Robust output feedback based iterative learning control for batch processes with input delay subject to time-varying uncertainties

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Abstract: A robust closed-loop iterative learning control (ILC) method is proposed in this paper for industrial batch processes with input delay subject to time-varying uncertainties. By introducing a novel two-dimensional (2D) state observer for predicting the augmented closed-loop 2D system states to describe batch operation characteristics, only measured outputs of current and previous cycles are used for robust feedback control and ILC design. Delay-dependent sufficient condition in terms of matrix inequalities is established by constructing a comprehensive 2D Lyapunov-Krasovskii functional candidate along with free-weighting matrices. By solving these matrix inequalities using a modified cone complementarity linearization (CCL) method, the closed-loop ILC controller is explicitly formulated together with an adjustable robust H infinity performance index. An illustrative example of injection molding machine is shown to demonstrate the effectiveness and merit of the proposed ILC method.

Key Words: Industrial batch process, input delay, time-varying uncertainties, iterative learning control (ILC), state predictor, robust H infinity control

1 INTRODUCTION

Iterative learning control (ILC) has been increasingly practiced in the recent years for industrial and chemical batch processes to realize perfect tracking and control optimization, owing to the use of historical cycle data from repetitive operation of a batch process. As surveyed in the recent papers [1,2], an amount of ILC control methods have been well developed in both continuous- and discrete-time domains that can realize perfect tracking for linear or nonlinear batch processes. As a matter of fact, ILC methods based on time-invariant process modeling cannot hold robust stability for time-varying batch processes, and therefore, cannot be used for robust tracking or on-line optimization from batch to batch. To cope with these problems, closed-loop feedback control has been explored to combine with an ILC scheme, resulting in the so-called feedback feed-forward ILC, i.e. two-dimensional (2D) control that determines the dynamical behavior along both the time and batchwise directions [3]. A number of robust ILC methods have been recently proposed to cope with structured and unstructured process uncertainties in batch process operation [2]. A few ILC methods have been developed using a 2D linear quadratic optimal control criterion combined with robust control theory to accommodate for a variety of process uncertainties [4-6]. In contrast, model predictive control based ILC schemes have also been proposed to meet implementation constraints [7, 8].

Time-delay, which widely exists in chemical and industrial processes, is a source of instability and performance degradation in closed-loop control systems and causes ILC design more difficult for batch processes with time-varying uncertainties. In frequency domain, an ILC algorithm based on Smith predictor control structure was proposed by Xu et al. [9], to improve the tracking performance for batch processes with input delay; Liu et al. [10] developed an IMC-based ILC method to cope with uncertain input delay.

* This work is supported in part by the National Thousand Talents Program of China, NSF China Grants 61473054, and the Fundamental Research Funds for the Central Universities of China.

In time domain, Xu et al. [11] proposed a dead-time compensation approach of ILC design to realize perfect tracking for batch processes with fixed input delay. Subsequently, Tan et al. [12] developed a phase lag compensation method for ILC design to cope with input delay. For batch processes with state delays, by transforming the process model into a 2D Fornasini-Marchsini (FM) model for describing batch operation, robust H infinity 2D iterative learning control methods [13, 14] were developed to accommodate for time-varying process uncertainties. However, the process states were required to be measured, which are not available in many engineering applications. It remains open as yet for closed-loop feedback ILC design based on only output measurement for batch processes with input delay and time-varying uncertainties.

In this paper, a robust closed-loop ILC method is proposed for batch processes with input delay and time-varying uncertainties. To facilitate closed-loop ILC design, the process together with batch operation is equivalently transformed into a 2D FM system description. For the convenience of implementation, only measured outputs of current and previous cycles are used for closed-loop ILC design. A novel 2D state observer is constructed for estimating the augmented 2D system states for the sake of control design. Delay-dependent sufficient matrix inequality conditions are derived by introducing a comprehensive 2D Lyapunov-Krasovskii functional candidate. By solving the matrix inequalities using a modified complementarity linearization (CCL) method, the closed-loop ILC controller is explicitly formulated, together with an adjustable robust H infinity performance index.

Throughout this paper, the following notations are used: $\Re^n$ denotes a n-dimensional Euclidean space. For any matrix $P \in \Re^{m \times m}$, $P > 0$ (or $P \geq 0$) means P is a positive definite (semidefinite) symmetric matrix, in which the symmetric elements are indicated by '*'. P^T denotes the transpose of P. The identity or zero vector (or matrix) with appropriate dimension is denoted by $\boldsymbol{I}$ or $\mathbf{0}$. For a

2D signal, $z(i,j)$, if $\|z(i,j)\|_2 = \sqrt{\sum_{i=0}^{n}\sum_{j=0}^{m}\|z(i,j)\|^2} < \infty$ for any integers n and m, then $z(i,j)$ is said to be in the $L_2[0, \infty)$ space of all square integrable functions.

2 PROBLEM FORMULATION AND 2D SYSTEM REPRESENTATION

A batch process with input delay subject to time-varying uncertainties is generally described by the following observable canonical discrete-time state-space model,

$$\begin{cases} x(t+1,k+1) = [A+\Delta A(t,k+1)]x(t,k+1) + [B+\Delta B(t,k+1)] \\ \qquad \times u(t-d,k+1), \\ y(t,k+1) = Cx(t,k+1), \quad 0 \le t \le T_{\mathrm{p}}, \\ x(0,k+1) = x(0), \quad k=0,1,\cdots. \end{cases} \tag{1}$$

where t and k denote the time and batch indices, respectively, and $k+1$ indicates the current batch (or cycle). Denote by $x(t,k+1) \in \Re^{n_x}$ the state variables, $u(t,k+1) \in \Re^{n_u}$ the control inputs, $y(t,k+1) \in \Re^{n_y}$ the process outputs. $x(0)$ is the initial resetting condition of each cycle, $\{A,B,C\}$ are the nominal system matrices with appropriate dimensions, and d is a constant delay. Denote by T_{p} the time period of each cycle. The matrices $\Delta A(t,k+1)$ and $\Delta B(t,k+1)$, respectively, indicates the uncertainties at time t in the $k+1$ cycle with the following structure:

$$\left[\Delta A(t,k+1),\ \Delta B(t,k+1)\right] = E\Delta(t,k+1)\left[F_a,\ F_b\right] \tag{2}$$

with $\Delta^T(t,k+1)\Delta(t,k+1) \le \boldsymbol{I}$, where E, F_a, F_b are known constant matrices with appropriate dimensions. Note that other process uncertainties such as from input actuator and output measurement may also be lumped into $\Delta A(t,k+1)$ and $\Delta B(t,k+1)$ for analysis.

Given a batch process described by (1), an ILC updating law of interest should have the following general form

$$\Sigma_{ILC}: \quad \begin{cases} u(t,k+1) = u(t,k) + r(t,k+1) \\ u(t,0) = 0, \quad 0 < t < T_p, \quad k = 0,1,\cdots \end{cases} \tag{3}$$

where $r(t,k+1)$ is referred to the updating law of iterative learning to be determined, $u(t,0)$ represents the initial value of iteration that is usually reset to zero for implementation.

The control objective in this paper is to determine a control law such that the system output can track the desired output profile (or target output trajectory) as close as possible against the process uncertainties and input delay.

To facilitate closed-loop ILC design to accommodate for time-varying uncertainties from cycle to cycle (in batchwise direction), we define output tracking error in the current cycle as

$$e(t,k+1) \triangleq y(t,k+1) - y_r(t) \tag{4}$$

where $y_r(t)$ denotes the desired output profile, and $y(t,k+1)$ the real output in the current cycle. Due to the existence of process response delay, the above tracking error should be rectified as

$$e(t,k+1) \triangleq y(t,k+1) - y_r(t-d) \tag{5}$$

Define a batchwise direction function of error as

$$\delta f(t,k+1) \triangleq f(t,k+1) - f(t,k) \tag{6}$$

where f may be state or output variable, and thus is in accordance with ILC updating law shown above.

It follows from (1) and (3) along with the definitions of (5) and (6) that

$$\begin{aligned} &e(t+1,k+1) = e(t+1,k) + C\delta x(t+1,k+1) \\ &\delta x(t+1,k+1) = \left[A+\Delta A(t,k+1)\right]\delta x(t,k+1) + \left[B+\Delta B(t,k+1)\right] \\ &\qquad \times r(t-d,k+1) + \omega(t,k+1) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \omega(t,k+1) \triangleq &\left[\Delta A(t,k+1) - \Delta A(t,k)\right]x(t,k) + \left[\Delta B(t,k+1) - \Delta B(t,k)\right] \\ &\times u(t-d,k) \end{aligned} \tag{8}$$

It is obvious that $\omega(t,k+1) \ne 0$ for any non-repetitive process uncertainties, and therefore, is viewed as a non-repetitive load disturbance to treat with. Thereby, an equivalent 2D system description of the considered batch process, together with an augmented state variable of tracking error information, can be expressed as

$$\begin{cases} x_g(t+1,k+1) = \left[A_1 + \Delta A_1\right]x_g(t,k+1) + A_2 x_g(t+1,k) + \left[B_g + \Delta B_g\right] \\ \qquad \times r(t-d,k+1) + D_g\omega(t,k+1) \\ y(t,k+1) = Gx_g(t,k+1) \\ z(t,k+1) = Lx_g(t,k+1) \end{cases} \tag{9}$$

where $x_g(t,k+1) \triangleq \left[\delta x^T(t,k+1) \quad e^T(t,k+1)\right]^T$, $L = [\mathbf{0} \quad 1]$

$$A_1 = \begin{bmatrix} A & 0 \\ CA & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} \mathbf{0} & 0 \\ \mathbf{0} & 1 \end{bmatrix},\ G = \begin{bmatrix} -C & 1 \\ \mathbf{0} & 1 \end{bmatrix}, D_g = \begin{bmatrix} \boldsymbol{I} \\ C \end{bmatrix},\ B_g = \begin{bmatrix} B \\ CB \end{bmatrix},$$

$$\Delta A_1 = \begin{bmatrix} \Delta A(t,k+1) & 0 \\ C\Delta A(t,k+1) & 0 \end{bmatrix} = \begin{bmatrix} E \\ CE \end{bmatrix}\Delta(t,k+1)\left[F_a \quad 0\right] \triangleq \overline{E}\Delta(t,k+1)\overline{F}_a,$$

$$\Delta B_g = \begin{bmatrix} \Delta B(t,k+1) \\ C\Delta B(t,k+1) \end{bmatrix} = \begin{bmatrix} E \\ CE \end{bmatrix}\Delta(t,k+1)F_b \triangleq \overline{E}\Delta(t,k+1)F_b. \tag{10}$$

From a view of control design, all the state variables in (9) are to be stabilized to final steady-state point of the origin in the state space. For batch process operation, individual measurements of all the process state variables are usually inaccurate or even unavailable in practice. Only measured output tracking errors in current and previous cycles are therefore used for ILC design to facilitate practical implementation. $z(t,k+1)$ indicates measured tracking error in the current cycle to be minimized against input delay and process uncertainties, i.e.,

$$\| z(t,k+1) \|_2 \le \gamma \| \omega(t,k+1) \|_2 \tag{11}$$

where γ is a prescribed level of disturbance attenuation, namely, a robust H infinity control performance level.

3 ROBUST 2D ILC DESIGN

Based on the equivalent 2D system described in (9), a novel 2D state predictor is proposed as

$$\begin{aligned} \tilde{x}(t+1,k+1) = &A_1\tilde{x}(t,k+1) + A_2\tilde{x}(t+1,k) + B_g r(t,k+1) + L_1\left[G\tilde{x}(t-d,k+1)\right. \\ &\left. - y(t,k+1)\right] + L_2\left[G\tilde{x}(t+1,k) - y(t+1+d,k)\right] \end{aligned} \tag{12}$$

where $\tilde{x}(t,k+1)$ is the predicted state of $x_g(t+d,k+1)$. The prediction error is computed by

$$\tilde{e}(t,k+1) \triangleq \tilde{x}(t-d,k+1) - x_g(t,k+1) \tag{13}$$

Note that the state predictor forecasts the augmented system states by d steps ahead. Hence, we have

$$\begin{aligned}\tilde{e}(t+1,k+1) &= A_1\tilde{e}(t,k+1) + [A_2 + L_2G]\tilde{e}(t+1,k) + L_1G\tilde{e}(t-d,k+1) \\ &\quad - \Delta A_1 x_g(t,k+1) - \Delta B_g r(t-d,k+1) - D_g\omega(t,k+1)\end{aligned} \tag{14}$$

Based on the predicted states, the following ILC updating law is considered

$$r(t,k+1) = K_1\tilde{x}(t,k+1) + K_2\tilde{x}(t+1,k) \tag{15}$$

where K_1 and K_2 are controller gains to be determined later.

Remark 1: The prediction of the augmented state $x_g(t,k+1)$ is used for robust feedback control design. In fact, the prediction accuracy is affected by model uncertainties especially for the case of long time delay.

Define an augmented state vector,

$$X(t,k+1) \triangleq \begin{bmatrix} x_g(t,k+1) + \tilde{e}(t,k+1) \\ \tilde{e}(t,k+1) \end{bmatrix} \tag{16}$$

Correspondingly, the closed-loop system is written as

$$\begin{cases} X(t+1,k+1) = \left[\hat{A}_1 + \Delta\hat{A}_1\right]X(t,k+1) + \left[\hat{A}_2 + \Delta\hat{A}_2\right]X(t+1,k) \\ \qquad + \hat{A}_d X(t-d,k+1) + \hat{D}_g\omega(t,k+1) \\ z(t,k+1) = [L \quad -L]X(t,k+1) \triangleq \hat{L}X(t,k+1) \end{cases}$$

(17)

where $\hat{D}_g = \begin{bmatrix}\mathbf{0} & -D_g^T\end{bmatrix}^T$,

$$\hat{A}_1 = \begin{bmatrix} A_1 + B_gK_1 & \mathbf{0} \\ \mathbf{0} & A_1 \end{bmatrix},\ \hat{A}_2 = \begin{bmatrix} A_2 + B_gK_2 & L_2G \\ \mathbf{0} & A_2 + L_2G \end{bmatrix},\ \hat{A}_d = \begin{bmatrix} \mathbf{0} & L_1G \\ \mathbf{0} & L_1G \end{bmatrix},$$

$$\begin{aligned}\Delta\hat{A}_1 &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\Delta A_1 - \Delta B_gK_1 & \Delta A_1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \bar{E} \end{bmatrix}\Delta(t,k+1)\begin{bmatrix} -\bar{F}_a - F_bK_1 & \bar{F}_a \end{bmatrix}, \\ &\triangleq \hat{E}\Delta(t,k+1)\hat{F}_a\end{aligned}$$

$$\Delta\hat{A}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\Delta B_gK_2 & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \bar{E} \end{bmatrix}\Delta(t,k+1)\begin{bmatrix} -F_bK_2 & \mathbf{0} \end{bmatrix} \triangleq \hat{E}\Delta(t,k+1)\hat{F}_b\,. \tag{18}$$

To derive a robust ILC updating law for the above closed-loop system, the following lemmas are briefed which play a crucial role in the derivation.

Lemma 1 [15]: For any constant matrix $M \in \mathbb{R}^{m\times m}$ with $M > 0$, integers $l_2 > l_1$, vector function $\chi : \{l_1, \cdots, l_2\} \to \mathbb{R}^m$, there stands

$$(l_2 - l_1 + 1)\sum_{i=l_1}^{l_2} \chi(i)^T M \chi(i) \ge \left(\sum_{i=l_1}^{l_2} \chi(i)\right)^T M \left(\sum_{i=l_1}^{l_2} \chi(i)\right) \tag{19}$$

Lemma 2 [18]: Given matrices X and Y, with appropriate dimensions, the following inequality holds for any scalar $\varepsilon > 0$ and matrix Θ that satisfies $\Theta^T\Theta \le \boldsymbol{I}$ with appropriate dimension

$$X\Theta Y + Y^T\Theta^T X^T \le \varepsilon XX^T + \varepsilon^{-1}Y^TY \tag{20}$$

The following theorem is given to analyze the robust stability of the closed-loop 2D system.

Theorem 1: The closed-loop 2D system (17) is asymptotically stable with the robust H infinity performance level in (11), if there exist matrices $P > P_0 > 0$, $R > 0$, $Q = Q^T$, $N \ge 0$, M, K_1, K_2, L_1, L_2 and scalar $\varepsilon > 0$ such that the following matrix inequalities are satisfied

$$\begin{bmatrix} \Omega_{11} & M_2^T + dN_{12} & \Omega_{13} & M_4^T + dN_{14} & \hat{L}^T & \hat{A}_1^T & d(\hat{A}_1^T - \boldsymbol{I}) & \hat{F}_a^T \\ * & \Omega_{22} & -M_2 + dN_{23} & dN_{24} & \mathbf{0} & \hat{A}_2^T & d\hat{A}_2^T & \hat{F}_b^T \\ * & * & \Omega_{33} & -M_4^T + dN_{34} & \mathbf{0} & \hat{A}_d^T & d\hat{A}_d^T & \mathbf{0} \\ * & * & * & -\gamma\boldsymbol{I} + dN_{44} & \mathbf{0} & \hat{D}_g^T & d\hat{D}_g^T & \mathbf{0} \\ * & * & * & * & -\gamma\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -P^{-1} + \varepsilon\hat{E}\hat{E}^T & \varepsilon d\hat{E}\hat{E}^T & \mathbf{0} \\ * & * & * & * & * & * & -dR^{-1} + \varepsilon d^2\hat{E}\hat{E}^T & \mathbf{0} \\ * & * & * & * & * & * & * & -\varepsilon\boldsymbol{I} \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} & M_1 \\ * & N_{22} & N_{23} & N_{24} & M_2 \\ * & * & N_{33} & N_{34} & M_3 \\ * & * & * & N_{44} & M_4 \\ * & * & * & * & R \end{bmatrix} \ge 0\ ,\quad \begin{bmatrix} P + R & -R \\ * & dQ + R \end{bmatrix} > 0\,, \tag{22}$$

where $M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}^T$,

$$\begin{aligned}\Omega_{11} &= -P_0 + Q + M_1 + M_1^T + dN_{11}, \\ \Omega_{13} &= -M_1 + M_3^T + dN_{13}, \\ \Omega_{22} &= -P + P_0 + dN_{22}, \\ \Omega_{33} &= -Q - M_3 - M_3^T + dN_{33},\end{aligned}\ ,\ N = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ * & N_{22} & N_{23} & N_{24} \\ * & * & N_{33} & N_{34} \\ * & * & * & N_{44} \end{bmatrix} \tag{23}$$

Proof: The following Lypunov-Krasovskii functional candidate is considered for the closed-loop system in (17),

$V(X(t,k+1)) = V_h(X(t,k+1)) + V_v(X(t,k+1))$,

$V_h(X(t,k+1)) = V_1(X(t,k+1)) + V_2(X(t,k+1)) + V_3(X(t,k+1))$,

$V_1(X(t,k+1)) = X^T(t,k+1)P_0X(t,k+1)$,

$$V_2(X(t,k+1)) = \sum_{i=t-d}^{t-1} X^T(i,k+1)QX(i,k+1)\,,$$

$$V_3(X(t,k+1)) = \sum_{j=-d}^{-1}\sum_{i=t+j}^{t-1} \delta^T(i,k+1)R\delta(i,k+1)\,,$$

$V_v(X(t,k+1)) = X^T(t,k+1)(P - P_0)X(t,k+1)$,

$\delta(i,k+1) \triangleq X(i+1,k+1) - X(i,k+1)$.

Concerning the condition of theorem 1, it needs to prove that there exists a scalar $\sigma > 0$, such that

$V_h(X(t,k+1)) \ge \sigma \| X(t,k+1) \|^2$

For this purpose, we note that $R > 0$ and lemma 1 implies

$$\begin{aligned}\sum_{j=-d}^{-1}\sum_{i=t+j}^{t-1} \delta^T(t,k+1)R\delta(i,k+1) &\ge \sum_{j=-d}^{-1} -\frac{1}{j}\left(\sum_{i=t+j}^{t-1}\delta(i,k+1)\right)^T R\left(\sum_{i=t+j}^{t-1}\delta(i,k+1)\right) \\ &= \sum_{j=-d}^{-1} -\frac{1}{j}[X(t,k+1) - X(t+j,k+1)]^T R[X(t,k+1) - X(t+j,k+1)] \\ &\ge \sum_{j=-d}^{-1} -\frac{1}{d}[X(t,k+1) - X(t+j,k+1)]^T R[X(t,k+1) - X(t+j,k+1)] \\ &= \frac{1}{d}\sum_{i=t-d}^{t-1}[X(t,k+1) - X(i,k+1)]^T R[X(t,k+1) - X(i,k+1)]\end{aligned}$$

It follows that

$$V_h(X(t,k+1)) = X^T(t,k+1)PX(t,k+1) + \sum_{i=t-d}^{t-1} X^T(i,k+1)QX(i,k+1) + \sum_{j=-d}^{-1}\sum_{i=t+j}^{t-1} \delta^T(i,k+1)R\delta^T(i,k+1)$$
$$\geq X^T(t,k+1)PX(t,k+1) + \sum_{i=t-d}^{t-1} X^T(i,k+1)QX(i,k+1) + \sum_{i=t-d}^{t-1}\frac{1}{d}\left[X(t,k+1)-X(i,k+1)\right]^T R\left[X(t,k+1)-X(i,k+1)\right]$$
$$= \frac{1}{d}\sum_{i=t-d}^{t-1}\left[X(t,k+1)-X(i,k+1)\right]^T \begin{bmatrix} P+R & -R \\ -R & dQ+R \end{bmatrix}\begin{bmatrix} X(t,k+1) \\ X(i,k+1) \end{bmatrix}$$

By using the second matrix inequality in (22), there exists a scalar $\sigma > 0$ such that

$$\begin{bmatrix} P+R & -R \\ -R & dQ+R \end{bmatrix} - \begin{bmatrix} \sigma \boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} > 0$$

This results in

$$V_h(X(t,k+1)) \geq \sigma \| X(t,k+1) \|^2$$

Next, we show that the unforced closed-loop 2D system is asymptotically stable with $\omega(t,k+1) = 0$. Define

$$\Delta V(X(t,k+1)) = \sum_{i=1}^{3} \Delta V_i(X(t,k+1)) + \Delta V_v(X(t,k+1)) \tag{24}$$

where

$$\Delta V_1(X(t,k+1)) = X^T(t+1,k+1)P_0X(t+1,k+1) - X^T(t,k+1)P_0X(t,k+1)$$
$$\Delta V_2(X(t,k+1)) = X^T(t,k+1)QX(t,k+1) - X^T(t-d,k+1)QX(t-d,k+1)$$
$$\Delta V_3(X(t,k+1)) = d\delta^T(t,k+1)R\delta(t,k+1) - \sum_{i=t-d}^{t-1}\delta^T(i,k+1)R\delta(i,k+1)$$
$$\Delta V_v(X(t,k+1)) = X^T(t+1,k+1)(P-P_0)X(t+1,k+1) - X^T(t+1,k)(P-P_0)X(t+1,k)$$

Note that the following two identities hold

$$2\hat{\phi}^T(t,k+1)\widehat{M}\left[X(t,k+1) - X(t-d,k+1) - \sum_{i=t-d}^{t-1}\delta(i,k+1)\right] = 0,$$

$$d\hat{\phi}^T(t,k+1)\widehat{N}\hat{\phi}(t,k+1) - \sum_{i=t-d}^{t-1}\hat{\phi}^T(t,k+1)\widehat{N}\hat{\phi}(t,k+1) = 0.$$

where

$$\hat{\phi}(t,k+1) = \left[X^T(t,k+1) \quad X^T(t+1,k) \quad X^T(t-d,k+1)\right]^T,$$

$$\widehat{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, \quad \widehat{N} = \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ * & N_{22} & N_{23} \\ * & * & N_{33} \end{bmatrix}.$$

By means of the above identities and simple algebraic manipulations, we have

$$\Delta V = \hat{\phi}^T(t,k+1)\left[\widehat{\Gamma}_1^T P\widehat{\Gamma}_1 + d\widehat{\Gamma}_2^T R\widehat{\Gamma}_2 + \widehat{T}\right]\hat{\phi}(t,k+1) - \sum_{i=t-d}^{t-1}\delta^T(i,k+1)R \times \delta(i,k+1) - 2\hat{\phi}^T(t,k+1)\widehat{M}\sum_{i=t-d}^{t-1}\delta(i,k+1) - \sum_{i=t-d}^{t-1}\hat{\phi}^T(t,k+1)\widehat{N}\hat{\phi}(t,k+1)$$
$$= \hat{\phi}^T(t,k+1)\left[\widehat{\Gamma}_1^T P\widehat{\Gamma}_1 + d\widehat{\Gamma}_2^T R\widehat{\Gamma}_2 + \widehat{T}\right]\hat{\phi}(t,k+1) - \sum_{i=t-d}^{t-1}\hat{\xi}^T(t,i)\begin{bmatrix} \widehat{N} & \widehat{M} \\ * & R \end{bmatrix}\hat{\xi}(t,i)$$

where $\hat{\xi}(t,i) \triangleq \left[\hat{\phi}^T(t,k+1) \quad \delta^T(i,k+1)\right]^T$ and

$$X(t+1,k+1) = \left[\widehat{A}_1 + \Delta\widehat{A}_1 \quad \widehat{A}_2 + \Delta\widehat{A}_2 \quad \widehat{A}_d\right]\hat{\phi}(t,k+1) \triangleq \widehat{\Gamma}_1\hat{\phi}(t,k+1),$$

$$\delta(t,k+1) = \left[\widehat{A}_1 + \Delta\widehat{A}_1 - \boldsymbol{I} \quad \widehat{A}_2 + \Delta\widehat{A}_2 \quad \widehat{A}_d\right]\hat{\phi}(t,k+1) \triangleq \widehat{\Gamma}_2\hat{\phi}(t,k+1),$$

$$\widehat{T} \triangleq \begin{bmatrix} -P_0 + Q + M_1 + M_1^T + dN_{11} & M_2^T + dN_{12} & -M_1 + M_3^T + dN_{13} \\ * & -P + P_0 + dN_{22} & -M_2 + dN_{23} \\ * & * & -Q - M_3 - M_3^T + dN_{33} \end{bmatrix}$$

Therefore, $\Delta V(X(t,k+1)) < 0$ is guaranteed based on the matrix inequality conditions in (21) and (22) by employing Schur complement and lemma 2.

Based on (24), the following inequality is obtained,

$$V_h(X(t+1,k+1)) + V_v(X(t+1,k+1)) \leq V_h(X(t,k+1)) + V_v(X(t+1,k)) \tag{25}$$

Using inequality (25) and following a similar procedure given in the proof in [16], we are sure of

$$\sum_{t+k \leq N+1} V(X(t,k+1)) \leq \sum_{t+k \leq N} V(X(t,k+1))$$

Consequently, we have $\lim_{t+k\to\infty} \| X(t,k+1) \| = 0$. Thus system (17) with $\omega(t,k+1) = 0$ is asymptotically stable.

In the sequel, we shall prove the H infinity performance level in (11) is satisfied for $\omega(t,k+1) \neq 0$. To this end, we define the control objective function as

$$J_{BP} = \sum_{t=0}^{T_p}\sum_{k=0}^{\infty}\left[\gamma^{-1}z^T(t,k+1)z(t,k+1) - \gamma\omega^T(t,k+1)\omega(t,k+1)\right] \tag{26}$$

Under the zero initial condition, we have

$$J_{BP} \leq \sum_{t=0}^{T_p}\sum_{k=0}^{\infty}\left[\gamma^{-1}z^T(t,k+1)z(t,k+1) - \gamma\omega^T(t,k+1)\omega(t,k+1) + \Delta V(X(t,k+1))\right]$$

Define the following notations

$$X(t+1,k+1) = \left[\widehat{A}_1 + \Delta\widehat{A}_1 \quad \widehat{A}_2 + \Delta\widehat{A}_2 \quad \widehat{A}_d \quad \widehat{D}_g\right]\phi(t,k+1) \triangleq \Gamma_1\phi(t,k+1)$$

$$\delta(t,k+1) = \left[\widehat{A}_1 + \Delta\widehat{A}_1 - \boldsymbol{I} \quad \widehat{A}_2 + \Delta\widehat{A}_2 \quad \widehat{A}_d \quad \widehat{D}_g\right]\phi(t,k+1) \triangleq \Gamma_2\phi(t,k+1)$$

$$\phi(t,k+1) = \left[X^T(t,k+1) \quad X^T(t+1,k) \quad X^T(t-d,k+1) \quad \omega^T(t,k+1)\right]^T$$

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix}, \quad N = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ * & N_{22} & N_{23} & N_{24} \\ * & * & N_{33} & N_{34} \\ * & * & * & N_{44} \end{bmatrix}.$$

Following the same routine as before, we have

$$J_{BP} \leq \sum_{t=0}^{T_p}\sum_{k=0}^{\infty}\left[\gamma^{-1}z^T(t,k+1)z(t,k+1) - \gamma\omega^T(t,k+1)\omega(t,k+1) + \Delta V(X(t,k+1))\right]$$
$$= \sum_{t=0}^{T_p}\sum_{k=0}^{\infty}\left\{\phi^T(t,k+1)\left[\Gamma_1^T P\Gamma_1 + d\Gamma_2^T R\Gamma_2 + T\right]\phi(t,k+1) - \sum_{i=t-d}^{t-1}\xi^T(t,i)\begin{bmatrix} N & M \\ * & R \end{bmatrix}\xi(t,i)\right\}$$

where $\xi(t,i) \triangleq \left[\phi^T(t,k+1) \quad \delta^T(i,k+1)\right]^T$ and

$$T \triangleq \begin{bmatrix} -P_0 + Q + M_1 + M_1^T + dN_{11} + \gamma^{-1}\hat{L}^T\hat{L} & M_2^T + dN_{12} & -M_1 + M_3^T + dN_{13} & M_4^T + dN_{14} \\ * & -P + P_0 + dN_{22} & -M_2 + dN_{23} & dN_{24} \\ * & * & -Q - M_3 - M_3^T + dN_{33} & -M_4^T + dN_{34} \\ * & * & * & -\gamma\boldsymbol{I} + dN_{44} \end{bmatrix}$$

So, using Schur complement and lemma 2, it follows that $J_{BP} \leq 0$ in terms of the constraints in (21) and (22), which guarantees the robust H infinity performance level shown in (11). This completes the proof. ■

Remark 2: In Theorem 1, free-weighting matrices are introduced to reduce the conservatism of the sufficient conditions. Moreover, the second inequality in (22) is less conservative compared to the use of a positive definite matrix Q along with $P > 0$ and $R > 0$.

Note that the sufficient condition in (21) is not a linear matrix inequality because of the occurrence of P^{-1} and R^{-1}. A feasible solution is to transform the original non-convex problem into an LMI-based nonlinear optimization problem which can be effectively solved by the CCL method [18].

Two slack variables T and S are introduced such that

$$PT = \boldsymbol{I}\ ,\ RS = \boldsymbol{I}$$

The original feasibility problem of matrix inequalities (21) and (22) can therefore be converted into the following LMI-based nonlinear optimization problem,

Minimize $Tr(PT + RS)$

s. t. (22) and

$$\begin{bmatrix} P & \boldsymbol{I} \\ * & T \end{bmatrix} > 0,\ \begin{bmatrix} R & \boldsymbol{I} \\ * & S \end{bmatrix} > 0, \tag{27}$$

$$\begin{bmatrix} \Omega_{11} & M_2^T + dN_{12} & \Omega_{13} & M_4^T + dN_{14} & \hat{L}^T & \hat{A}_1^T & d(\hat{A}_1^T - \boldsymbol{I}) & \hat{F}_a^T \\ * & \Omega_{22} & -M_2 + dN_{23} & dN_{24} & \boldsymbol{0} & \hat{A}_2^T & d\hat{A}_2^T & \hat{F}_b^T \\ * & * & \Omega_{33} & -M_4^T + dN_{34} & \boldsymbol{0} & \hat{A}_d^T & d\hat{A}_d^T & \boldsymbol{0} \\ * & * & * & -\gamma \boldsymbol{I} + dN_{44} & \boldsymbol{0} & \hat{D}_g^T & d\hat{D}_g^T & \boldsymbol{0} \\ * & * & * & * & -\gamma \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -T + \varepsilon \hat{E}\hat{E}^T & \varepsilon d\hat{E}\hat{E}^T & \boldsymbol{0} \\ * & * & * & * & * & * & -dS + \varepsilon d^2 \hat{E}\hat{E}^T & \boldsymbol{0} \\ * & * & * & * & * & * & * & -\varepsilon \boldsymbol{I} \end{bmatrix} < 0 \tag{28}$$

To solve the above optimization problem, a numerical algorithm is given below:

Step 1. Given the values of d and γ, solve (22), (27) and (28) to find a feasible solution of $\{P_0^0, Q^0, P^0, T^0, R^0, S^0, M^0, N^0, K_1^0, K_2^0, L_1^0, L_2^0\}$. Set $k = 0$.

Step 2. Solve the following minimization problem for all variables

(Op) Minimize $Trace(P^k T + PT^k + R^k S + RS^k)$

s.t. (22), (27) and (28).

Set $P^{k+1} = P$, $T^{k+1} = T$, $R^{k+1} = R$, and $S^{k+1} = S$.

Step 3. With the obtained K_1, K_2, L_1, L_2, if the LMI conditions in (22) and (29) are satisfied, then exit.

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & M_4^T + dN_{14} & \hat{L}^T & \hat{A}_1^T P & d(\hat{A}_1^T - \boldsymbol{I})R & \boldsymbol{0} \\ * & \Lambda_{22} & -M_2 + dN_{23} & dN_{24} & \boldsymbol{0} & \hat{A}_2^T P & d\hat{A}_2^T R & \boldsymbol{0} \\ * & * & \Lambda_{33} & -M_4^T + dN_{34} & \boldsymbol{0} & \hat{A}_d^T P & d\hat{A}_d^T R & \boldsymbol{0} \\ * & * & * & -\gamma \boldsymbol{I} + dN_{44} & \boldsymbol{0} & \hat{D}_g^T P & d\hat{D}_g^T R & \boldsymbol{0} \\ * & * & * & * & -\gamma \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -P & \boldsymbol{0} & P\hat{E} \\ * & * & * & * & * & * & -dR & dR\hat{E} \\ * & * & * & * & * & * & * & -\varepsilon \boldsymbol{I} \end{bmatrix} < 0 \tag{29}$$

where

$$\Lambda_{11} = -P_0 + Q + M_1 + M_1^T + dN_{11} + \varepsilon \hat{F}_a^T \hat{F}_a,$$

$$\Lambda_{12} = M_2^T + dN_{12} + \varepsilon \hat{F}_a^T \hat{F}_b,$$

$$\Lambda_{13} = -M_1 + M_3^T + dN_{13},$$

$$\Lambda_{22} = -P + P_0 + dN_{22} + \varepsilon \hat{F}_b^T \hat{F}_b,$$

$$\Lambda_{33} = -Q - M_3 - M_3^T + dN_{33}.$$

Otherwise, set $k = k + 1$ .Go to **Step 2**. If the LMI conditions in (22) and (29) are not satisfied within a prescribed number of iterations, then increase γ and go to **Step 1.**

4 ILLUSTRATION

Consider the cyclic injection velocity/nozzle pressure (V/P) transfer process of an injection molding machine as depicted in Fig.1,

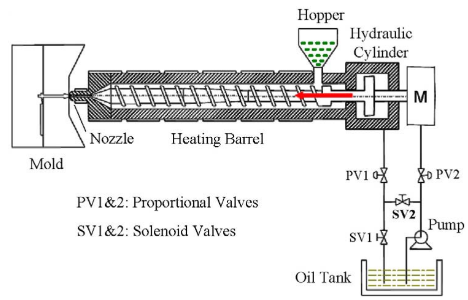

Fig.1 Schematic of the injection molding process

The nozzle packing pressure response to the hydraulic control valve opening was modeled in [4] as

$$\begin{cases} x(t+1, k+1) = \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta A \right) x(t, k+1) + \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta B \right) \\ \qquad \times u(t-d, k+1) \\ y(t, k+1) = [1,\ 0]\, x(t, k+1) \end{cases}$$

Batch-to-batch time-varying process uncertainties were estimated as

$$\Delta A(t) = \begin{bmatrix} 0.0804\delta(t) & 0 \\ -0.0304\delta(t) & 0 \end{bmatrix},\quad \Delta B(t) = \begin{bmatrix} 0.062\delta(t) \\ -0.0464\delta(t) \end{bmatrix}$$

where $\delta(t)$ is a time-varying factor along either the time or batch-wise direction and $|\delta(t)| \le 1$. Note that the process input delay d was assumed to be zero in [4, 6] for ILC design, and the influence from nonlinear dynamics of the hydraulic control valve was lumped into these norm-bounded process uncertainties.

The target profile is taken the same as in these references,

$$Y_r = \begin{cases} 200, & 0 \le t \le 100; \\ 200 + 5(t - 100), & 100 < t \le 120; \\ 300, & 120 < t \le T_p = 200. \end{cases}$$

The initial part of Y_r, corresponding to a step change of the setpoint, is smoothed by a user-specified pre-filter of $G_f(z) = (z^{-1} + z^{-2}) / (3 - z^{-1})$ for practical tracking.

By performing the optimization problem (Op) for different input delays and the process uncertainty ranges, we obtain the minimal H infinity robust performance level $\gamma^* = 55.38$ when $d = 1$ and $|\delta(t)| \le 1$; $\gamma^* = 297.42$ when $d = 2$ and $|\delta(t)| \le 0.7$; $\gamma^* = 127.01$ when $d = 5$ and $|\delta(t)| \le 0.1$. The corresponding ILC controller and state predictor are listed in Table 1 for the case of $d = 2$ and $|\delta(t)| \le 0.7$.

Table 1. ILC controller and state predictor for $\gamma^* = 297.42$

L_1	L_2	K_1	K_2
$\begin{bmatrix} 0.3280 & -0.3284 \\ -0.2260 & 0.2263 \\ 0.2185 & -0.2189 \end{bmatrix}$	$\begin{bmatrix} 0.0005 & -0.0021 \\ -0.0013 & 0 \\ 0.4111 & -0.5485 \end{bmatrix}$	$[-1.4258, -0.8937, -0.0149]$	$[-0.0576, 0.0716, -0.2366]$

The tracking results are shown in Fig. 2, and the output tracking errors for 200 cycles in terms of the index, $ATE(k) = \sum_{t=1}^{T_p} |e(t,k)| / T_p$, is plotted in Fig.3,

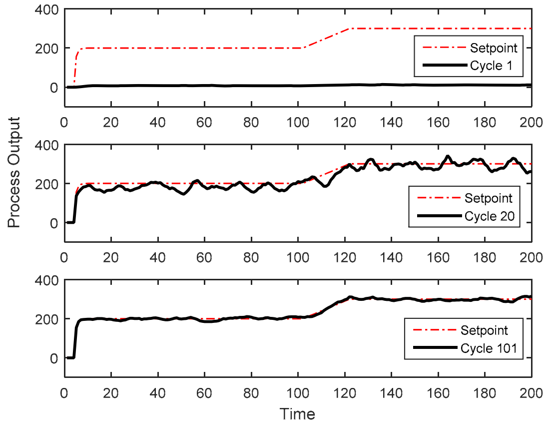

Fig.2 Tracking performance in the batch operation

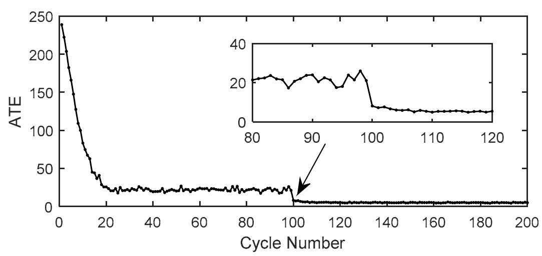

Fig.3 Plot of ATE performance index

It is seen that the closed-loop 2D ILC system maintains good robust stability in both the time and batchwise directions. Note that when the uncertainty range of the process transfer matrices is reduced to $|\delta(t)|\leq 0.1$, the output tracking error is reduced accordingly, as illustrated from the 101st to 200th cycle in Fig.3, while the tracking performance in the 101st cycle is shown in Fig.2. It is therefore demonstrated that the proposed ILC method can be well applied for robust tracking and on-line optimization against batch-to-batch time-varying process uncertainties.

5 CONCLUSION

For industrial batch processes with input delay subject to time-varying process uncertainties from batch to batch, a robust output feedback based ILC method has been proposed. Based on an equivalent 2D system description of the batch process operation, the design of ILC updating law is converted to the stabilization of an equivalent 2D system. A noteworthy merit is that a novel 2D state predictor is introduced to forecast the future state of the equivalent 2D system, so as to facilitate the robust feedback control design. Only measured outputs of current and previous cycles are therefore used to implement the proposed ILC scheme for the convenience of practical application. The application to an illustrative example from the recent references has well demonstrated the effectiveness of the proposed ILC method, in particular for online performance optimization.

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