

Model-Free adaptive Predictive Control for Non-circular Cutting Derived CNC System

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Abstract: Model-free adaptive predictive control method based on non-circular cutting derived CNC system is proposed in this paper, the MATLAB simulation of the permanent magnet synchronous linear motor tool feed driving system verify the model-free adaptive predictive control algorithm to improve the control performance of the system, but also get the higher position tracking accuracy than the PID control algorithm. Accordingly, the disadvantage that complex adjustment, low processing efficiency and processing accuracy in the process of system can be overcome to some extent, when the data-driven control method is applied to the non-circular cross-section parts processing.

Key Words: Non-circular cutting, Model-free adaptive predictive control, Permanent magnet synchronous linear motor, Data-driven

1 INTRODUCTION¹

With the rapid development of large ships, motorcycle, aerospace, automotive, biotechnology, medicine and other modern industry, the higher machining accuracy of the non-circular cross section has been put forward. While the middle-convex and varying oval piston is a typical non-circular part, therefore precision motion control of non-circular cutting tool feed systems become a hot topic in the field. In the cutting process, nonlinear cutting force, tool vibration, and other disturbances have an effect on the system, in addition, variation of the parameters in the linear motor also affect its normal operation^[1-4]. In recent years, lots of research for middle-convex and varying piston CNC lathe system have been done in domestic, since whose thrust is too small, the actuator is also widespread poor linearity and dynamic characteristics, and it is sensitive to the sudden load, the spindle speed is too low, the accuracy can't be guaranteed, there exists a large gap compared with foreign machine tool industry^[5]. In view of the above problem, advanced control strategies must be suppressed or compensate for these disturbances, in

addition to the research and the use of high-performance hardware. In order to make the controller have high steady-state tracking accuracy, fast dynamic response, strong anti-interference ability, and good robustness, data-driven control of the non-circular turning dynamic system control may be a better choice. Data-driven control is the control theory and method, in which the controller is designed merely using online or offline I/O data of the controlled system or using knowledge from the data processing without explicitly or implicitly using information from the mathematical model of the controlled process, and whose stability, convergence, and robustness can be guaranteed by rigorous analysis under certain reasonable assumptions^[6]. Typical data-driven control methods including PID control, model-free adaptive control, iterative learning control, etc. It is well known that the traditional PID control algorithm has large lag, relatively complex adjustment parameters, poor control performance, and low control accuracy. Therefore, in this paper, Model-Free Adaptive Predictive Control based on Compact Form Dynamic Linearization (CFDL-MFAPC) is applied in non-circular cutting tool feed linear motor servo system, and compared with PID control simulation results, verify that the control performance of the control algorithm is higher, and improve the position

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tracking accuracy of non-circular turning tool feeding linear servo system. For discrete-time nonlinear systems, model-free adaptive control uses a new dynamic linearization method and a new concept called Pseudo Partial Derivative (PPD), an equivalent dynamic linearization data model of the closed-loop system at each operation point is set up, then the controller is designed based on the equivalent virtual data model, and theory analysis of the control system is done, so as to realize the adaptive control of nonlinear system, PPD parameters can be estimated by using only I/O measurement data of the controlled plant, model-free adaptive predictive control comprehensively utilize advantages between predictive control and model-free adaptive control^[7-12].

2 THE SYSTEM DESIGN OF MODEL-FREE ADAPTIVE PREDICTIVE CONTROL

A Single Input Single Output (SISO) nonlinear discrete-time system can be expressed as

$$\begin{aligned} y(t+1) &= f(y(t), y(t-1), \dots, y(t-n_y), \\ &u(t), u(t-1), \dots, u(t-n_u)) \end{aligned} \quad (1)$$

Where $u(t) \in \mathbf{R}$ and $y(t) \in \mathbf{R}$ are the input and output at sample time t , n_u and n_y are two unknown positive integers, and $f(\dots): \mathbf{R}^{n_u+n_y+2} \mapsto \mathbf{R}$ is an unknown nonlinear function.

Since nonlinear function $f(\dots)$ is unknown, the output sequence of system can't be predicted directly. However, the system (1) can be converted into the following equivalent CFDL data model under certain assumptions: $\Delta y(t+1) = \phi_c(t)\Delta u(t)$, where $\phi_c(t) \in \mathbf{R}$ is the PPD of the system.

According to the above incremental data model, the following one-step-ahead output prediction equation can be established:

$$y(t+1) = y(t) + \phi_c(t)\Delta u(t) \quad (2)$$

On the basis of (2), P-step-ahead prediction equations can be obtained as follows:

$$\begin{aligned} y(t+1) &= y(t) + \phi_c(t)\Delta u(t) \\ y(t+2) &= y(t+1) + \phi_c(t+1)\Delta u(t+1) \\ &= y(t) + \phi_c(t)\Delta u(t) + \phi_c(t+1)\Delta u(t+1) \\ &\vdots \\ y(t+P) &= y(t+P-1) + \phi_c(t+P-1)\Delta u(t+P-1) \\ &= y(t+P-2) + \phi_c(t+P-2)\Delta u(t+P-2) \\ &\quad + \phi_c(t+P-1)\Delta u(t+P-1) \\ &\vdots \\ &= y(t) + \phi_c(t)\Delta u(t) + \dots + \phi_c(t+P-1)\Delta u(t+P-1) \end{aligned} \quad (3)$$

Make

$$\Delta \mathbf{U}_p(t) = [\Delta u(t), \dots, \Delta u(t+P-1)]^T,$$

$$\mathbf{Y}_p(t+1) = [y(t+1), \dots, y(t+P)]^T,$$

$$\mathbf{E}(t) = [1, 1, \dots, 1]^T,$$

$$\mathbf{A}(t) = \begin{bmatrix} \phi_c(k) & 0 & 0 & 0 & 0 & 0 \\ \phi_c(t) & \phi_c(t+1) & 0 & 0 & & \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \phi_c(t) & \dots & \phi_c(t+P_u-1) & & & \\ \vdots & & \vdots & \ddots & & 0 \\ \phi_c(t) & \phi_c(t+1) & \dots & \phi_c(t+P_u-1) & \dots & \phi_c(t+P_u-1) \end{bmatrix}_{P \times P}$$

Where $\Delta \mathbf{U}_p(t)$ denotes the control input

increment vector, $\mathbf{Y}_p(t+1)$ is the P-step-ahead prediction vector of the system output.

The formula (3) can be abbreviated as:

$$\mathbf{Y}_p(t+1) = \mathbf{E}(t)y(t) + \mathbf{A}(t)\Delta \mathbf{U}_p(t) \quad (4)$$

When $\Delta u(t+j-1) = 0, j > P_u$, prediction equation

(4) becomes

$$\mathbf{Y}_p(t+1) = \mathbf{E}(t)y(t) + \mathbf{A}_1(t)\Delta \mathbf{U}_p(t) \quad (5)$$

Where P_u is the control input horizon,

$$\mathbf{A}_1(t) = \begin{bmatrix} \phi_c(t) & 0 & 0 & 0 \\ \phi_c(t) & \phi_c(t+1) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \phi_c(t) & \phi_c(t+1) & \dots & \phi_c(t+P_u-1) \\ \vdots & \vdots & \dots & \vdots \\ \phi_c(t) & \phi_c(t+1) & \dots & \phi_c(t+P_u-1) \end{bmatrix}_{P \times P_u}$$

$$\Delta \mathbf{U}_{P_u}(t+1) = [\Delta u(t), \dots, \Delta u(t+P_u-1)]^T$$

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2.1 The Basis of Predictive Control Algorithm

Consider the following criterion function of control input:

$$J = \sum_{i=1}^P (y(t+i) - y^*(t+i))^2 + \lambda \sum_{j=0}^{P_u-1} \Delta u^2(t+j) \quad (6)$$

Where $\lambda > 0$ is a weight factor, and $y^*(t+i)$ is the desired output at time $t+i$, $i = 1, \dots, P$.

Make $\mathbf{Y}_p^*(t+1) = [y^*(t+1), \dots, y^*(t+P)]^T$, criterion function (6) becomes

$$J = [\mathbf{Y}_p^*(t+1) - \mathbf{Y}_p(t+1)]^T [\mathbf{Y}_p^*(t+1) - \mathbf{Y}_p(t+1)] + \lambda \Delta \mathbf{U}_{P_u}^T(t) \Delta \mathbf{U}_{P_u}(t) \quad (7)$$

According to (5) and (7), and using the optimality

condition $\frac{\partial J}{\partial \mathbf{U}_{P_u}(t)} = 0$, the control law can be given as

follows:

$$\Delta \mathbf{U}_{P_u}(t) = [\mathbf{A}_1^T(t) \mathbf{A}_1(t) + \lambda \mathbf{I}]^{-1} \mathbf{A}_1^T(t) [\mathbf{Y}_p^*(t+1) - \mathbf{E}(t)y(t)] \quad (8)$$

Thus, the control input at current time t is described by

$$u(t) = u(t-1) + \mathbf{g}^T \Delta \mathbf{U}_{P_u}(t) \quad (9)$$

Where $\mathbf{g} = [1, 0, \dots, 0]^T$.

When $P_u = 1$, equation (9) becomes

$$u(t) = u(t-1) + \frac{1}{\hat{\phi}_c^2(t) + \lambda / P} \frac{1}{P} [\hat{\phi}_c(t) \sum_{i=1}^P (y^*(t+i) - y(t))] \quad (10)$$

λ is an important parameter, and its proper selection can guarantee the stability of the controlled system and obtain good output performance.

2.2 Design of Predictive Control Scheme

Integrating control algorithm, parameter estimation algorithm and prediction algorithm, the CFDL-MFAPC scheme can be designed as follows^[9]

$$\hat{\phi}_c(t) = \hat{\phi}_c(t-1) + \frac{\eta \Delta u(t-1)}{\mu + \Delta u(t-1)^2} [\Delta y(t) - \hat{\phi}_c(t-1) \Delta u(t-1)] \quad (11)$$

$$\hat{\phi}_c(t) = \hat{\phi}_c(1), \text{ if } |\hat{\phi}_c(t)| \leq \varepsilon \text{ or } |\Delta u(t-1)| \leq \varepsilon$$

$$\text{sign}(\hat{\phi}_c(t)) \neq \text{sign}(\hat{\phi}_c(1)) \quad (12)$$

$$\theta(t) = \theta(t-1) + \frac{\hat{\phi}(t-1)}{\delta + \|\hat{\phi}(t-1)\|^2} [\hat{\phi}_c(t) - \hat{\phi}^T(t-1)\theta(t-1)] \quad (13)$$

$$\theta(t) = \theta(1), \text{ if } \|\theta(t)\| \geq M \quad (14)$$

$$\begin{aligned} \hat{\phi}_c(t+j) = & \theta_1(t) \hat{\phi}_c(t+j-1) + \theta_2(t) \hat{\phi}_c(t+j-2) + \\ & \dots + \theta_{n_p}(t) \hat{\phi}_c(t+j-n_p), \end{aligned}$$

$$j = 1, 2, \dots, P_u - 1 \quad (15)$$

$$\hat{\phi}_c(t+j) = \hat{\phi}_c(1), \text{ if } \left| \hat{\phi}_c(t+j) \right| \leq \varepsilon \text{ or}$$

$$\text{sign}(\hat{\phi}_c(t+j)) \neq \text{sign}(\hat{\phi}_c(1)), \quad j = 1, 2, \dots, P_u - 1 \quad (16)$$

$$\Delta \mathbf{U}_{P_u}(t) = [\hat{\mathbf{A}}_1^T(t) \hat{\mathbf{A}}_1(t) \lambda \mathbf{I}]^{-1} \hat{\mathbf{A}}_1^T(t) [\mathbf{Y}_p^*(t+1) - \mathbf{E}(t)y(t)] \quad (17)$$

$$u(t) = u(t-1) + \mathbf{g}^T \Delta \mathbf{U}_{P_u}(t) \quad (18)$$

Where M and ε are positive constants, $\hat{\phi}_c(t+j)$ and $\hat{\mathbf{A}}_1(t)$ are the respectively estimated values of $\phi_c(t+j)$ and $\mathbf{A}_1(t)$, $j = 1, 2, \dots, P_u - 1$, and $\lambda > 0$, $\mu > 0$, $\delta \in (0, 1]$, $\eta \in (0, 1]$.

Where equation (12) makes the pseudo partial derivative estimation algorithm (11) have a strong ability to track time-varying parameters, (14) ensures that the boundedness of $\hat{\mathbf{A}}_1(t)$, and equation (16) guarantees that the sign of prediction parameters is invariable.

The control scheme has P_u parameters to be adjusted online, it is designed only using I/O data of the controlled system, and it has nothing to do with the model and order of the controlled system. It is essentially different from the traditional predicted control. Further, the control horizon need to satisfy $P_u \leq P$. P_u can be set to be 1 in simple systems.

In order to be able to contain the dynamic characteristics of the controlled system, prediction horizon should be chosen sufficiently large. In a time-delay system, it should be greater than the dead

time at least. In practice, for a time-delay unknown system, it is generally selected as 4 to 10.

λ can change the dynamics of the closed-loop system, in theory, the greater the λ , the slower the system response, and the smaller the overshoot, more stable the response, and vice versa. Generally, n_p is chosen to $2\sim 7$ ^[7], which is set to be 3 in this paper.

3 SIMULATION STUDY

The voice coil linear motor is applied in the non-circular cutting tool feed linear servo system, whose voltage balance equation can be derived by the working principle.

$$u = L \frac{di}{dt} + Ri + Blv \quad (19)$$

Where u , L , i , R are the terminal voltage, inductance, current, resistance of the motor coil, B is the air-gap magnetic field intensity of the motor, l is the effective length of the coil, and v is the movement speed of the motor rotor.

The kinetic equilibrium equation of the motor rotor is described as follows

$$\begin{cases} F = ma \\ F = NBil \end{cases} \quad (20)$$

Where F is the driving force of the motor, m is the motor rotor mass, a is the acceleration of the motor rotor, and N is the motor coil turns.

According to the above equation, the transfer function of the voice coil motor can be obtained as

$$G(s) = \frac{Y(s)}{I(s)} = \frac{NBl}{m} \cdot \frac{1}{s^2} \quad (21)$$

which is a second-order system.

The control plant of the voice coil motor servo control system also contains the motor drive except for the voice coil motor, thus the control system is a three-loop control system, and its structure diagram as follows

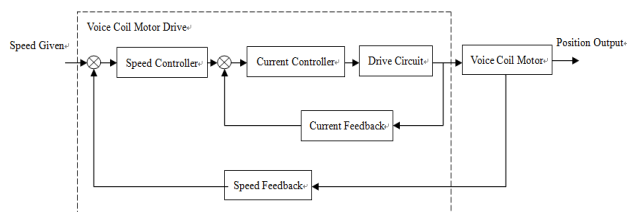


Fig.1 Velocity loop and current loop of the voice coil

motor

The mathematical model of the system can be expressed as

$$G(s) = \frac{K(T_z s + 1)}{(T_{p1}s + 1)(T_{p2}s + 1)(T_{p3}s + 1)} \quad (22)$$

The parameter of the system can be calculated by using system identification toolbox of the MATLAB tool, and meanwhile the model can be rewritten as

$$G(s) = \frac{7437.7769 (s-928.2)}{(s+474) (s+474) (s+6.404)} \quad (23)$$

The zero-order holder and discretization are employed in the transfer function model, which can be transformed into the form of difference equation.

$$y(k) = 2.236y(k-1) - 1.6246y(k-2) + 0.385y(k-3) + 0.0018u(k-1) - 0.0036u(k-2) - 0.0025u(k-3) \quad (24)$$

During the operation of the non-circular cutting tool feed linear servo system, the change of the motor parameters directly affects its normal operation, linear motor has cogging and end effect, as well as the friction between the linear guides. In order to improve the performance of the motor control system for precision control effect, at the same time in order to illustrate the validity and superiority of the proposed scheme, the classical PID algorithm and CFDL-MFAPC algorithm have carried on the simulation research, and then analysis of the effects of two methods of position control based on simulation results.

The desired position input signal of the tool feed linear motor is a sine wave, the amplitude of the given desired input is 1 and the frequency is 0.5Hz, the sampling period is 0.001s, then the motor will achieve the tool reciprocation. The Fig.2 and Fig.3 show the simulation results using the PID controller with $K_p = -10$, $K_i = 0.1$, and $K_d = 0$.

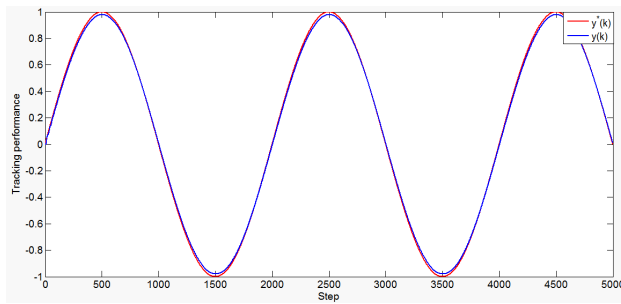


Fig.2 Tracking performance of PID

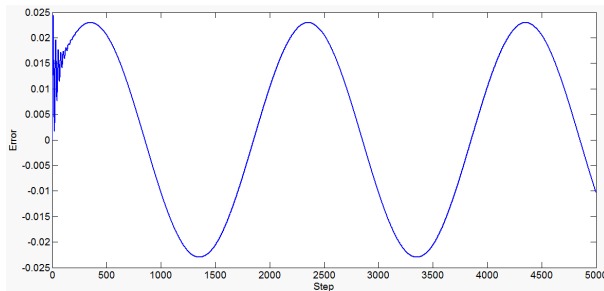


Fig.3 Position error of PID

The simulation results show that the system tracking error reaches the maximum at both ends of the reciprocating motion of the motor, about 0.023, the tracking performance is good, and there is no obvious delay.

Fig.4 and Fig.5 show the simulation results using the CFDL-MFAPC controller with $\varepsilon = 10^{-5}$, $\delta = 1$, $\eta = 1$, $\mu = 1$, $M = 10$, $P = 10$, and $\lambda = 45$.

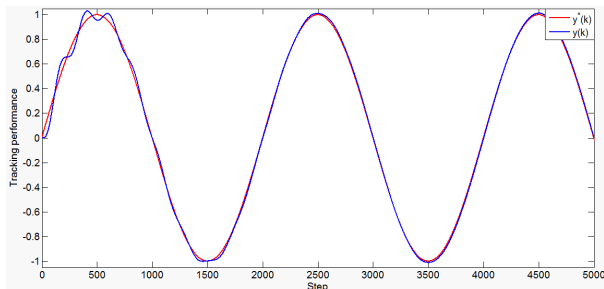


Fig.4 Tracking performance of CFDL-MFAPC

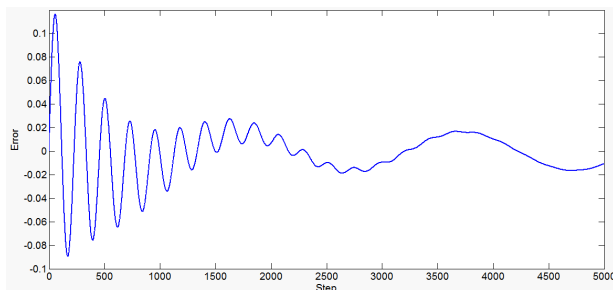


Fig.5 Position error of CFDL-MFAPC

The simulation results show that the system tracking error reaches the maximum at both ends of the reciprocating motion of the motor, about 0.0165, the tracking performance is also good, and there is no obvious delay.

When the amplitude of the given desired input remains 1, frequency increases to 2.5 Hz, Fig.6 and Fig.7 show the simulation results using the PID controller with $K_p = -9$, $K_i = 0.2$, and $K_d = 1$.

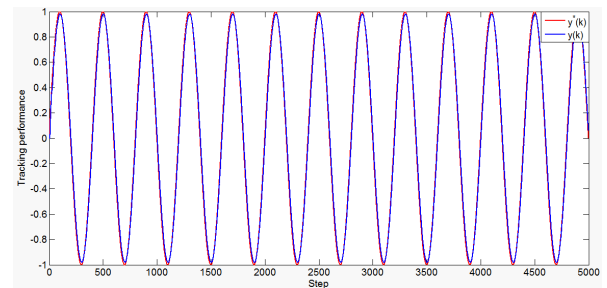


Fig.6 Tracking performance of PID

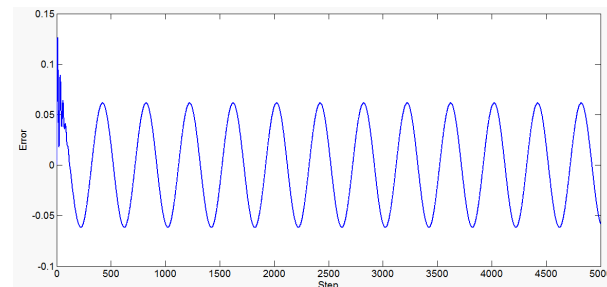


Fig.7 Position error of PID

From the simulation results, the system tracking error reaches the maximum at both ends of the reciprocating motion of the motor, about 0.061, the tracking performance is poor, and there is an obvious delay.

However, the CFDL-MFAPC controller with $P = 250$, and $\lambda = 8700$ at the moment, the simulation results are as follows

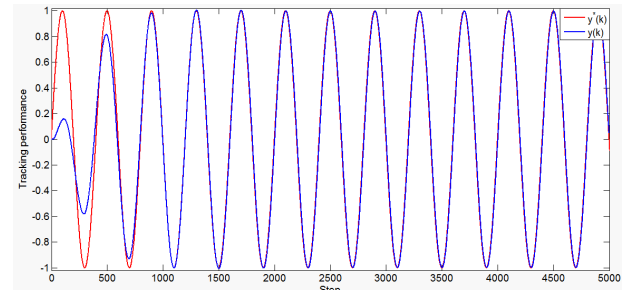


Fig.8 Tracking performance of CFDL-MFAPC

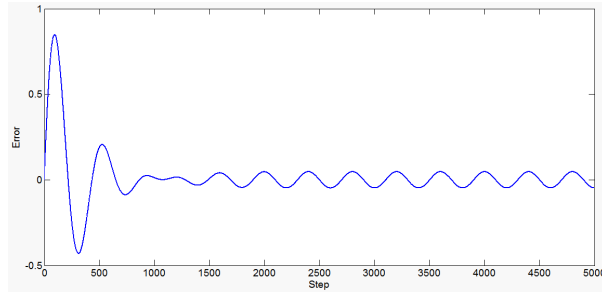


Fig.9 Position error of CFDL-MFAPC

From the simulation results, the system tracking error reaches the maximum at both ends of the reciprocating motion of the motor, about 0.047, the tracking performance is still satisfactory, and there is no obvious delay.

4 CONCLUSION

The effectiveness and superiorities of CFDL-MFAPC scheme have been demonstrated by the above simulation results. When the model-free adaptive predictive control is applied to the non-circular cutting derived CNC system, the input voltage and the output position data systems are only used in the control process, therefore the design of the controller is model-free, and regardless of the structure of the motor, the stability of the system and the output performance can be achieved by selecting the appropriate λ . In the case of low frequency, PID controller can control stability, but the error is relatively large, the adjustment of the parameters is complex, and the control performance is very sensitive to the change of the parameters. With the increase of frequency, the large delay and great error of the system can't even be eliminated through the adjustment of PID parameters.

Compared with PID control, adjustment of the weighting factor λ is very convenient for the model-free adaptive predictive control, and the control performance of the system is very insensitive to the variation of the λ , the position error of the system is also very small even if increasing the frequency, more importantly, which can be eliminated by selecting proper prediction horizon P.

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