A New Design Method of a Cascade Iterative Learning Control (ILC) for the Batch/Repetitive Processes

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Abstract: In industrial applications, many processes are constructed by two or more sub-processes connected in serial to perform a given task repetitively or periodically, termed as the cascaded batch/repetitive processes. In this paper, an integrated design method of a novel cascade control is developed for this kind of processes under the two-dimensional (2D) model predictive iterative learning control (ILC) infrastructure. The resulted control system essentially consists of a real time cascade model predictive control along time direction as the inner-loop control and an iterative learning control along the cycle direction as the outer-loop control. The proposed design algorithm not only realizes the integrated design for all control loops but also guarantees the optimal control in terms of the 2D control performances. Compared to the non-cascade ILC schemes, the proposed algorithm is essentially a 2D cascaded control with superior control performances that are clearly illustrated by the simulation results on the injection molding processes.

Key Words: Iterative learning control (ILC), Batch processes, Cascade control, Two-dimensional (2D) systems, Model predictive control (MPC)

1 INTRODUCTION

Iterative learning control (ILC), originally proposed to control the robot arms by Arimoto[1], is an advanced control strategies which mimics the human learning mechanism and, therefore, has the capability of improving the control performance from trail to trail without much requirement for prior process knowledge. Due to the good control performance and applicability, this control scheme has been studied extensively in recently decades with its application extending from mechanical systems to many batch or repetitive processes [2-4].

Generally, a process is called a cascaded process if it consists of two (or more) sub-processes with the output of one sub-process fed forward as the input to another, as illustrated by the dashed block in Fig.1. For this kind of process, the conventional cascade control schemes composed by an inner-loop control and an out-loop control, as shown in Fig. 1, are widely adopted. As there are two control loops to be designed for the different control objectives, in the conventional design methods, the two control loops are usually designed or turned independently to ensure the control performance. The optimal control of the whole system, however, may not be guaranteed under these design methods.

In this paper, a class of cascaded processes with repetitive or periodically operation nature, which commonly exist in the current industry and machinery, are considered. Typical examples of this kind of process include the batch reaction processes with jacket temperature control, batch food sterilization processes with the steam as the heating and pressurization media, biological batch fermentation processes with the feeding volume controlled by flow valves, injection molding processes controlled by hydraulic valves and so on. These processes all has the cascaded structure in the process schematic and share the batch/repetitive nature in the process operation and dynamic, which is termed as cascaded batch/repetitive processes. Due to the large operating range and significant nonlinear dynamics of the cascaded batch/repetitive processes, the conventional cascade control schemes may not guarantee both the rapid system response and precise control, and also there is no cycle to cycle improvement of the control performance.

Regarding the batch/repetitive operation of these processes, it is reasonable to introduce the ILC to the control practice. Although ILC scheme has been widely applied to the industrial processes with repetitive nature, little progress and few insightful investigations have been made for the application of ILC in the cascaded batch/repetitive processes. In 2002, Robertsson *et al.*[5] first proposed to solve the trajectory tracking problem of the pipette manipulator using two cascade ILC loops, and, for a class of cascaded nonlinear process with unknown periodic time function and nonlinear uncertainties, Qu *et a.*[6] developed an ILC scheme to ensure the asymptotic sterilization of control system and convergent of the learning error. Tan Ying *et al.*[7] designed a cascade control system with two ILC loops for a class of exactly repeatable cascade processes, and provided the convergence analysis for the control system. In 2013, Seel *et al.*[8] developed a cascade control system for a nonpenetrating blood pressure control device, where the inner-loop controller, a pole-placement feedback control, and the outer loop controller, an ILC, are designed independently.

Model predictive control (MPC) has been widely applied to modern industry and proved to be an effective advanced

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control algorithm[9,10]. For the cascaded batch/repetitive processes, this paper developed an integrated design method for a novel ILC control system under the MPC infrastructure by optimizing a 2D predictive cost function. The analysis for the control law reveals that this control scheme is essentially equivalent to a 2D feedback cascade control scheme composed by three cascaded control loops, where the two inner-loop controls form the real-time cascade control along the time direction ensuring the within batch control performance, and the outer-loop control is an ILC along the cycle/batch direction guaranteeing the convergence of the control performance along the cycle/batch index. The proposed algorithm realizes an integrated design for these three control loops, and then guarantees an optimal control in the time and cycle/batch directions simultaneously. Compared with the non-cascaded ILC or the traditional cascade control schemes, the proposed cascade ILC scheme can provide better control performances. The numerical simulations and comparisons with non-cascade ILC scheme [11] for the packing phase of injection molding demonstrates the superiority, robustness and applicability of the proposed control scheme.

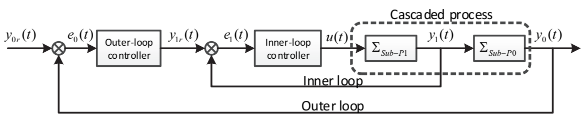


Fig.1: Block diagram of the cascade control system.

2 PROBLEM FORMULATION

2.1 Cascaded batch/repetitive processes

Without loss of the generality, consider a cascaded batch/repetitive process consists of two sub-processes described by the following discrete time Controlled Auto-Regression Moving Average (CARMA) model

$$\Sigma_{Sub-P0}: A_0\left(q_t^{-1}\right) y_0(t,k) = B_0\left(q_t^{-1}\right) y_1(t,k) + d_0(t,k) \quad (1)$$

$$\Sigma_{Sub-P1}: A_1\left(q_t^{-1}\right) y_1(t,k) = B_1\left(q_t^{-1}\right) u(t,k) + d_1(t,k) \quad (2)$$

$$t = 0,1,\cdots,T; k = 1,2,\cdots$$

where t,k represents the time and batch/cycle index, respectively, T is the constant time duration of each cycle/batch, $u(t,k), y_1(t,k), d_1(t,k)$ are the input, output and unknown disturbance of sub-process Σ_{Sub-P1}, respectively, $y_1(t,k), y_0(t,k), d_0(t,k)$ are the input, output and unknown disturbance of sub-process Σ_{Sub-P0}, respectively, q_t^{-1} represents the unit backward shifting operator in the time direction, and $A_0\left(q_t^{-1}\right), B_0\left(q_t^{-1}\right)$ and $A_1\left(q_t^{-1}\right), B_1\left(q_t^{-1}\right)$ are all operator polynomials characterizing the dynamics of sub-processes Σ_{Sub-P0} and Σ_{Sub-P1}, respectively, formulated by

$$A_0\left(q_t^{-1}\right) = 1 + a_{0,1} q_t^{-1} + \cdots + a_{0,n_0} q_t^{-n_0} \quad (3)$$

$$B_0\left(q_t^{-1}\right) = b_{0,1} q_t^{-1} + b_{0,2} q_t^{-2} + \cdots + b_{0,m_0} q_t^{-m_0} \quad (4)$$

$$A_1\left(q_t^{-1}\right) = 1 + a_{1,1} q_t^{-1} + \cdots + a_{1,n_1} q_t^{-n_1} \quad (5)$$

$$B_1\left(q_t^{-1}\right) = b_{1,0} + b_{1,1} q_t^{-1} + b_{1,2} q_t^{-2} + \cdots + b_{1,m_1} q_t^{-m_1} \quad (6)$$

where integrals $\{n_0, m_0\}$ and $\{n_1, m_1\}$ are the input and output orders of the models (1) and (2), respectively.

It is noted that sub-process Σ_{Sub-P0} is described by a strict well-posed discrete time model while sub-process Σ_{Sub-P1} is modeled by a well-posed discrete time model, since sub-process Σ_{Sub-P0} usually represents a slow response process suffering from severe disturbances transmitted from sub-process Σ_{Sub-P1} which has relatively fast response to the control input and external disturbance.

For the above cascaded process, the traditional cascade control schemes are generally used in many industrial applications. The repetitive nature of the process, however, is not considered. To improve the control performance from cycle to cycle, in this paper, a 2D cascade ILC scheme based on a 2D performance index will be developed under the framework of MPC.

2.2 Objective function

As the cascaded processes considered in this paper is assumed to be a batch/repetitive process, all the signals of the process vary not only along the time direction but also along cycle/batch direction. For the simplicity of the description, the time and cycle wise backward difference operators are, respectively, denoted by $\Delta_t = 1 - q_t^{-1}$ and $\Delta_k = 1 - q_k^{-1}$ where q_k^{-1} indicates the cycle/batch-wise unit backward shifting operator. Then the predictive cost function considered in this paper is as follows

$$J\left(t,k,\{u(t+j,k)\}\right) = = \frac{1}{2}\sum_{i=1}^{N} \alpha_i \left(y_r(t+i) - \hat{y}_0(t+i \mid t,k)\right)^2$$
$$+ \frac{1}{2}\sum_{i=0}^{N-1} \beta_i \left(\Delta_k \hat{y}_1(t+i \mid t,k)\right)^2 + \frac{1}{2}\sum_{i=0}^{M-1} \gamma_i \left(\Delta_k \Delta_t u(t+i,k)\right)^2 \quad (7)$$

where $y_r(t)$ is the reference trajectory, $\hat{y}_j(t+i \mid t,k); j = 0,1$ represent the output predictions of the two sub-processes at the sampling time $t+i$ in the k th cycle based on the information up to time t, and $\{\alpha_i \geq 0\}_{i=1,2,\cdots,N}$, $\{\beta_i \geq 0\}_{i=0,1,\cdots,N-1}$ and $\{\gamma_i > 0\}_{i=0,1,\cdots,M-1}$ are all weighting factors.

Compared with the quadratic cost functions commonly used in the conventional MPC, cost function (7) is essentially a 2D cost function because not only the predictive control errors along time direction, i.e. $\hat{e}(t+i \mid t,k) = y_r(t+i) - \hat{y}(t+i \mid t,k)$, but also the cycle/batch wise predictive increments of the intermediate variable and the 2D predictive increments of the control inputs are penalized. It will be seen in the next section that the minimization of this 2D cost function will naturally lead to a 2D cascade optimal control law that relies on the process information of current cycle and previous cycles.

3 INTEGRATED DESIGN METHOD

3.1 Predictive models

Obviously, models (1) and (2) can also be expressed equivalently as follows:

$$\Sigma_{Sub-P0}: A_0\left(q_t^{-1}\right) y_0(t,k) = B_0\left(q_t^{-1}\right)\Delta_k y_1(t,k) + A_0\left(q_t^{-1}\right) y_0(t,k-1) + \Delta_k d_0(t,k) \quad (8)$$

$$\Sigma_{Sub-P1}: A_1'\left(q_t^{-1}\right)\Delta_k y_1(t,k) = B_1\left(q_t^{-1}\right)\Delta_k\Delta_t u(t,k) + \Delta_k\Delta_t d_1(t,k) \quad (9)$$

where $A_1'\left(q_t^{-1}\right)$ is a monic operator polynomial indicated by

$$A_1'\left(q_t^{-1}\right) = \Delta_t A_1\left(q_t^{-1}\right) = 1 + a_{1,1}' q_t^{-1} + \cdots + a_{1,n_1+1}' q_t^{-n_1-1} \quad (10)$$

Denote

$$\mathbf{f}\left(\Big|_{t+j}^{t-i}\right) = \begin{pmatrix} f(t-i) & f(t-i+1) & \cdots & f(t+j-1) & f(t+j) \end{pmatrix}^T \quad f \in \{u, y_0, y_1, d_0, d_1\} \quad (11)$$

Thus, the predictive outputs of the two sub-processes can be computed by models (8) and (9), respectively, as follows:

$$\left(\mathbf{A}_{00} \;\vdots\; \mathbf{A}_{01}\right)\begin{pmatrix} \mathbf{y}_0\left(\Big|_{t}^{t-n_0}, k\right) \\ \mathbf{y}_0\left(\Big|_{t+N}^{t+1}, k\right) \end{pmatrix} = \left(\mathbf{B}_{00} \;\vdots\; \mathbf{B}_{01}\right)\begin{pmatrix} \Delta_k \mathbf{y}_1\left(\Big|_{t-1}^{t-m_0}, k\right) \\ \Delta_k \mathbf{y}_1\left(\Big|_{t+N-1}^{t}, k\right) \end{pmatrix} + \left(\mathbf{A}_{00} \;\vdots\; \mathbf{A}_{01}\right)\begin{pmatrix} \mathbf{y}_0\left(\Big|_{t}^{t-n_0}, k-1\right) \\ \mathbf{y}_0\left(\Big|_{t+N}^{t+1}, k-1\right) \end{pmatrix} + \Delta_k\Delta_t \mathbf{d}_0\left(\Big|_{t+N}^{t+1}, k\right) \quad (12)$$

$$\left(\mathbf{A}_{10} \;\vdots\; \mathbf{A}_{11}\right)\begin{pmatrix} \Delta_k \mathbf{y}_1\left(\Big|_{t-1}^{t-n_1-1}, k\right) \\ \Delta_k \mathbf{y}_1\left(\Big|_{t+N-1}^{t}, k\right) \end{pmatrix} = \left(\mathbf{B}_{10} \;\vdots\; \mathbf{B}_{11}\right)\begin{pmatrix} \Delta_k\Delta_t \mathbf{u}\left(\Big|_{t-1}^{t-m_1}, k\right) \\ \Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+N-1}^{t}, k\right) \end{pmatrix} + \Delta_k\Delta_t \mathbf{d}_1\left(\Big|_{t+N-1}^{t+1}, k\right) \quad (13)$$

where

$$\left(\mathbf{A}_{00} \;\vdots\; \mathbf{A}_{01}\right) = \left(\begin{array}{ccccc|ccccc} a_{0,n_0} & a_{0,n_0-1} & a_{0,n_0-2} & \cdots & a_{0,1} & 1 & 0 & \cdots & 0 & 0 \\ 0 & a_{0,n_0} & a_{0,n_0} & \cdots & a_{0,2} & a_{0,1} & 1 & \cdots & 0 & 0 \\ 0 & 0 & a_{0,n_0} & \cdots & a_{0,3} & a_{0,2} & a_{0,1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & * & * & \cdots & a_{0,1} & 1 \end{array}\right)_{N\times(N+n_0)} \quad (14)$$

$$\left(\mathbf{B}_{00} \;\vdots\; \mathbf{B}_{01}\right) = \left(\begin{array}{ccccc|ccccc} b_{0,m_0} & b_{0,m_0-1} & b_{0,m_0-2} & \cdots & b_{0,2} & b_{0,1} & 0 & \cdots & 0 & 0 \\ 0 & b_{0,m_0} & b_{0,m_0-1} & \cdots & b_{0,3} & b_{0,2} & b_{0,1} & \cdots & 0 & 0 \\ 0 & 0 & b_{0,m_0} & \cdots & b_{0,4} & b_{0,3} & b_{0,2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & * & * & \cdots & b_{0,2} & b_{0,1} \end{array}\right)_{N\times(N+m_0-1)} \quad (15)$$

$$\left(\mathbf{A}_{10} \;\vdots\; \mathbf{A}_{11}\right) = \left(\begin{array}{ccccc|ccccc} a_{1,n_1+1}' & a_{1,n_1}' & a_{1,n_1-1}' & \cdots & a_{1,1}' & 1 & 0 & \cdots & 0 & 0 \\ 0 & a_{1,n_1+1}' & a_{1,n_1}' & \cdots & a_{1,2}' & a_{1,1}' & 1 & \cdots & 0 & 0 \\ 0 & 0 & a_{1,n_1+1}' & \cdots & a_{1,3}' & a_{1,2}' & a_{1,1}' & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & * & * & \cdots & a_{1,1}' & 1 \end{array}\right)_{N\times(N+n_1+1)} \quad (16)$$

$$\left(\mathbf{B}_{10} \;\vdots\; \mathbf{B}_{11}\right) = \left(\begin{array}{ccccc|ccccc} b_{1,m_1} & b_{1,m_1-1} & b_{1,m_1-2} & \cdots & b_{1,1} & b_{1,0} & 0 & \cdots & 0 & 0 \\ 0 & b_{1,m_1} & b_{1,m_1-1} & \cdots & b_{1,2} & b_{1,1} & b_{1,0} & \cdots & 0 & 0 \\ 0 & 0 & b_{1,m_1} & \cdots & b_{1,3} & b_{1,2} & b_{1,1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & * & * & \cdots & b_{1,1} & b_{1,0} \end{array}\right)_{N\times(N+m_1)} \quad (17)$$

It is noticed that $\mathbf{A}_{01}$ and $\mathbf{A}_{11}$ are both non-singular matrices. Then, from Equations (12) and (13), one can get:

$$\mathbf{y}_0\left(\Big|_{t+N}^{t+1}, k\right) = \mathbf{G}_0\Delta_k \mathbf{y}_1\left(\Big|_{t+N-1}^{t}, k\right) + F_0(t,k) + H_0(t,k) + \Delta_k\Delta_t \mathbf{d}_0\left(\Big|_{t+N}^{t+1}, k\right) \quad (18)$$

$$\Delta_k \mathbf{y}_1\left(\Big|_{t+N-1}^{t}, k\right) = \mathbf{G}_1\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+N-1}^{t}, k\right) + F_1(t,k) + \Delta_k\Delta_t \mathbf{d}_1\left(\Big|_{t+N-1}^{t+1}, k\right) \quad (19)$$

where

$$\mathbf{G}_0 = \mathbf{A}_{01}^{-1}\mathbf{B}_{01} \quad (20)$$

$$F_0(t,k) = \mathbf{A}_{01}^{-1}\mathbf{B}_{00}\Delta_k \mathbf{y}_1\left(\Big|_{t-1}^{t-m_0}, k\right) - \mathbf{A}_{01}^{-1}\mathbf{A}_{00}\mathbf{y}_0\left(\Big|_{t}^{t-n_0}, k\right) \quad (21)$$

$$H_0(t,k) = \mathbf{A}_{01}^{-1}\mathbf{A}_{00}\mathbf{y}_0\left(\Big|_{t}^{t-n_0}, k-1\right) + \mathbf{y}_0\left(\Big|_{t+N}^{t+1}, k-1\right) \quad (22)$$

$$\mathbf{G}_1 = \mathbf{A}_{11}^{-1}\mathbf{B}_{11} \quad (23)$$

$$F_1(t,k) = \mathbf{A}_{11}^{-1}\mathbf{B}_{10}\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t-1}^{t-m_1}, k\right) - \mathbf{A}_{11}^{-1}\mathbf{A}_{10}\Delta_k \mathbf{y}_1\left(\Big|_{t-1}^{t-n_1-1}, k\right) \quad (24)$$

It is reasonable to assume that all elements of the unknown disturbance vectors $\Delta_k\Delta_t \mathbf{d}_0\left(\Big|_{t+N}^{t+1}, k\right)$ and $\Delta_k\Delta_t \mathbf{d}_1\left(\Big|_{t+M-1}^{t+1}, k\right)$ are independent random noise, then the optimal predictive models that can be obtained are as follows

$$\widehat{\mathbf{y}}_0\left(\Big|_{t+N}^{t+1} \,|\, t, k\right) = \mathbf{G}_0\Delta_k \widehat{\mathbf{y}}_1\left(\Big|_{t+N-1}^{t} \,|\, t, k\right) + F_0(t,k) + H_0(t,k) \quad (25)$$

$$\Delta_k \widehat{\mathbf{y}}_1\left(\Big|_{t+N-1}^{t} \,|\, t, k\right) = \mathbf{G}_1\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+N-1}^{t}, k\right) + F_1(t,k) \quad (26)$$

Unfortunately, the predictive model (26) is only applicable when $M = N$. For the case that $M < N$ with assumption that $\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+N-1}^{t+M}, k\right) \equiv \mathbf{0}$, the last $N-M$ columns in gain matrix $\mathbf{G}_1$ should be neglected, resulting in a more general form of predictive model for variable $\Delta_k y_1(t,k)$:

$$\Delta_k \widehat{\mathbf{y}}_1\left(\Big|_{t+N-1}^{t} \,|\, t, k\right) = \mathbf{G}_1\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+M-1}^{t}, k\right) + F_1(t,k) \quad (27)$$

3.2 Iterative learning control law

Rewrite the cost function (7) in the matrix form

$$J\left(t, k, u\left(\Big|_{t+M-1}^{t}, k\right)\right) = \frac{1}{2}\widehat{\mathbf{e}}\left(\Big|_{t+N}^{t+1} \,|\, t, k\right)^T \mathbf{Q}\widehat{\mathbf{e}}\left(\Big|_{t+N}^{t+1} \,|\, t, k\right) + \frac{1}{2}\Delta_k \widehat{\mathbf{y}}_1\left(\Big|_{t+N-1}^{t} \,|\, t, k\right)^T \mathbf{R}\Delta_k \widehat{\mathbf{y}}_1\left(\Big|_{t+N-1}^{t} \,|\, t, k\right) + \frac{1}{2}\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+M-1}^{t}, k\right)^T \mathbf{S}\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+M-1}^{t}, k\right) \quad (28)$$

where the weighting matrices $\mathbf{Q},\mathbf{R},\mathbf{S}$, are all diagonal matrices as follows

$$\mathbf{Q} = diag\left\{\alpha_1, \alpha_2, \cdots, \alpha_N\right\} \quad (29)$$

$$\mathbf{R} = diag\left\{\beta_0, \beta_1, \cdots, \beta_{N-1}\right\} \quad (30)$$

$$\mathbf{S} = diag\left\{\gamma_0, \gamma_1, \cdots, \gamma_{M-1}\right\} \quad (31)$$

Based on predictive models (25)(27) and cost function (28), the optimal control law can be derived:

$$\Delta_k\Delta_t \mathbf{u}\left(\Big|_{t+M-1}^{t}, k\right) = \mathbf{K}\Big[\mathbf{G}_0^T\mathbf{Q}\left(\mathbf{y}_r\left(\Big|_{t+N}^{t+1}\right) - F_0(t,k) - H_0(t,k)\right)$$

$$-\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)F_1(t,k)\Big] \quad (32)$$

where $\mathbf{K}=\left(\mathbf{G}_1^T\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0\mathbf{G}_1+\mathbf{G}_1^T\mathbf{R}\mathbf{G}_1+\mathbf{S}\right)^{-1}\mathbf{G}_1^T$. Let K_1 be the first row of matrix $\mathbf{K}$, then the optimal control at any time t in any cycle k can be determined by:

$$\Delta_k\Delta_t u(t,k)=K_1\Big[\mathbf{G}_0^T\mathbf{Q}\left(\mathbf{y}_r\left(\left|_{t+N}^{t+1}\right.\right)-F_0(t,k)-H_0(t,k)\right)$$
$$-\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)F_1(t,k)\Big] \quad (33)$$

The above control law exploits the information from previous sampling time in both the current cycle and the previous cycle. From viewpoint of the 2D system, it is actually a 2D feedback control law with a 2D integral action, where the 2D feedback control ensures the stability of the closed-loop system along time and cycle/batch directions, while the 2D integral action ensure the convergence of the steady-state control errors along these two directions. Moreover, the above control law can also be regarded as an ILC law, since it can be re-expressed equivalently as follows:

$$\Delta_t u(t,k)=\Delta_t u(t,k-1)+K_1\Big[\mathbf{G}_0^T\mathbf{Q}\left(\mathbf{y}_r\left(\left|_{t+N}^{t+1}\right.\right)-F_0(t,k)\right.$$
$$\left.-H_0(t,k)\right)-\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)F_1(t,k)\Big] \quad (34)$$

which is a standard ILC formulation depending on the process information along the 2D time.

4 SYSTEM STRUCTURE ANALYSIS

Since $\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}$ is a non-singular matrix, according to Equ. (21) and (22), the control law (33) can be decomposed into two control laws as follows:

$$\Sigma_{inner-fb}:\Delta_t u(t,k)=K_1\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)\left(\mathbf{y}_{r1}(t,k)-F_1'(t,k)\right) \quad (35)$$

$$\Sigma_{outer}:\quad \Delta_k\mathbf{y}_{r1}(t,k)=\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)^{-1}\mathbf{G}_0^T\mathbf{Q}\left(\mathbf{y}_r\left(\left|_{t+N}^{t+1}\right.\right)\right.$$
$$\left.-F_0(t,k)-H_0(t,k)\right) \quad (36)$$

where

$$F_1'(t,k)=\mathbf{A}_{11}^{-1}\mathbf{B}_{10}\Delta_t\mathbf{u}\left(\left|_{t-1}^{t-m_1}\right.,k\right)-\mathbf{A}_{11}^{-1}\mathbf{A}_{10}\mathbf{y}_1\left(\left|_{t-1}^{t-n_1-1}\right.,k\right) \quad (37)$$

Let $K_{10}=K_1\left(\mathbf{G}_0^T\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)$, $\mathbf{K}_{11}=\mathbf{A}_{11}^{-1}\mathbf{B}_{10}$, $\mathbf{K}_{12}=-\mathbf{A}_{11}^{-1}\mathbf{A}_{10}$, then the control law $\Sigma_{inner-fb}$ can be rewritten as:

$$\Sigma_{inner-fb}:\ \Delta_t u(t,k)=K_{10}\Big[\mathbf{y}_{r1}(t,k)-\mathbf{K}_{11}\Delta_t\mathbf{u}\left(\left|_{t-1}^{t-m_1}\right.,k\right)$$
$$-\mathbf{K}_{12}\mathbf{y}_1\left(\left|_{t-1}^{t-n_1-1}\right.,k\right)\Big] \quad (38)$$

It is obvious that control law $\Sigma_{inner-fb}$ only depends on the real-time input and output information of sub-process Σ_{Sub-P1}, which is essentially a real-time inner-loop control law, meanwhile, $\mathbf{y}_{r1}(t,k)$ should be regarded as the reference trajectory determined by outer-loop control law Σ_{outer}. Let $\mathbf{L}=\left(\mathbf{G}_0^{\mathrm{T}}\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right)^{-1}\mathbf{G}_0^{\mathrm{T}}\mathbf{Q}$, $\mathbf{K}_{01}=\mathbf{A}_{01}^{-1}\mathbf{B}_{00}$, and $\mathbf{K}_{02}=-\mathbf{A}_{01}^{-1}\mathbf{A}_{00}$. From Equ. (21) and (22), outer-loop control law Σ_{outer} can be rewritten as follows:

$$\Sigma_{outer}:\ \Delta_k\mathbf{y}_{r1}(t,k)=\mathbf{L}\Big[\mathbf{e}\left(\left|_{t+N}^{t+1}\right.,k-1\right)-\mathbf{K}_{01}\Delta_k\mathbf{y}_1\left(\left|_{t-1}^{t-m_0}\right.,k\right)$$
$$-\mathbf{K}_{02}\Delta_k\mathbf{y}_0\left(\left|_{t}^{t-n_0}\right.,k\right)\Big] \quad (39)$$

Obviously, this control law only depends on the input and output information of sub-process Σ_{Sub-P0}. It is, therefore, an outer-loop control, together with inner-loop control law $\Sigma_{inner-fb}$, forming a cascade control system. In addition, it is also noted that control law (39) is essentially a 2D feedback control exploits not only the historical input and output information of the current cycle, but also the input, output as well as control error information of the previous cycle. According to Equ. (39), this 2D feedback control law can be further decomposed into two control laws cascaded as follows:

$$\Sigma_{outer-fb}:\ \mathbf{y}_{r1}(t,k)=\mathbf{L}\Big[\mathbf{y}_{r0}(t,k)-\mathbf{K}_{01}\mathbf{y}_1\left(\left|_{t-1}^{t-m_0}\right.,k\right)$$
$$-\mathbf{K}_{02}\mathbf{y}_0\left(\left|_{t}^{t-n_0}\right.,k\right)\Big] \quad (40)$$

$$\Sigma_{outer-ilc}:\ \mathbf{y}_{r0}(t,k)=\mathbf{y}_{r0}(t,k-1)+\mathbf{e}\left(\left|_{t+N}^{t+1}\right.,k-1\right) \quad (41)$$

where $\Sigma_{outer-fb}$ represents a feedback control law only dependent on the real-time information of sub-process Σ_{Sub-P0}, while $\Sigma_{outer-ilc}$ is essentially a simple ILC law located in the outer-loop only depending on the control error over the predictive horizon of the previous cycle. From 2D system point of view, control law $\Sigma_{outer-ilc}$ can be considered as a cycle/batch wise feedback control law as well.

Based on the above analysis, the whole control system can be schematically shown by Fig.2. It can be seen that the whole control system contains a real-time cascade feedback control law in the inner loop and an ILC law in the outer loop. The real-time cascade feedback control can ensure the process control performance in the time direction, while the ILC law guarantees the improvement of the control performance along the cycle/batch direction. The design algorithm proposed in this paper realizes an integrated design and unified optimization for this kind of control scheme.

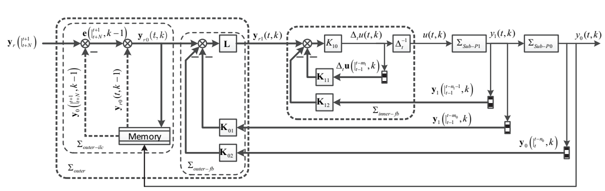
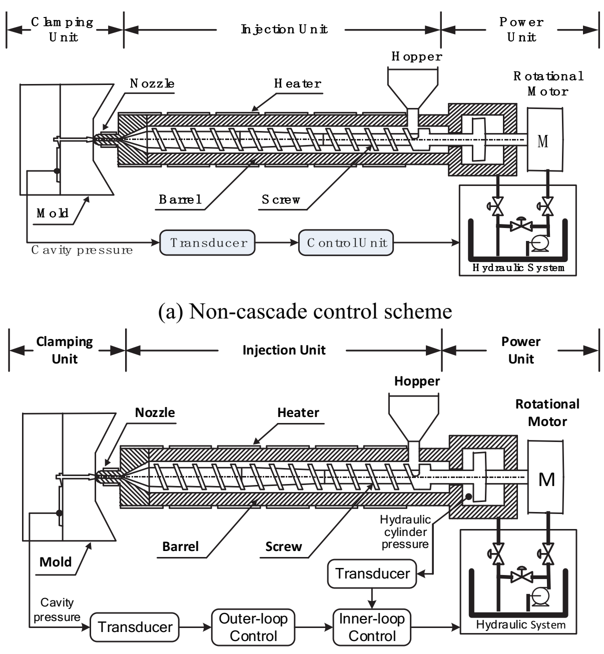


Fig.2: Schematic of the closed-loop control system.

5 NUMERICAL ILLUSTRATION

Injection molding is a major polymer processing technology that transforms plastic granules into different shapes of products. It is a typical batch/repetitive process that contains three phases including filling, packing-holding and cooling [12]. Key process variables of each phase must be precisely controlled to follow the preset optimal trajectories to ensure a good product quality and a high production rate.

Packing is an important phase of the molding process. During packing, material is slowly packed into the mold cavity to compensate for the material shrinkage associated with cooling and solidification. It is suggested to control the mold cavity pressure during packing phase, and in the previous studies [11,12], the single loop feedback control strategies are adopted to control the cavity pressure by manipulating the hydraulic pressure and flow rate. This kind of control is essentially a non-cascade control as schematically shown in Fig. 3(a). Considering the fact that for most of the injection molding machines, the hydraulic cylinder pressure of the screw is also a measurable variable, it is reasonable to construct a cascaded system with the hydraulic cylinder pressure as the feedback information of the inner loop, and the cavity pressure as the feedback information of the outer loop, as schematically shown in Fig. 3(b). This packing pressure cascaded system is adopted in this section as the application example to illustrate the design process and performance of the proposed algorithm.



(a) Non-cascade control scheme

(b) Cascade control scheme

Fig. 3. Different schematics of the cavity pressure control for injection molding

The control valve opening, the hydraulic cylinder pressure and the cavity pressure signals were acquired during the injection molding process, then the following sub-processes models can be obtained through traditional model identification methodology,

$$\Sigma_{Sub-P0}: \quad \left(1-1.607q_t^{-1}+0.6086q_t^{-2}\right)y_0(t,k) = \left(1.239q_t^{-1}-0.9282q_t^{-2}\right)y_1(t,k)+d_0(t,k) \qquad (42)$$

$$\Sigma_{Sub-P1}: \quad \left(1-0.85q_t^{-1}\right)y_1(t,k)=u(t,k)+d_1(t,k) \qquad (43)$$

where Σ_{Sub-P0} is a slow response process with high order dynamics and high steady-state gain modelling the dynamical characteristic from hydraulic cylinder pressure to cavity pressure, while Σ_{Sub-P1} is a first order fast response process modelling the dynamical responses from valve opening to hydraulic cylinder pressure, and $d_0(t,k), d_1(t,k)$ are both unknown disturbances, ideally, including all the unavoidable nonlinear disturbance, measurement noise and unmodeled nonlinear dynamics. It is assumed in the simulation that:

$$d_0(t,k)=5\sin(0.05t) \qquad (44)$$

$$d_1(t,k)=2n(t,k) \qquad (45)$$

where $n(t,k)$ is a random noise uniformly distributed within [-1,1]. Obviously, $d_0(t,k)$ represents the repeatable disturbance with significant nonlinearity while $d_1(t,k)$ represents the non-repeatable measurement noise.

An accurate process model is fairly difficult to achieve in the realities. Therefore, for simplicity and applicability, the common practice in industrial applications is to use the simplified linear model identified based on the input and output data of the processes. For the packing process simulated by models (42) and (43), it is assumed that only the first order models can be approximately obtained in practice by using the traditional identification algorithm:

$$\Sigma_{Sub-M0}: \quad \left(1-0.995q_t^{-1}\right)y_0(t,k)=q_t^{-2}y_1(t,k) \qquad (46)$$

$$\Sigma_{Sub-M1}: \quad \left(1-0.8q_t^{-1}\right)y_1(t,k)=u(t,k) \qquad (47)$$

It is noted from these models that the model/plant mismatches are significant. To demonstrate the robustness of the proposed control algorithm, the control design is conducted only based on models (46) and (47). For the comparison purpose, two control schemes are designed under the framework of MPC, one is the non-cascade ILC scheme developed in our previous work[11], termed as Solution 1, and another is the control scheme proposed in this paper, termed as Solution 2.

According to models (46) and (47), the following simplified I/O model is used as the process model from the valve opening to cavity pressure for the non-cascaded controller design in Solution 1:

$$\Sigma_M: \quad \left(1-0.995q_t^{-1}\right)\left(1-0.8q_t^{-1}\right)y_0(t,k)=q_t^{-2}u(t,k) \qquad (48)$$

The predictive cost function without penalty on the intermediate variable, i.e. hydraulic cylinder pressure, is adopted

$$J\left(t,k,\{u(t+j,k)\}\right)=\frac{1}{2}\sum_{i=1}^{N}\alpha_i\left(y_r(t+i)-\hat{y}_0(t+i\,|\,t,k)\right)^2 +\frac{1}{2}\sum_{i=0}^{M-1}\gamma_i\left(\Delta_k\Delta_t u(t+i,k)\right)^2 \qquad (49)$$

The design parameters for these two solutions are given in Table 1.

Table 1. Design parameters for Solution 1 and Solution 2

Design parameters	Solution 1	Solution 2
N	5	5
M	2	2
$\alpha_i; i=1,2,\cdots,5$	1	1
$\beta_i; i=0,1$	--	10
$\gamma_i; i=0,1$	100	100

To evaluate the evolution of the control performance from cycle to cycle, the Summed Absolute Error (SAE), defined as below, is used as the rigorous index:

$$SAE(k)=\sum_{t=1}^{T}\left|y_r(t)-y(t,k)\right| \tag{50}$$

According to the responses of the control system, Fig.4 plots the SAE versus the cycle/batch number of Solution 1. It is seen that the control system is totally unstable due to the significant model/plant mismatch and disturbance in process. Fig. 5 gives the output responses of Solution 2. It is clearly shown that the closed-loop system tracks the set point profile almost perfectly even after two cycle iterations even though there are significant model/plant mismatches and unknown disturbances. Fig. 6 plots the SAE versus the cycle/batch number of Solution 2, which clearly demonstrates the improvement of the control performance along the cycle/batch direction. All above simulation results and comparisons clearly illustrates the superiority, robustness and applicability of the control design algorithm proposed in this paper.

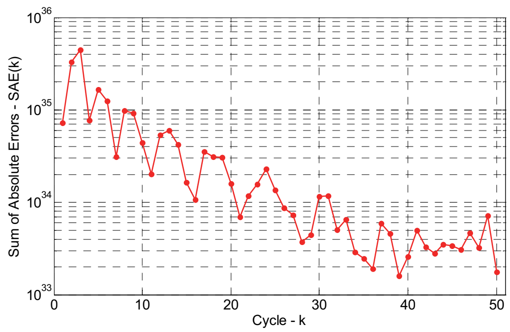


Fig. 4: SAE values of Solution 1.

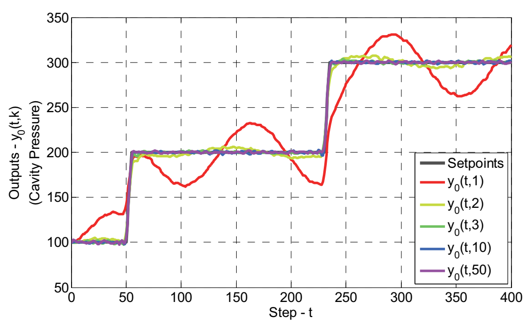


Fig. 5: Control output responses of Solution 2.

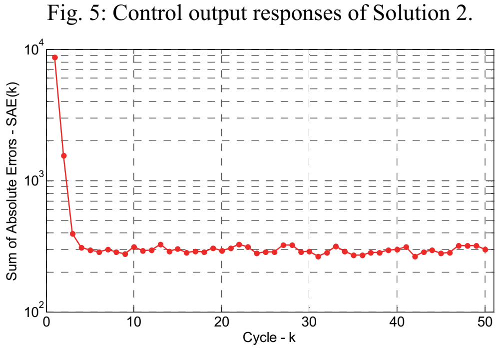


Fig. 6: SAE values of Solution 2

6 CONCLUSIONS

In this paper, a new cascade ILC scheme is proposed for a class of cascaded batch/repetitive processes under the framework of 2D MPC. From the control structure analysis, it is shown that the design algorithm leads to the cascade ILC scheme with three control loops cascaded. The two inner control loops are real-time cascade feedback control to ensure the control performance along the time axis, meanwhile, the outer-loop is an ILC to guarantee the performance improvement along cycle/batch direction. The control design algorithm developed under the framework of 2D system provides not only a unified design for all these control loops but also an overall optimization for control performances both in the time and the cycle/batch directions. Through a numerical illustration on the packing pressure control of the injection molding process, the design is proved to have superior performance against the conventional non-cascade ILC schemes.

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