integrates batch-axis information and time-axis information into one uniform frame. The control law is obtained through the solution of a MPC optimization with time-varying prediction horizon, which leads to superior tracking performance and robustness against disturbance and uncertainty. Moreover, the model identification with online updated parameter algorithm is employed to eliminate the model-plant mismatch and match the real plant better. Next, the convergence analysis of the proposed integrated model predictive control system is given rigorous description and proof. Lastly, the effectiveness of the proposed method is verified by an example.

Key Words: batch process; integrated model predictive control (MPC); model identification

1 INTRODUCTION

Batch processes have been used increasingly in the production of low volume and high value added products, such as special chemicals, pharmaceuticals, and heat treatment processes for metallic or ceramic products[1]. However, with strong nonlinearity and dynamic characteristics, optimal control of batch processes is more challenging than that of continuous processes and thus it needs new non-traditional techniques.

Batch processes have the characteristic of repetition, and thus iterative learning control (ILC) can be used in the optimization control of batch processes[2]. But for ILC, only the batch-to-batch performance is taken for consideration but not the performance of real-time feedback. As a result, it is difficult to guarantee the performance of the batch process when real-time uncertainties and disturbances exist. Therefore, an integrated optimization control system is required to derive the maximum benefit from batch processes, in which the performance of time-axis and batch-axis are both analyzed synchronously. However, most reported two dimension control results[3-5] assume that the batch processes are linear or can be locally linearized for the feasibility of proof and analysis. How to extend 2D controller to a more general frame for nonlinear batch processes is still challenging.

To solve these problems, inspired by MPC with time-varying prediction horizon and ILC, a novel integrated model prediction control (MPC) strategy with model identification for batch processes is proposed in this paper, which adopts a more general frame of control system for nonlinear batch processes instead of the traditional 2D controller for linear or linearized batch process. Moreover, the new control strategy with model identification not only created a feedback controller in time-axis with iterative learning convergence in batch-axis, but also improved the accuracy of the model through online parameter modification. We also made the first attempt to give rigorous description and proof of the convergence of the proposed control system. Lastly, a simulation example illustrated the performance and applicability of the method.

2 SYSTEM DESCRIPTION

In this paper, the discussed nonlinear batch process can be described by the following discrete-time state-space representation.

$$\begin{cases} x_k(t+1) = f(x_k(t), u_k(t), t) \\ y_k(t) = g(x_k(t), t) \\ x_k(0) = x_0,\ t = 1, 2, \cdots, T;\ k = 1, 2, \cdots \end{cases} \tag{1}$$

where t and k denote time step and cycle index, respectively. $x_k(t) \in R^n$, $u_k(t) \in R^m$ and $y_k(t) \in R^l$ are, respectively, the state, the control input and the batch process output at time t in k-th cycle, and x_0 is the initial state of each cycle. $f(\cdot,\cdot,\cdot): R^n \times R^m \times R^n \times R^+ \to R^n$ and $g(\cdot,\cdot,\cdot): R^n \times R^+ \to R^l$ represent the system dynamics, T is the duration of each batch.

3 INTEGRATED MPC STRATEGY WITH MODEL IDENTIFICATION

3.1. Model Identification with Online Updated Parameter Algorithm

In this paper, Neuro-fuzzy model (NFM)[6] is employed to identify the proposed batch process. The NFM is described by the function $\hat{\mathbf{Y}}_k = \mathbf{\Phi}(\mathbf{U}_k)\mathbf{W}_k$, where $\mathbf{W}_k = \left[w_1(k), w_2(k), \cdots w_N(k)\right]^T$ are model adjustable parameters, and $\mathbf{\Phi}(\mathbf{U}_k)$ is a matrix decided by $\mathbf{U}_k$. It is evident that $\mathbf{U}_k$ is the variable of function $\mathbf{\Phi}(\cdot)$. More specificity, $\hat{\mathbf{Y}}_k = \mathbf{\Phi}(\mathbf{U}_k)\mathbf{W}_k$ can be written as

$$
\begin{aligned}
\hat{\mathbf{Y}}_k &= \sum_{i=1}^{N} \hat{\alpha}_i \cdot f_i(\mathbf{U}_k) \\
&= \left(f_1(\mathbf{U}_k), f_2(\mathbf{U}_k), ..., f_N(\mathbf{U}_k)\right) \cdot (\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_N)^T \\
&= \mathbf{\Phi}(\mathbf{U}_k) \cdot \mathbf{W}_k
\end{aligned}
$$

And the function $\mathbf{\Phi}(\mathbf{U}_k)$ is

$$
\mathbf{\Phi}(\mathbf{U}_k) = \frac{\left[\mu_1(V(\mathbf{U}_k)), \mu_2(V(\mathbf{U}_k)), \cdots, \mu_N(V(\mathbf{U}_k))\right]}{\sum_{j=1}^{N} \mu_j(V(\mathbf{U}_k))}
$$

where $V(\mathbf{U}_k) = [\mathbf{Y}_k, \mathbf{U}_k] = [v_1, v_2, \cdots, v_M]$, v_i is the i-th input variable of the NFM $(i = 1, 2, \cdots M)$, $\mu_j(V(\mathbf{U}_k))$ denotes Gaussian membership function $\mu_j(V(\mathbf{U}_k)) = \exp\left(-\sum_{i=1}^{M} \frac{(v_i - c_{ji})^2}{\sigma_j^2}\right)$, and c_{ji} and σ_j are center and width, respectively.

Owing to the model-plant mismatch, the process output may not be same as the one predicted by the model. The offset between the measured output and the model prediction is termed as model prediction error defined by

$$
\hat{e}_k(t) = y_k(t) - \hat{y}_k(t) \tag{2}
$$

And the tracking error is defined as

$$
e_k(t) = y_d(t) - y_k(t) \tag{3}
$$

Based on our previous work[6], assuming that the matrixes $\mathbf{W}_k(t)$ and $\mathbf{\Phi}(\bar{\mathbf{U}}_k(t))$ are both bounded, the following parameter updated strategy is convergent along the batch direction while eliminating model-plant mismatch online.

$$
\begin{aligned}
\mathbf{W}_k(t) = &\left(p_1 \cdot \mathbf{\Phi}^T(\bar{\mathbf{U}}_k(t))\mathbf{\Phi}(\bar{\mathbf{U}}_k(t)) - p_2 \cdot \mathbf{I}_N\right)^{-1} \cdot \\
&\left(p_1 \cdot \mathbf{\Phi}^T(\bar{\mathbf{U}}_k(t))\bar{\mathbf{Y}}_k(t) + p_2 \cdot \mathbf{W}_{k-1}(t)\right)
\end{aligned} \tag{4}
$$

where

$$
\bar{\mathbf{U}}_k(t) = \begin{bmatrix} u_k(0) \\ u_k(1) \\ \vdots \\ u_k(t) \end{bmatrix}_{(t+1)\times 1}, \bar{\mathbf{Y}}_k(t) = \begin{bmatrix} y_k(0) \\ y_k(1) \\ \vdots \\ y_k(t) \end{bmatrix}_{(t+1)\times 1},
$$

$$
\mathbf{P}_1 = p_1 \cdot \mathbf{I}_{t+1}, \mathbf{P}_2 = p_2 \cdot \mathbf{I}_N, t = 0, 1, \cdots, T-1,
$$

where $p_2 = \beta k^2$, p_1 and β are both positive real numbers.

3.2. Integrated MPC System

For the k-th batch, the integrated MPC strategy with time-varying prediction horizon applied along time axis can be given by the following quadratic cost function:

$$
\begin{aligned}
\min J(\mathbf{U}_k(t\,|\,t), k, t) = &\left\|\mathbf{Y}_d(t+1) - \hat{\mathbf{Y}}_{P_t}(t+1\,|\,t)\right\|_{\mathbf{Q}}^2 + \\
&\left\|\mathbf{U}_k(t\,|\,t) - \mathbf{U}_{k-1}(t)\right\|_{\mathbf{R}}^2 + \left\|\Delta\mathbf{U}_k(t\,|\,t)\right\|_{\bar{\mathbf{R}}}^2
\end{aligned} \tag{5}
$$

Subject to

$$
\begin{aligned}
&\hat{\mathbf{Y}}_{P_t}(t+1\,|\,t) = \hat{\mathbf{Y}}_{P_t}(\mathbf{U}_k(t\,|\,t)) \\
&\mathbf{U}^{low} \le \mathbf{U}_k(t\,|\,t) \le \mathbf{U}^{up} \\
&\mathbf{Y}^{low} \le \mathbf{Y}_k \le \mathbf{Y}^{up}
\end{aligned} \tag{6}
$$

where

$P_t = T - t,\ t = 0, 1, \cdots, T-1,$

$$
\mathbf{Y}_d(t+1) = \begin{bmatrix} y_d(t+1) \\ y_d(t+2) \\ \vdots \\ y_d(T) \end{bmatrix}_{P_t\times 1}, \hat{\mathbf{Y}}_{P_t}(t+1\,|\,t) = \begin{bmatrix} \hat{y}(t+1\,|\,t) \\ \hat{y}(t+2\,|\,t) \\ \vdots \\ \hat{y}(T\,|\,t) \end{bmatrix}_{P_t\times 1},
$$

$$
\mathbf{U}_k(t\,|\,t) = \begin{bmatrix} u_k(t\,|\,t) \\ u_k(t+1\,|\,t) \\ \vdots \\ u_k(T-1\,|\,t) \end{bmatrix}_{P_t\times 1}, \mathbf{U}_{k-1}(t) = \begin{bmatrix} u_{k-1}(t) \\ u_{k-1}(t+1) \\ \vdots \\ u_{k-1}(T-1) \end{bmatrix}_{P_t\times 1},
$$

$$
\Delta\mathbf{U}_k(t\,|\,t) = \begin{bmatrix} u_k(t\,|\,t) - u_k(t-1\,|\,t-1) \\ u_k(t+1\,|\,t) - u_k(t\,|\,t) \\ \vdots \\ u_k(T-1\,|\,t) - u_k(T-2\,|\,t) \end{bmatrix}_{P_t\times 1},
$$

$\mathbf{U}^{low}$ and $\mathbf{U}^{up}$ are the lower and upper bounds of the input sequence, $\mathbf{Y}^{low}$ and $\mathbf{Y}^{up}$ are the lower and upper bounds of the output sequence. $\mathbf{Q}$, $\mathbf{R}$ and $\bar{\mathbf{R}}$ are weighting matrices defined as $\mathbf{Q} = q \times \mathbf{I}_{P_t}$, $\mathbf{R} = r \times \mathbf{I}_{P_t}$ and $\bar{\mathbf{R}} = \bar{r} \times \mathbf{I}_{P_t}$, where q, r and $\bar{r}$ are all positive real numbers.

It should be noted that above MPC prediction horizon P_t is time-varying which ranges from current instant t to the final instant of a batch and the control horizon is the same length as prediction horizon.

In summary, the following steps describe the algorithm of integrated MPC strategy.

Algorithm 1.

Step1 Identify data-based model based on historical batch operation date points. Let $k = 1$ and initialize $\mathbf{U}_k$, $\mathbf{Q}$, $\mathbf{R}$ and $\bar{\mathbf{R}}$, Set $t = 1$.

Step2 At the t-th instant of the k-th batch, update the parameters $\mathbf{W}_k(t)$ of data-based model according to Eq. (4). Solve the optimization problem Eq. (5) to achieve $u_k(t|t)$ as the actual control signal, and then measure the corresponding output $y_k(t+1)$.

Step3 If $t<T$, set $t=t+1$ and go back to Step 2, else set $k=k+1,\ t=1$ and go to Step 2.

4 CONVERGENCE ANALYSIS

Theorem 1. Consider a batch process described by Eq. (1) controlled by Algorithm 1. With the strategy of model identification according to Eq. (4), the control sequence $\mathbf{U}_k$ of integrated MPC policy will converge to a constant sequence along batch cycle, whose increment corresponds to zero, namely $\Delta\mathbf{U}_k^{ILC}=\mathbf{U}_{k+1}^{ILC}-\mathbf{U}_k\to\mathbf{0}$ as $k\to\infty$.

Proof:

In the domain of batch-axis, we can rewrite Eq. (5)

$$\begin{aligned}&J\left(\mathbf{U}_k(t|t),k,t\right)\\&=J\left(u_k(t|t),k,t\right)+J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\ \mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t),k,t\right)\\&=\left\|y_d(t+1)-\hat{y}(t+1|t)\right\|_{\mathbf{Q}}^2+\left\|u_k(t|t)-u_{k-1}(t)\right\|_{\mathbf{R}}^2+\left\|\Delta u_k(t|t)\right\|_{\bar{\mathbf{R}}}^2+\\&\ J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\ \mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t),k,t\right)\end{aligned} \tag{7}$$

At the t-th instant of the k-th batch, the P_t dimension solution to the integrated MPC optimization problem in Eq. (5) $\mathbf{U}_k(t|t)$ is the optimal solution of the t-th instant.

Thus, we have

$$J\left(\mathbf{U}_k(t+1|t+1),k,t\right)\le J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\ \mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t),k,t\right) \tag{8}$$

According to Eqs. (7), (8), we get

$$\begin{aligned}&J\left(\mathbf{U}_k(t+1|t+1),k,t\right)+\left\|y_d(t+1)-\hat{y}(t+1|t)\right\|_{\mathbf{Q}}^2+\\&\left\|u_k(t|t)-u_{k-1}(t)\right\|_{\mathbf{R}}^2+\left\|\Delta u_k(t|t)\right\|_{\bar{\mathbf{R}}}^2\le J\left(\mathbf{U}_k(t|t),k,t\right)\end{aligned} \tag{9}$$

Specially, at the initial instant $t=0$, the following inequality holds.

$$J\left(\mathbf{U}_k(0|0),k,0\right)\le J(\mathbf{U}_{k-1},k,0)=\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_{k-1}\right\|_{\bar{\mathbf{R}}}^2 \tag{10}$$

From Eqs. (9), (10), after a batch, we have the following final form of inequality

$$\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\mathbf{U}_k-\mathbf{U}_{k-1}\right\|_{\mathbf{R}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2\le\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_{k-1}\right\|_{\bar{\mathbf{R}}}^2 \tag{11}$$

Furthermore, we get

$$0\le\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2\le\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_{k-1}\right\|_{\bar{\mathbf{R}}}^2 \tag{12}$$

$\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2$ decreases as k increasing and has lower bound. Therefore, the limit of $\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2$ exists, namely

$$\lim_{k\to\infty}\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2=\lim_{k\to\infty}\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_{k-1}\right\|_{\bar{\mathbf{R}}}^2 \tag{13}$$

From Eq. (11), we obtain

$$\begin{aligned}&\lim_{k\to\infty}\left\|\mathbf{U}_k-\mathbf{U}_{k-1}\right\|_{\mathbf{R}}^2\le\\&\lim_{k\to\infty}\left[\left(\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_k\right\|_{\bar{\mathbf{R}}}^2\right)-\left(\left\|\mathbf{Y}_d-\hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2+\left\|\Delta\mathbf{U}_{k-1}\right\|_{\bar{\mathbf{R}}}^2\right)\right]\end{aligned} \tag{14}$$

Eqs. (13), (14) show $\lim_{k\to\infty}\left\|\mathbf{U}_k-\mathbf{U}_{k-1}\right\|_{\mathbf{R}}^2\le 0$. That is to say, the proposed integrated MPC policy converges to zero along batch cycle, namely $\lim_{k\to\infty}\left\|\mathbf{U}_k-\mathbf{U}_{k-1}\right\|_{\mathbf{R}}^2=0$. This completes the proof. Q.E.D.

5 EXAMPLE

To demonstrate the effectiveness of the proposed scheme, we implemented it to control product quality of a typical nonlinear batch reactor, in which a first-order irreversible exothermic reaction $A\xrightarrow{k_1}B\xrightarrow{k_2}C$ takes place[7,8]. This process can be described by the following dynamic equations

$$\begin{aligned}\dot{x}_1&=-k_1\exp(-E_1/T)x_1^2\\\dot{x}_2&=k_1\exp(-E_1/T)x_1^2-k_2\exp(-E_2/T)x_2\end{aligned} \tag{15}$$

where x_1 and x_2 are respectively the reactant concentrations of A and B, and T denotes reactor temperature.

The reactor temperature is normalized by using $u=(T-T_{\min})/(T_{\max}-T_{\min})$. u is control variable bounded between $[0,1]$, and $x_2(t)$ is the output variable. The control objective is to maximize the yield of the reactor by choosing an appropriate reactor temperature policy u in the time interval $0\le t\le t_f$. The nominal operating conditions are: $x_1(0)=1$, $x_2(0)=0$.

In the simulation, we choose NFM model as the prediction model, and a NFM model is firstly constructed to represent the mapping relationship of $u \to x_2$, respectively. The control object is to drive system output $y = x_2$ approximating to $y_d(t_f) = 0.61$.

$\mathbf{R}$ is considered to be dynamic parameter[9] as follow:

$$\mathbf{R} = r \cdot \mathbf{I}_T, r = \frac{\tau_1}{1 + e^{\tau_2 \cdot \left| y_d(t_f) - y_k(t_f) \right|}} \tag{16}$$

Eq. (16) indicates that r would be increasing while $\left| y_d(t_f) - y_k(t_f) \right|$ decreasing. Thus if the final system output is close enough to the desired output, the change of control input would be very small. Initial values and parameters in the simulation are chosen as $\mathbf{U}_0 = 0,\ \tau_1 = 20,\ \tau_2 = 1\times10^3,\ \ q = 1\times10^3,\ \ \overline{r} = 0.1$.

The trajectories of the proposed controller system are shown in Figs. 1-3, respectively. To test the robustness, two additional cases are considered: the internal uncertainties and external disturbances. As seen from Fig. 4 and Fig. 5, the proposed control strategy has fast convergence rate and can maintain good performance despite the existence of noise.

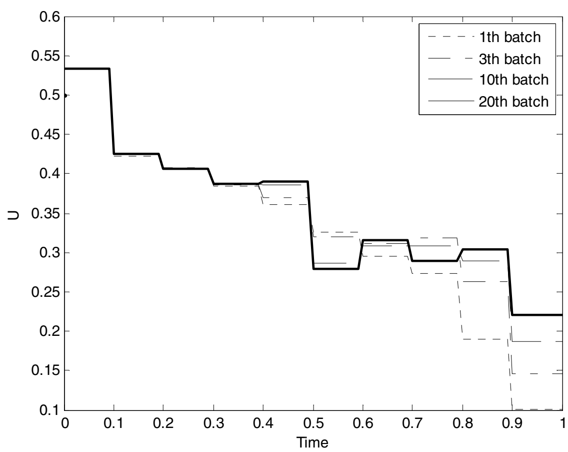


Fig. 1 Input trajectories at 1st, 3rd, 10th, 20th batches

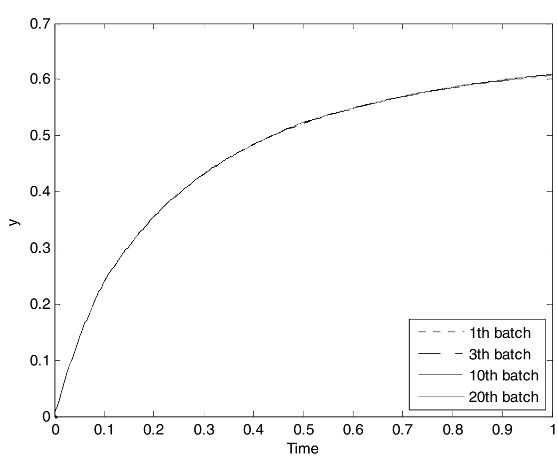


Fig. 2 Output trajectories at 1st, 3rd, 10th, 20th batches

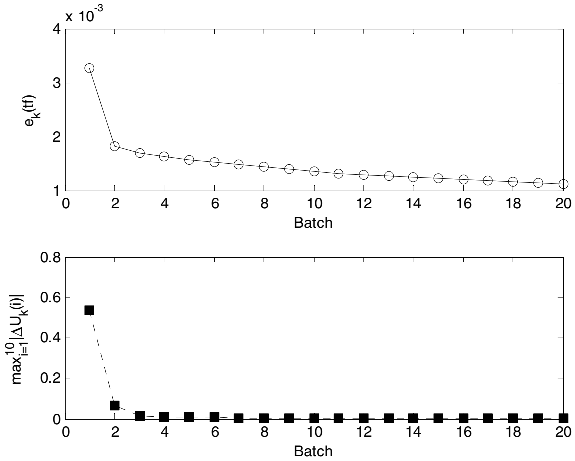
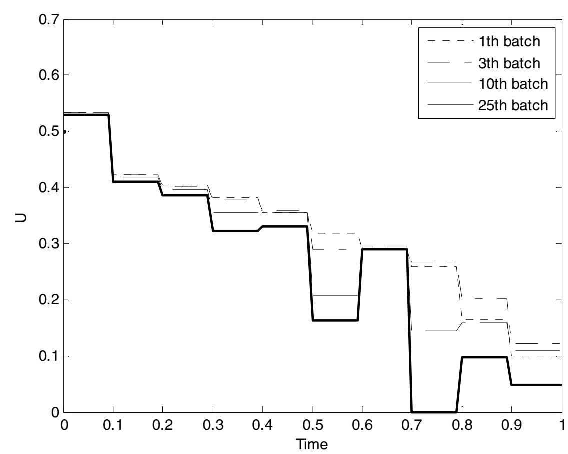


Fig.3 Curve of error and curve of $\max_{t=1}^{10}\left(\left|\Delta U_k(i)\right|\right)$

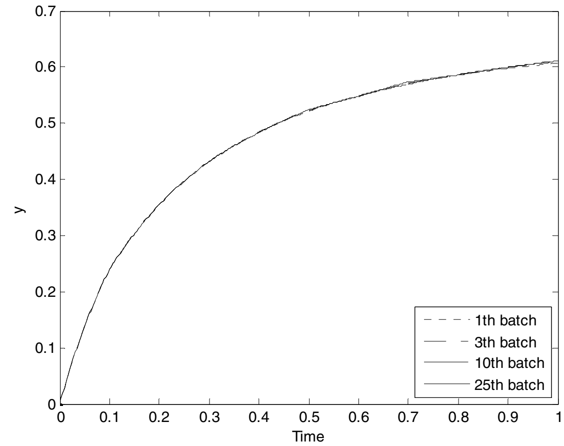
Case 1. The parameter of batch processes is varied to simulate the internal uncertainties. This case is simulated by changing parameter E_2 along batch axis.

$$E_2 = E_{20}\left[1.1 - 0.1\exp(-0.01k)\right]$$

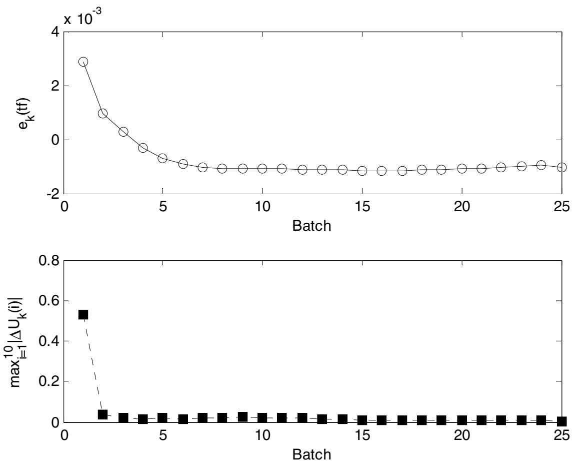
where E_{20} is the nominal value of E_2. Eq.5-2 indicates when batch index k increases from 1 to ∞, the corresponding dynamic parameter E_2 varies ranging from E_{20} to $1.1E_{20}$.



(a) Input trajectories at 1st, 3rd, 10th, 20th batches



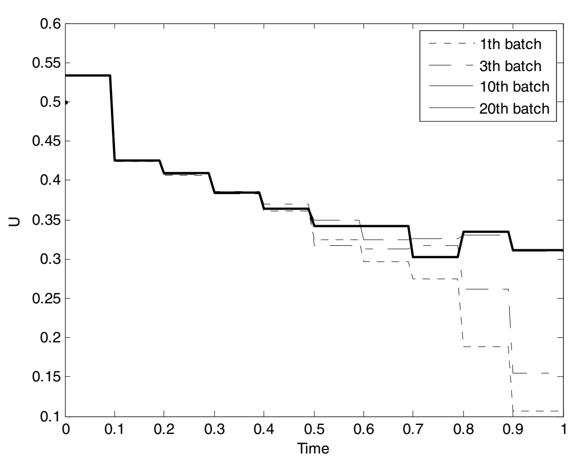
(b) Output trajectories at 1st, 3rd, 10th, 20th batches



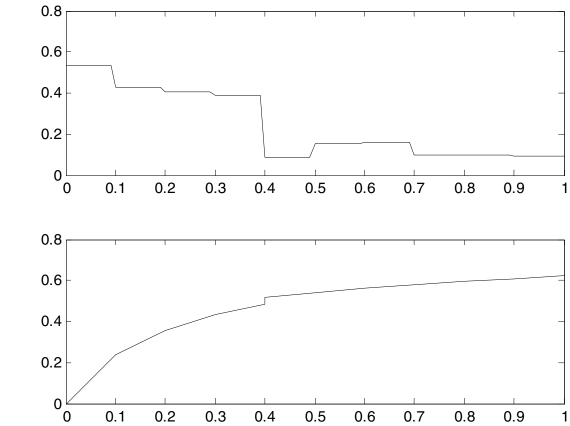
(c) Curve of error and curve of $\max_{t=1}^{10}\left(\left|\Delta U_k(i)\right|\right)$

Fig. 4 Trajectories of controller system under internal uncertainties

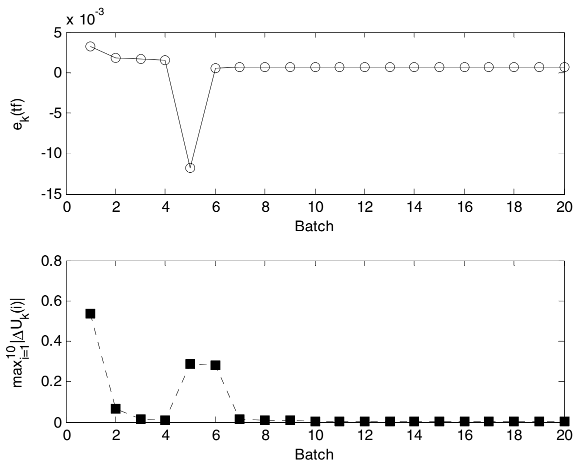
Case 2. The output of batch processes is corrupted by $5\% \cdot y_d$ external disturbance at instant $t=4$ of the 5-th batch in this case.



(a) Input trajectories at 1st, 3rd, 10th, 20th batches



(b) 5-th batch input and output trajectory



(c) Curve of error and curve of $\max_{t=1}^{10}\left(\left|\Delta U_k(i)\right|\right)$

Fig. 5 Trajectories of controller system under external disturbances

6 CONCLUSION

In this paper, a novel integrated model predictive control strategy with model identification was proposed, which not only combined realtime feedback with batch convergence into one frame, but also improved the accuracy of the model through online identification. Moreover, we made the first attempt to give rigorous description and proof of the convergence of the proposed integrated model predictive control system. Lastly, an example illustrated the performance and applicability of the proposed integrated optimization control. Simulation results demonstrate that the proposed integrated control strategy has good tracking performance and robustness.

REFERENCES

[1] D. Bonvin, Optimal operation of batch reactors: a personal view, J. Process Contr.8, 355–368 (1998).

[2] J.H. Lee, Lee.F K.S., Iterative learning control applied to batch processes: an overview, Control Eng. Pract. 15 (10), 1306–1318 (2007).

[3] X.-D. Li, T.W.S. Chow, J.K.L. Ho, 2-D system theory based iterative learning control for linear continuous systems with time delays, IEEE Trans. Circuits Syst. I,Reg. Papers 52 (7), 1421–1430 (2005).

[4] T. Liu, F. Gao, Robust two-dimensional iterative learning control for batch processes with state delay and time-varying uncertainties, Chem. Eng. Sci. 65, 6134–6144 (2010).

[5] C. Chen, Z.-H. Xiong, Y. Zhong, Design and analysis of integrated predictive iterative learning control for batch process based on two-dimensional system theory[J], Chinese Journal of Chemical Engineering 22, 762–768 (2014).

[6] L. Jia, J.P. Shi, M.-S. Chiu, Integrated neuro-fuzzy model and dynamic r-parameter based quadratic criterion-iterative learning control for batch process control technique, Neurocomputing 98, 24–33 (2012).

[7] W.H. Ray, Advanced Process Control, McGraw-Hil, New York, 1981.

[8] Z.-H. Xiong, J. Zhang, Product quality trajectory tracking in batch processes using iterative learning control based on time-varying perturbation models, Am. Chem. Soc. 42 (26), 6802–6814 (2003).

[9] L. Jia, T. Yang, M.-S. Chiu, An integrated iterative learning control strategy with model, Journal of Process Control 23, 1332– 1341 (2013).