# Stability Analysis of Full Form Dynamic Linearization Controller Based Data-driven Model Free Adaptive Control

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**Abstract:** Full form dynamic linearization controller based data-driven model free adaptive control algorithm has been studied in control community. This method merely requires the I/O data of the plant to design the controller and is easy for implementation. However, there still lacks necessary research on the conditions which can guarantee the stability of the closed loop system. This paper presents a stability analysis result for a class of nonlinear system. With some mild assumptions, the convergence of the output regulating error was derived by rigorous mathematical method, which guarantees the correctness of the proposed method in theory.

**Key Words:** Dynamic Linearization Controller, Data-driven, Model Free Adaptive Control, Nonlinear System

## 1 INTRODUCTION

With the development of modern industry, the plants and processes become more and more complicated, which makes the control tasks difficult to complete. The control methods could be categorized into model based control (MBC) method and data-driven control (DDC) method, based on whether the plant model is required as a prior knowledge for controller design.

In MBC methods, the first principle model of the plant should be built and identified. The MBC controller will be designed and analyzed based on the identified model. From the controller design process of MBC, we could find out that the control performance severely depends on the accuracy of the identified model. However, for the modern industry, the complicated systems make the modelling task difficult, which results in the questionable control performance of the MBC controller. Noted that, operating data of the controlled system could reflect their real-time dynamics. And thanks to the informatization development, these data of the plant could be stored for either online or offline controller design and analysis. Hence, it is intuitive to study whether these plant data could be utilized for controller design directly, and avoid the complicated and annoying modeling process, which is the basic idea of DDC method. In this kind of method, the controller structure design and parameter tuning of DDC methods are merely based on the input and output data of the plant, and the physical model is not required anymore. Moreover, considering the unmodeled dynamics or internal disturbance, DDC methods are also more suitable. A recent survey, which discussed the relationship and differences between MBC and DDC methods could be found in [1].

With the idea of utilizing data of the plant, MFAC (model free adaptive control) was proposed as a DDC method for a class of discrete-time nonlinear system. Instead of identifying the global plant model, MFAC builds the local data model to mimic the plant behavior. The data model is built by equivalent dynamic linearization technique along the operation points, which simplifies the complex global modeling and the controller design process. With the local data model, the controller will be designed to minimizing the output regulating error. The equivalent dynamic linearization data models can be summarized as three types, i.e., compact form, partial form and full form dynamic linearization, which could be chosen according to the complexity degree of the plant. Right now, due to its characteristics of independence to physical model, MFAC has been widely studied and applied in control community. The proof of the convergence property of the corresponding closed-loop control system using partial form dynamic linearization was given in [2]. The MFAC method for MIMO system was developed in [3]. A preliminary study about the robust issue of MFAC was shown in [4]. The theoretical framework and typical application of MFAC were introduced systematically in [5], where the MFAC for complex interconnected systems and the modularized and symmetric similarity design conception were investigated. Later, a novel MFAC that is based on the dynamic linearization of ideal controller has been emerged to open up another promising research direction [6, 7], which is expected to build up a unified framework of DDC method. With the development of the theory of MFAC, it has been successfully implemented in

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many practical applications, such as: pump speed control in artificial heart [8], displacement control of the pneumatic muscle actuator [9], dynamic voltage restorer [10], linear servo systems [11], main stream pressure control of power plant [12], levitation control of an electromagnetic system [13].

Since for most of the control system, the control input in the past instants will also affects the system output, the controller output $u(k)$ should explicitly relies on the historical output tracking errors and control inputs. Thus, full form dynamic linearization controller based MFAC (FFDLc-MFAC) was proposed in [14]. Even though the extensive numerical examples showed that the proposed method is effective for a large class of nonlinear system, there still lacks necessary research on the conditions which can guarantee the stability of the closed loop system. Hence, we investigate the stability issue of this method for a class of nonlinear system in this paper.

The outline of this paper is arranged as follows. In Section II, full form dynamic linearization controller based model free adaptive control scheme is recalled. Then, the stability analysis of the closed-loop system is studied in Section III. Conclusions are given in Section IV.

Throughout the paper, the difference operator $\delta$ for a signal $\psi_i(k)$ leads $\delta\psi_i(k)=\psi_i(k)-\psi_{i-1}(k)$ .

## 2 PRELIMENARY OF FFDLc-MFAC

The FFDLc-MFAC has been in [14]. Different from CFDLc- or PFDLc-MFAC, the controller output $u(k)$ explicitly relies on the historical output tracking errors and control inputs, which is more general because for most of the control system, the control input in the past instants will also affects the system output.

Consider a nonlinear controlled plant,

$$y(k+1)=f(y(k),\cdots,y(k-n_y),u(k),\cdots,u(k-n_u)), \qquad (1)$$

where $f(\cdot)$ is an unknown nonlinear function; $n_y$, $n_u$ are orders of output $y$ and input $u$ , respectively; $k$ is sampling instant.

Generally, the ideal nonlinear controller for plant (1) is with the form as follows:

$$u(k)=g(e(k+1),\cdots,e(k-n_e+2),u(k-1),\cdots,u(k-n_c)), \qquad (2)$$

where $g(\cdot)$ is an unknown smooth nonlinear function; $e(k)=y_d(k)-y(k)$ is the output tracking error; $y_d(k)$, $y(k)$ are the desired output and actual output of the system, respectively; $n_c$, $n_e$ are two unknown constants.

**Assumption 1.** Controller $g(\cdot)$ is a smooth nonlinear function and its partial derivatives, $\partial g(\cdot)/\partial e(k-m+2)$ and $\partial g(\cdot)/\partial u(k-n)$ are continuous and bounded, where $k\in\mathbb{N}$; $m=1,\cdots,L_e$; $n=1,\cdots,L_u$; $L_e$, $L_c\in\mathbb{N}$ .

**Assumption 2.** Controller $g(\cdot)$ satisfies the generalized *Lipschitz* condition, that is,

$$|\Delta u(k)|\le\alpha\left\|\Delta\xi(k+1)\right\|+\beta,\quad\forall k\in\mathbb{N}, \qquad (3)$$

where $\Delta\xi^T(k+1)=[\Delta e(k+1),\cdots,\Delta e(k-L_e+2),\Delta u(k-1),\cdots,\Delta u(k-L_c)]$; $\alpha,\beta$ are unknown bounded constants.

**Remark 1.** The true values of $\alpha,\beta$ are not required when designing the controller but their estimations of the upper bound would help to set the initial condition for the parameter update algorithm.

**Theorem 1.** If nonlinear controller (2) satisfies *Assumption 1* and *Assumption 2*, and $\left\|\Delta\xi(k+1)\right\|\neq 0$ holds, there exists a time-varying parameter vector, called pseudo-gradient (PG), such that controller (2) can be transformed into the following equivalent dynamic linearization data model, called full form dynamic linearization controller (FFDLc):

$$\Delta u(k)=\boldsymbol{\psi}^T(k)\Delta\xi(k+1),\quad\forall k\in\mathbb{N}, \qquad (4)$$

where, $\boldsymbol{\psi}^T(k)=[\psi_1(k),\cdots,\psi_{Le+Lc}(k)]$; $\left\|\boldsymbol{\psi}(k)\right\|\le\underline{b}$; $\underline{b}>0$.

**Proof.** The details could be referred to [14].

**Remark 2.** For nonlinear plants with different complexities, if the orders $n_e,n_c$ of ideal controller are known, LLC can set to be the same values as them. If $n_e,n_c$ are too large, reduced time-varying controller could be obtained by setting relatively small LLC. In fact, to realize the controller (4) in practice, all the existed identification techniques with data-driven fashion could be applied to estimate the one-step-ahead output tracking error and the PG.

Supposed there exists an ideal controller that can make the system output follow the desired value in one-step-ahead instant, the output tracking error $e(k+1)$ will be zero. Setting $e(k+1)=0$ in (4), the practical controller could be rewritten as follows,

$$\Delta u(k)=\boldsymbol{\psi}^T(k)\Delta\overline{\xi}(k), \qquad (5)$$

where $\Delta\overline{\xi}^T(k)=[-e(k),\cdots,\Delta e(k-L_e+2),\Delta u(k-1),\cdots,\Delta u(k-L_c)]$ .

Consider the following performance index

$$J=\left(y_d(k+1)-y(k+1)\right)^2+\lambda_k\left\|\boldsymbol{\psi}(k)-\hat{\boldsymbol{\psi}}(k-1)\right\|^2, \qquad (6)$$

where $\lambda_k>0$ is weighting sequence; $y_d(k+1)$ is the desired system output.

Using the compact form dynamic linearization data model of plant (1) (CFDLp) [5],

$$\Delta y(k+1)=\phi(k)\Delta u(k), \qquad (7)$$

and controller (5), the control scheme is obtained as follows by minimizing (6).

$$\Delta\hat{\phi}(k)=\frac{\Delta u(k-1)}{\mu+\Delta u(k-1)^2}\left(\Delta y(k)-\hat{\phi}(k-1)\Delta u(k-1)\right) \qquad (8)$$

$$\hat{\phi}(k)=\hat{\phi}(1)\text{ , if }\hat{\phi}(k)<\underline{\hat{\sigma}}\text{ or }\hat{\phi}(k)>\varpi \qquad (9)$$

$$\Delta\hat{\boldsymbol{\psi}}(k)=\frac{\hat{\phi}(k)\Delta\overline{\xi}(k)\left(y_d(k+1)-y(k)-\hat{\phi}(k)\hat{\boldsymbol{\psi}}^T(k-1)\Delta\overline{\xi}(k)\right)}{\lambda_k+\left(\hat{\phi}(k)\left\|\Delta\overline{\xi}(k)\right\|\right)^2} \qquad (10)$$

$$\hat{\boldsymbol{\psi}}^T(k) = -\frac{b}{2}[1,0,\cdots,0]_{1\times(Le+Lc)},$$

$$\text{if}\begin{cases} |\hat{\psi}_i(k)| > \underline{b}/2, i=1,\cdots,L_e; \\ or\ |\hat{\psi}_{Le+1}(k-1)+\cdots+\hat{\psi}_{Le+Lc}(k-1)| > \tau^{Lc}; \\ or\ \left|\hat{\phi}(k)\|\Delta\overline{\boldsymbol{\xi}}(k)\|^2 + \delta\hat{\psi}_2(k-1)\lambda_k\right| \\ \quad \le \left(|\delta\hat{\psi}_3(k-1)|+\cdots+|\delta\hat{\psi}_{Le}(k-1)|+|\hat{\psi}_{Le}(k-1)|\right)\lambda_k; \\ or\ \hat{\psi}_1(k) > -\overline{b}; \end{cases} \tag{11}$$

$$\Delta u(k) = \boldsymbol{\psi}^T(k)\Delta\overline{\boldsymbol{\xi}}(k). \tag{12}$$

## 3 STABILITY ANALYSIS

In this section, we study the stability property of the closed loop system, and convergence of the output regulating error is guaranteed with mild assumptions.

**Assumption 3.** The ideal controller parameters satisfy the condition $\psi_1(k) \le -\overline{b}, \forall k \in \mathbb{Z}^+$, where $\overline{b} > 0$.

From *Assumption 3* and (4), it has

$$-\underline{b} \le \psi_1(k) \le -\overline{b} < 0,\ \forall k \in \mathbb{Z}^+. \tag{13}$$

**Lemma 1**[15]**.** Let

$$\mathbf{M} = \left[\begin{array}{cccc:c} m_1 & m_2 & \cdots & m_{L-1} & m_L \\ \hdashline & \mathbf{I}_{(L-1)\times(L-1)} & & & \mathbf{0}_{(L-1)\times 1} \end{array}\right], \tag{14}$$

if $\sum_{i=1}^{L}|m_i| < 1$, then the spectral radius of matrix $\mathbf{M}$ satisfies $s(\mathbf{M}) < 1$.

**Theorem 2.** Nonlinear system (1) is controlled by scheme (8)-(12) for regulation problem. If *Assumption 1* - *Assumption 3* are satisfied and if the following conditons hold,

(1) $\lambda_k > \lambda_{\min} \ge 0$, (15)

(2) $0 < \underline{\sigma} \le \phi(k) \le \varpi$, (16)

(3) $\underline{b}\varpi L_e \le 1$, (17)

there exist a minimal weighting factor $\lambda_{\min}$ and a samll enough $\tau$, such that, the output regulating error converges to zero asymptotically. That is $\lim_{k\to\infty} e(k+1) = 0$.

**Proof.** From (7) and (12), the dynamics of output regulating error could be rewritten as

$$e(k+1) = e(k) - \phi(k)\Delta u(k) = e(k) - \phi(k)\hat{\boldsymbol{\psi}}^T(k)\Delta\overline{\boldsymbol{\xi}}(k). \tag{18}$$

According to (10) and (11), there are two cases in parameters estimation.

*1. The resetting conditoin is activated.*

From reseting condition (11) and (18), we have

$$e(k+1) = (1-\phi(k)\underline{b}/2)e(k). \tag{19}$$

And from (16)(17), it derives

$$0 < \frac{\sigma b}{2} \le \frac{\phi(k)\underline{b}}{2} \le \frac{\varpi \underline{b}}{2} \le \frac{\varpi \underline{b} L_e}{2} \le \frac{1}{2} < 1. \tag{20}$$

Thus, there exists a constant $c$, such that,

$$0 < 1 - \frac{\phi(k)\underline{b}}{2} \le 1 - \frac{\sigma b}{2} = c < 1. \tag{21}$$

Taking the absolution value on both sides of (19) and from (21), it has

$$|e(k+1)| = |1-\phi(k)\underline{b}/2||e(k)| \le c|e(k)|. \tag{22}$$

*2. The resetting conditoin is unactivated.*

In this case,

$$|\delta\hat{\psi}_i(\cdot)| \le \underline{b},\ i = 2,\cdots,L_e. \tag{23}$$

where,

$$\delta\hat{\psi}_i(k-1) = \hat{\psi}_i(k-1) - \hat{\psi}_{i-1}(k-1),\ i = 2,3,\cdots,L_e. \tag{24}$$

Since $y_d(k+1) = y_d$, substituting (10) into (19), it gives

$$e(k+1) = e(k) - \frac{\phi(k)\left(\hat{\phi}(k)\left\|\Delta\overline{\boldsymbol{\xi}}(k)\right\|^2 e(k) + \lambda_k\hat{\boldsymbol{\psi}}^T(k-1)\Delta\overline{\boldsymbol{\xi}}(k)\right)}{\lambda_k + \left(\hat{\phi}(k)\left\|\Delta\overline{\boldsymbol{\xi}}(k)\right\|\right)^2}. \tag{25}$$

Let,

$$\kappa(k) = \frac{\lambda_k}{\lambda_k + (\hat{\phi}(k)\|\Delta\overline{\boldsymbol{\xi}}(k)\|)^2}, \tag{26}$$

$$\gamma(k) = \frac{\kappa(k)\left(\hat{\phi}(k)\|\Delta\overline{\boldsymbol{\xi}}(k)\|^2 + \delta\hat{\psi}_2(k-1)\lambda_k\right)}{\lambda_k}, \tag{27}$$

Then, (25) could be rewritten as

$$\mathbf{e}_{Le}(k+1) = \mathbf{C}(k)\mathbf{e}_{Le}(k) + \Delta\boldsymbol{U}(k-1), \tag{28}$$

where

$$\mathbf{e}_{Le}(k) = [e(k),\cdots,e(k-L_e+1)]^T;$$

$$\mathbf{C}(k) = \left[\begin{array}{cccc:c} c_1(k) & c_2(k) & \cdots & c_{Le-1}(k) & c_{Le}(k) \\ \hdashline & \mathbf{I}_{(Le-1)\times(Le-1)} & & & \mathbf{0}_{(Le-1)\times 1} \end{array}\right],$$

$$c_1(k) = 1 - \gamma(k)\phi(k),$$
$$c_i(k) = -\phi(k)\kappa(k)\delta\hat{\psi}_{i+1}(k-1),\ i = 2,\cdots,L_e-1,$$
$$c_{Le}(k) = \phi(k)\kappa(k)\hat{\psi}_{Le}(k-1);$$

$$\Delta\boldsymbol{U}(k-1) = \phi(k)\Theta_{Lc}\mathbf{A}(k)\Delta\mathbf{u}_{Lc}(k-1), \tag{29}$$

$$\Theta_{Lc} = \left[\begin{array}{c:c} 1 & \mathbf{0}_{1\times(Lc-1)} \\ \hdashline \mathbf{0}_{(Lc-1)\times 1} & \mathbf{0}_{(Lc-1)\times(Lc-1)} \end{array}\right],$$

$$\mathbf{A}(k) = \left[\begin{array}{cccc:c} a_1(k) & a_2(k) & \cdots & a_{Lc-1}(k) & a_{Lc}(k) \\ \hdashline & \mathbf{I}_{(Lc-1)\times(Lc-1)} & & & \mathbf{0}_{(Lc-1)\times 1} \end{array}\right],$$

$$a_i(k) = \kappa(k)\hat{\psi}_{Le+i}(k-1),\ i = 1,\cdots,L_c;$$

$$\Delta\mathbf{u}_{Lc}(k) = [\Delta u(k),\cdots,\Delta u(k-L_c+1)]^T.$$

To prove $\mathbf{e}_{Le}(k+1)$ is convergent, we consider the relation between $\Delta\mathbf{u}_{Lc}(k-1)$ and regulating error firstly.

From control law (12), it has

$$\Delta\mathbf{u}_{Lc}(k)=\mathbf{A}(k)\Delta\mathbf{u}_{Lc}(k-1)+\boldsymbol{E}(k)\,, \tag{30}$$

where,

$$\boldsymbol{E}(k)=\Theta_{Le}\mathbf{D}(k)\mathbf{e}_{Le}(k),$$

$$\mathbf{D}(k)=\left[\begin{array}{cccc|c} d_1(k) & d_2(k) & \cdots & d_{Le-1}(k) & d_{Le}(k) \\ \hline & \mathbf{I}_{(Le-1)\times(Le-1)} & & & \mathbf{0}_{(Le-1)\times 1}\end{array}\right], \tag{31}$$

$$d_1(k)=\gamma(k),$$
$$d_i(k)=\kappa(k)\delta\hat{\psi}_{i+1}(k-1),\quad i=2,\cdots,L_e-1,$$
$$d_{Le}(k)=-\kappa(k)\hat{\psi}_{Le}(k-1).$$

From (31), the characteristic equation of $\mathbf{D}(k)$ could be written as,

$$z^{Le}-d_1(k)z^{Le-1}-\cdots-d_{Le}(k)z^0=0\,. \tag{32}$$

Note that,

$$\sum_{i=1}^{Le}|d_i(k)|=\frac{\left|\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2+\delta\hat{\psi}_2(k-1)\lambda_k\right|}{\lambda_k+\left(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|\right)^2}+\frac{\left|\delta\hat{\psi}_3(k-1)\lambda_k\right|+\cdots+\left|\delta\hat{\psi}_{Le}(k-1)\lambda_k\right|+\left|\hat{\psi}_{Le}(k-1)\lambda_k\right|}{\lambda_k+\left(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|\right)^2}. \tag{33}$$

Since $|\hat{\psi}_i(k-1)|\le \underline{b}/2,\ \ i=1,\cdots,L_e$, it has

$$\begin{aligned}&\frac{\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2+\left|\delta\hat{\psi}_2(k-1)\lambda_k\right|+\cdots+\left|\delta\hat{\psi}_{Le}(k-1)\lambda_k\right|}{\lambda_k+\left(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|\right)^2}+\frac{\left|\hat{\psi}_{Le}(k-1)\lambda_k\right|}{\lambda_k+\left(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|\right)^2}\\ &\le\frac{\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2+\lambda_k\underline{b}_3L_e}{\lambda_k+(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2}.\end{aligned} \tag{34}$$

Let

$$g(\lambda_k)=\frac{\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2+\lambda_k\underline{b}L_e}{\lambda_k+(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2},$$

since $\lambda_k$ is an independent variable of function $g(\lambda_k)$, it has,

$$\partial g(\lambda_k)/\partial\lambda_k=\left(\underline{b}L_e\hat{\phi}(k)-1\right)\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2\,. \tag{35}$$

From (16) and (17), we have $(\underline{b}L_e\hat{\phi}(k)-1)\le(\underline{b}L_e\varpi-1)\le 0$ and $\partial g(\lambda_k)/\partial\lambda_k\le 0$. Hence, $g(\lambda_k)$ is a non-increasing function. Then, there exist sufficiently large $\lambda_{\min}$ and sufficiently small $\underline{b}_{\min}$, when $\lambda_k\ge\lambda_{\min}$ and $\underline{b}_3\le\underline{b}_{\min}$, it has

$$g(\lambda_k)<\min\left(\frac{0.5^{Le}}{\varpi},\frac{0.5^{Le}}{\varpi\|\Theta_{Le}\|_v\|\Theta_{Lc}\|_v},0.5^{Le}\right). \tag{36}$$

From (34), (35) and (36), (33) satisfies following condition,

$$\sum_{i=1}^{Le}|d_i(k)|<t_1^{Le}=\min\left(\frac{0.5^{Le}}{\bar{b}},\frac{0.5^{Le}}{\bar{b}\|\Theta_{Le}\|_v\|\Theta_{Lc}\|_v},0.5^{Le}\right)\le 0.5^{Le} \tag{37}$$

Therefore, according to *Lemma 1*, the roots of characteristic equation (32) satisfy $|z|<1$. Hence,

$$|z|^{Le}=\sum_{i=1}^{Le}|d_i(k)||z|^{Le-i}\le\sum_{i=1}^{Le}|d_i(k)|\,. \tag{38}$$

It means

$$|z|\le\left(\sum_{i=1}^{Le}|d_i(k)|\right)^{\frac{1}{Le}}<t_1\le 0.5\,.$$

Thus, there always exists a sufficiently small positive constant $\varepsilon_1$, such that,

$$\|\mathbf{D}(k)\|_v\le s\left(\mathbf{D}(k)\right)+\varepsilon_1\le\left(\sum_{i=1}^{Le}|d_i(k)||z|^{Le-i}\right)^{\frac{1}{Le}}+\varepsilon_1<t_1<1\,.$$

Letting $d=t_1\|\Theta_{Le}\|_v$, it has,

$$\begin{aligned}\|\boldsymbol{E}(k)\|_v&=\|\Theta_{Le}\mathbf{D}(k)\mathbf{e}_{Le}(k)\|_v\\ &\le\|\Theta_{Le}\|_v\|\mathbf{D}(k)\|_v\|\mathbf{e}_{Le}(k)\|_v<d\|\mathbf{e}_{Le}(k)\|_v\end{aligned} \tag{39}$$

Analogously, for matrix $\mathbf{A}(k)$, we have

$$g'(\lambda_k)=\sum_{i=1}^{Lc}|a_i(k)|=\frac{\left(|\hat{\psi}_{Le+1}(k-1)|+\cdots+|\hat{\psi}_{Le+Lc}(k-1)|\right)\lambda_k}{\lambda_k+\left(\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|\right)^2}\,. \tag{40}$$

And $g'(\lambda_k)$ it is a non-decreasing function with respect to $\lambda_k$. Hence, it has

$$g'(\lambda_k)\le g'(\infty)=|\hat{\psi}_{Le+1}(k-1)|+\cdots+|\hat{\psi}_{Le+Lc}(k-1)|\,. \tag{41}$$

Letting $t_2^{Lc}=|\hat{\psi}_{Le+1}(k-1)|+\cdots+|\hat{\psi}_{Le+Lc}(k-1)|$, and from (40), (41) and resetting condition in (11), we have

$$\sum_{i=1}^{Lc}|a_i(k)|\le t_2^{Lc}<\tau^{Lc}\,. \tag{42}$$

Thus, there exists a sufficiently small positive constant $\varepsilon_2$ and $\tau$, such that, eigenvalues of matrix $\mathbf{A}(k)$ satisfy $|z|<1$ according to *Lemma 1*. Then is has

$$\|\mathbf{A}(k)\|_v\le s\left(\mathbf{A}(k)\right)+\varepsilon_2\le t_2+\varepsilon_2<\tau+\varepsilon_2<1\,.$$

Let $a=\tau+\varepsilon_2$ and take the compatible norm on both sides of (30). Since $\Delta\mathbf{u}_{Lc}(k)=0,\ k\le 0$, it has

$$
\begin{aligned}
&\|\Delta\mathbf{u}_{Lc}(k-1)\|_v \\
&\le \|\mathbf{A}(k-1)\|_v \|\Delta\mathbf{u}_{Lc}(k-2)\|_v + \|\boldsymbol{E}(k-1)\|_v \\
&< a\|\Delta\mathbf{u}_{Lc}(k-2)\|_v + d\|\mathbf{e}_{Le}(k-1)\|_v \\
&< a^2\|\Delta\mathbf{u}_{Lc}(k-3)\|_v + ad\|\mathbf{e}_{Le}(k-2)\|_v + d\|\mathbf{e}_{Le}(k-1)\|_v \\
&\vdots \\
&< d\sum_{i=1}^{k-1} a^{k-i-1}\|\mathbf{e}_{Le}(i)\|_v .
\end{aligned} \tag{43}
$$

Letting $b = \bar{b}\|\Theta_{Lc}\|_v$ and substituting (43) into (29), it has

$$
\begin{aligned}
&\|\Delta\boldsymbol{U}(k-1)\|_v \\
&= \|\phi(k)\Theta_{Lc}\mathbf{A}(k)\Delta\mathbf{u}_{Lc}(k-1)\|_v \\
&\le \phi(k)\|\Theta_{Lc}\|_v \|\mathbf{A}(k)\|_v \|\Delta\mathbf{u}_{Lc}(k-1)\|_v \\
&< bad\sum_{i=1}^{k-1} a^{k-i-1}\|\mathbf{e}_{Le}(i)\|_v .
\end{aligned} \tag{44}
$$

Next, we will use (28) and (44) to derive the dynamics of regulating error.

Firstly, considering the matrix $\mathbf{C}(k)$ in (28) and using (34) (37), we have

$$
\begin{aligned}
&\sum_{i=1}^{Le}|c_i(k)| \\
&= 1-\phi(k)\frac{\left|\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2 + \delta\hat{\psi}_2(k-1)\lambda_k\right|}{\lambda_k + (\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2} \\
&\quad + \phi(k)\frac{|\delta\hat{\psi}_3(k-1)\lambda_k| + \cdots + |\delta\hat{\psi}_{Le}(k-1)\lambda_k|}{\lambda_k + (\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2} \\
&\quad + \phi(k)\frac{|\hat{\psi}_{Le}(k-1)\lambda_k|}{\lambda_k + (\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2} \\
&\ge 1-\phi(k)\frac{\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2 + |\hat{\psi}_{Le}(k-1)|\lambda_k}{\lambda_k + (\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2} \\
&\quad - \phi(k)\frac{\left(|\delta\hat{\psi}_2(k-1)| + \cdots + |\delta\hat{\psi}_{Le}(k-1)|\right)\lambda_k}{\lambda_k + (\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|)^2} \\
&\ge 1-\bar{b}\min\left(\frac{0.5^{Le}}{\bar{b}}, \frac{0.5^{Le}}{\bar{b}\|\Theta_{Le}\|_v\|\Theta_{Lc}\|_v}, 0.5^{Le}\right) \ge 0.5^{Le}.
\end{aligned} \tag{45}
$$

Then, from resetting condition, we have

$$
\begin{aligned}
&\left|\hat{\phi}(k)\|\Delta\bar{\xi}(k)\|^2 + \delta\hat{\psi}_2(k-1)\lambda_k\right| \\
&> \left(|\delta\hat{\psi}_3(k-1)| + \cdots + |\delta\hat{\psi}_{Le}(k-1)| + |\hat{\psi}_{Le}(k-1)|\right)\lambda_k .
\end{aligned}
$$

Hence, from the equation part of (45), it has $\sum_{i=1}^{Le}|c_i(k)| < 1$. Similar to the derivation before, there exists a sufficiently small positive constant $\varepsilon_3$, such that,

$$
0.5 \le s(\mathbf{C}(k)) \le \|\mathbf{C}(k)\|_v \le s(\mathbf{C}(k)) + \varepsilon_3 \le t_3 + \varepsilon_3 < 1 . \tag{46}
$$

Letting $c = t_3 + \varepsilon_3$ and taking the compatible norm on both sides of (28), we have following inequality by using (44) and (46),

$$
\begin{aligned}
&\|\mathbf{e}_{Le}(k+1)\|_v \le \|\mathbf{C}(k)\|_v\|\mathbf{e}_{Le}(k)\|_v + \|\Delta\boldsymbol{U}(k-1)\|_v \\
&< c\|\mathbf{e}_{Le}(k)\|_v + \|\Delta\boldsymbol{U}(k-1)\|_v \\
&< c^2\|\mathbf{e}_{Le}(k-1)\|_v + c\|\Delta\boldsymbol{U}(k-2)\|_v + \|\Delta\boldsymbol{U}(k-1)\|_v \\
&\vdots \\
&< c^k\|\mathbf{e}_{Le}(1)\|_v + \sum_{j=1}^{k-1} c^{k-j-1}\|\Delta\boldsymbol{U}(j)\|_v \\
&< c^k\|\mathbf{e}_{Le}(1)\|_v + \sum_{j=1}^{k-1} c^{k-j-1} bad\sum_{i=1}^{j} a^{j-i}\|\mathbf{e}_{Le}(i)\|_v .
\end{aligned} \tag{47}
$$

Let

$$
g(k+1) = c^k\|\mathbf{e}_{Le}(1)\|_v + \sum_{j=1}^{k-1} c^{k-j-1} bad\sum_{i=1}^{j} a^{j-i}\|\mathbf{e}_{Le}(i)\|_v ,
$$

(47) could be rewritten as

$$
\|\mathbf{e}_{Le}(k+1)\|_v < g(k+1). \tag{48}
$$

Obviously, if $g(k+1)$ monotonically converges to zero, output regulating error will converge to zero at the same time. And from the definition of $g(k+1)$, it is easy to have

$$
\begin{aligned}
&g(k+2) = c^{k+1}\|\mathbf{e}_{Le}(1)\|_v + \sum_{j=1}^{k} c^{k-j} dba\sum_{i=1}^{j} a^{j-i}\|\mathbf{e}_{Le}(i)\|_v \\
&= cg(k+1) + c^0 dba\sum_{i=1}^{k} a^{k-i}\|\mathbf{e}_{Le}(i)\|_v \\
&= cg(k+1) + dba\sum_{i=1}^{k-1} a^{k-i}\|\mathbf{e}_{Le}(i)\|_v + dba\|\mathbf{e}_{Le}(k)\|_v \\
&< cg(k+1) + c^0 dba\sum_{i=1}^{k-1} a^{k-i}\|\mathbf{e}_{Le}(i)\|_v + dbag(k).
\end{aligned} \tag{49}
$$

From the definition of $b, d$ in (39) and (44), it has $bd \le 0.5$ by using (37). And form definition of $c$ and (46), it has $c \ge 0.5$ . Then, $bd \le c$ . Hence, (49) could be rewritten as

$$
\begin{aligned}
&g(k+2) < cg(k+1) + c^0 dba\sum_{i=1}^{k-1} a^{k-i}\|\mathbf{e}_{Le}(i)\|_v + cag(k) \\
&= cg(k+1) + a\left(c^0 dba\sum_{i=1}^{k-1} a^{k-i-1}\|\mathbf{e}_{Le}(i)\|_v + cg(k)\right) \\
&< (c+a)g(k+1).
\end{aligned} \tag{50}
$$

Since $a = t_2 + \varepsilon_2$ and from (42), there exists a sufficiently small positive constant $\varepsilon_4$ . When $\tau \le \varepsilon_4$ , we have

$$
c + a = c + t_2 + \varepsilon_2 < c + \tau + \varepsilon_2 \le c + \varepsilon_4 + \varepsilon_2 < 1. \tag{51}
$$

Substituting (51) into (50), it has,

$$
\begin{aligned}
&\lim_{k\to\infty} g(k+2) < \lim_{k\to\infty}(c+a)g(k+1) < \cdots < \lim_{k\to\infty}(c+a)^k g(2) \\
&= \lim_{k\to\infty}(c+a)^k\left(c^2\|\mathbf{e}_{Le}(1)\|_v + bad\|\mathbf{e}_{Le}(1)\|_v\right) = 0.
\end{aligned} \tag{52}
$$

Apparently, it has $\lim_{k\to\infty} e(k+1) = 0$ from (22) and (52). ■

## 4 CONCLUSIONS

In this paper, the stability issue of FFDLc-MFAC is investigated in theory. With some mild assumptions, the convergence of the output regulating error was derived by rigorous mathematical method, which guarantees the correctness of the previously proposed method. Compared with CFDLc- or PFDLc-MFAC, the controller output $u(k)$ explicitly relies on the historical output tracking errors and control inputs, which is more general since for most of the control system, the control input in the past instants will also affects the system output. In a word, this method does not rely on the mathematical model of the plant and is easy for implementation, which is a promising control method for modern industrial process.

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