Model Free Adaptive Predictive Control Approach for Phase Splits of Urban Traffic Network

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Abstract: This paper proposes a novel model free adaptive predictive control strategy for phase splits of urban traffic network. The accurate model of the traffic network is difficult to establish. By using dynamic linearization approach, the nonlinear time-varying traffic network is described as an equivalent simplified data model. Then, model free adaptive predictive control (MFAPC) scheme is developed by using the equivalent data model. The robustness of the MFAPC based phase splits strategy to time-varying parameters and traffic demands is verified through simulation results.

Key Words: Model free adaptive Control, Signal Splits, Traffic Network

1. INTRODUCTION

Traffic congestion has become a common scene in many metropolises. There are two well-known and widely used coordinated traffic-responsive control strategies for urban traffic network, SCOOT [1] and SCATS [2]. It has shown that they work very well if traffic conditions are undersaturated, but their control performance may deteriorate during peak hours since severe traffic congestion persists [3].

Store and forward (SF) model [4] was proposed by Gazis and Potts for saturated urban traffic network. Several SF model based optimization methods are proposed for phase splits of urban traffic networks, e.g., offline infinite horizon linear quadratic optimization techniques [5], constrained quadratic optimal control [3] and open-loop constrained nonlinear control [6]. However, SF model is based on assumptions that traffic conditions do not change significantly and traffic parameters are known as a prior. In practice, traffic conditions and traffic parameters are time-varying and difficult to identify.

Model predictive control (MPC) has high control performance and robustness to model uncertainties and/or external disturbances. Several MPC based optimization methods are developed for phase splits of urban traffic network [7-10]. In [7], mixed-integer linear programming (MILP) based MPC was proposed to increase the online feasibility of the proposed MPC method. Based on the results in [7], efficient network-wide MPC-based control methods are investigated [8]. In [9], a decentralized multi-agent MPC is proposed for decoupled urban traffic network. Taking the unknown but bounded uncertainty into account, a centralized robust MPC is developed for urban traffic network in [10]. Although MPC methods [7-10] improve the robustness to traffic model uncertainties and external disturbances, their control performance depends on the model accuracy. Practically, an accurate traffic flow model of certain traffic network is difficult to establish and maybe very sensitive to some parameters of the traffic network, including turning ratios, exit rates and saturation flows. Thus, it is desirable to design a model-free or data-driven control strategy for phase splits in urban traffic network.

Model free adaptive control (MFAC) [11, 12] methodology proposed by Hou is designed and analyzed based on the dynamic linearization technique (DLT) of a class of discrete-time nonlinear systems. Theoretical analysis and extensive field applications have shown its effectiveness and applicability to unknown discrete-time nonlinear systems. In this paper, a novel model-free adaptive predictive control (MFAPC) based phase splits strategy is proposed for the oversaturated urban traffic network. Motivated by SF model in [4], more realistic time-varying SF models of an isolated junction with four phases and urban traffic network are derived. By DLT in [11, 12], the time-varying SF model of the urban traffic network are transformed into an equivalent simplified data model. Then, a model free adaptive predictive control scheme is designed for phase splits of all junctions in urban traffic network. Different from the SF model based control strategies, the proposed MFAPC method does not require an accurate model of the network and a designed nominal control scheme. An isolated junction is simulated on MATLAB to

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show the efficiency of the proposed MFAPC based phase splits strategy.

2. DYNAMICS OF URBAN TRAFFIC NETWORK

2.1 Isolated Junction

Consider the isolated junction with four phases shown in Fig. 1. The dynamics of the link l is given as follows

$$x_l(k+1) = x_l(k) + T\left(q_l(k) - s_l(k) + d_l(k) - h_l(k)\right) \quad (1)$$

where $x_l(k)$, $q_l(k)$, $h_l(k)$, $d_l(k)$ and $s_l(k)$ are the number of vehicles, inflow, outflow, demand and exit flow of the controlled link l, $l \in \{1,2,3,4\}$. T is the sampling time and $k = 0,1,\cdots$ the discrete-time index. The exit flow is set to be $s_l = t_l(k)q_l(k)$ with an unknown exit rates $t_l(k)$. The number of vehicles within link l is constrained by

$$0 \le x_l(k) \le x_{l,max} \quad (2)$$

where $x_{l,max}$ is the maximum admissible number of vehicles.

Denote $S_l(k)$ as the unknown time-varying saturation flow and u_i as the green time of phase i with following constraints

$$u_{i,\min} \le u_i \le u_{i,\max}, \quad (3)$$

$$\sum_{i\in\{1,2,3,4\}} u_i + L = C, \quad (4)$$

where L is the fixed lost time and C is the cycle time.

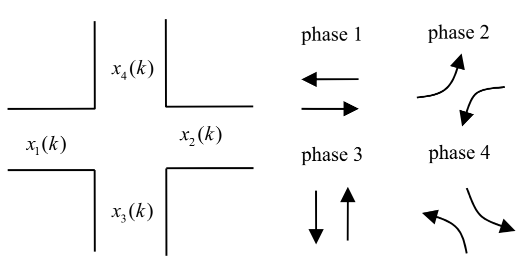

Fig. 1 Isolated junction with four links and phase specification

Let v_l denote the set of phases where link l has right of way. For oversaturated junction, if the link l has right of way, outflow $h_l(k)$ is equal to $S_l(k)$, and equal to zero otherwise. Thus, outflow $h_l(k)$ is represented as follows

$$h_l(k) = \sum_{i\in v_l} u_i(k)S_l(k)/C. \quad (5)$$

Substituting (5) into (1) gives

$$\begin{aligned} x_l(k+1) &= x_l(k) + T\left(\left(1-t_l(k)\right) + d_l(k) - h_l(k)\right) \\ &= x_l(k) + T\left(\left(1-t_l(k)\right)q_l(k) + d_l(k) - \sum_{i\in v_l} u_i(k)S_l(k)/C\right). \end{aligned} \quad (6)$$

Let $\mathbf{x}(k) = \left[x_1(k),\cdots,x_4(k)\right]^T$, $\mathbf{u}(k) = \left[u_1(k),\cdots,u_4(k)\right]^T$, $\mathbf{d}(k) = \left[d_1(k),\cdots,d_4(k)\right]^T$, $\mathbf{q}(k) = \left[q_1(k),\cdots,q_4(k)\right]^T$.

The dynamics of the isolated junction is represented by

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \Theta(k)\mathbf{q}(k) + B(k)\mathbf{u}(k) + T\mathbf{d}(k) \quad (7)$$

where $\Theta(k) = T\begin{bmatrix} 1-t_1(k) & & & \\ & 1-t_2(k) & & \\ & & 1-t_3(k) & \\ & & & 1-t_4(k) \end{bmatrix}$,

$$B(k) = -\frac{T}{C}\begin{bmatrix} S_1(k) & S_1(k) & 0 & 0 \\ S_2(k) & S_2(k) & 0 & 0 \\ 0 & 0 & S_3(k) & S_3(k) \\ 0 & 0 & S_4(k) & S_4(k) \end{bmatrix}.$$

2.2 Traffic Network

Let I_j, O_j, F_j denote the sets of incoming links, outgoing links, and phases of junction j, respectively. The offset and the cycle time for all junctions are assumed to be fixed.

For traffic network, the inflow $q_l(k)$ in (6) can be calculated by

$$\begin{aligned} q_l(k) &= \sum\nolimits_{w\in I_j} t_{w,l}(k)h_w(k) \\ &= \sum\nolimits_{w\in I_j} t_{w,l}(k)u_w(k)S_w(k)/C, \end{aligned} \quad (8)$$

where $t_{w,l}(k)$ is unknown turning ratio towards link l from the links that enter junction j.

Substituting (5) and (8) into (1) gives

$$\begin{aligned} x_l(k+1) &= x_l(k) \\ &+ T\left(\left(1-t_l(k)\right)\sum\nolimits_{w\in I_j}\frac{t_{w,l}(k)u_w(k)S_w(k)}{C} - \frac{\sum_{i\in v_l, v_l\in O_j} u_i(k)S_l(k)}{C}\right) \\ &+ Td_l(k). \end{aligned} \quad (9)$$

By organizing all equations of controlled links, the dynamics of the traffic network can be represented as a linear time-varying state-space model

$$\mathbf{x}(k+1) = \mathbf{x}(k) + B(k)\mathbf{u}(k) + T\mathbf{d}(k) \quad (10)$$

where $\mathbf{x}(k) = \left[x_1(k),\cdots,x_N(k)\right]^T$, $\mathbf{u}(k) = \left[u_1(k),\cdots,u_N(k)\right]^T$, $\mathbf{d}(k) = \left[d_1(k),\cdots,d_N(k)\right]^T$ are system state (number of vehicles), control input (green time), disturbance (unknown non-controlled traffic demand) vectors, N is the number of controlled links. $B(k)$ is an unknown time-varying matrix that consists of three basic elements, namely, turning ratios, exit rates, and saturation flows.

3. MFAPC BASED SIGNAL SPLITS STRATEGY

3.1 Dynamic Linearization

Urban traffic network (10) essentially is as a multi-input and multi-output (MIMO) system with time-varying control input gain matrix. According to DLT of MIMO discrete-time nonlinear systems in [11, 12], if traffic demand $\mathbf{d}(k)$ is slowly time-varying, (10) becomes

$$\Delta\mathbf{x}(k+1)=\Phi(k)\Delta\mathbf{u}(k) \tag{11}$$

where $\mathbf{\Phi}(k)\in\mathbf{R}^{N\times N}$ is the pseudo Jacobian matrix (PJM) of the traffic network.

Let n_u be the control input horizon. If $\Delta\mathbf{u}(k+j-1)=0$ holds for $j>n_u$, n-step ahead prediction equations are derived from (11) as follows

$$\begin{cases}\mathbf{x}(k+1)=\mathbf{x}(k)+\Phi(k)\Delta\mathbf{u}(k)\\ \mathbf{x}(k+2)=\mathbf{x}(k)+\Phi(k)\Delta\mathbf{u}(k)+\Phi(k+1)\Delta\mathbf{u}(k+1)\\ \quad\vdots\\ \mathbf{x}(k+n_u)=\mathbf{x}(k)+\Phi(k)\Delta\mathbf{u}(k)+\Phi(k+1)\Delta\mathbf{u}(k+n_u-1)\\ \quad\vdots\\ \mathbf{x}(k+n)=\mathbf{x}(k)+\Phi(k)\Delta\mathbf{u}(k)+\Phi(k+1)\Delta\mathbf{u}(k+1)\\ \qquad+\cdots+\Phi(k+n_u-1)\Delta\mathbf{u}(k+n_u-1),\end{cases} \tag{12}$$

Let $\bar{\mathbf{x}}(k+1)=\left[\mathbf{x}(k+1)^T,\cdots,\mathbf{x}(k+n)^T\right]^T$,

$$\Delta\bar{\mathbf{u}}(k)=\left[\Delta\mathbf{u}(k)^T,\cdots,\Delta\mathbf{u}(k+n_u-1)^T\right]^T,\ \mathbf{M}=\begin{bmatrix}I\\ \vdots\\ I\end{bmatrix},$$

$$\mathbf{A}(k)=\begin{bmatrix}\Phi(k) & 0 & 0 & 0\\ \Phi(k) & \Phi(k+1) & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ \Phi(k) & \Phi(k+1) & \cdots & \Phi(k+n_u-1)\\ \vdots & \vdots & \cdots & \vdots\\ \Phi(k) & \Phi(k+1) & \cdots & \Phi(k+n_u-1)\end{bmatrix}.$$

Equation (12) is rewritten in a compact form

$$\bar{\mathbf{x}}(k+1)=\boldsymbol{M}\mathbf{x}(k)+\mathbf{A}(k)\Delta\bar{\mathbf{u}}(k). \tag{13}$$

3.2 Model Free Adaptive Predictive Control

Consider cost function of control input as follows

$$J=\left\|\bar{\mathbf{x}}(k+1)\right\|^2+\lambda\left\|\Delta\bar{\mathbf{u}}(k)\right\|^2 \tag{14}$$

Substituting (13) into (14) and optimizing (14) yield the control law

$$\Delta\bar{\mathbf{u}}(k)=-\left[\mathbf{A}^T(k)\mathbf{A}(k)+\lambda\mathbf{I}\right]^{-1}\mathbf{A}^T(k)\mathbf{M}\mathbf{x}(k). \tag{15}$$

The control input at current time k is constructed according to the receding horizon principle from (15)

When $n_u=1$, control algorithm becomes

$$\Delta\mathbf{u}(k)=-\left[\frac{\lambda}{n}\mathbf{I}+\Phi(k)^T\Phi(k)\right]^{-1}\Phi(k)^T\mathbf{x}(k). \tag{16}$$

Since $\mathbf{A}(k)$ in (15) contains unknown PJM parameters $\Phi(k),\Phi(k+1)\cdots,\Phi(k+n_u-1)$, estimation and prediction algorithm should be designed before the control algorithm is used in practice. The following modified projection algorithm is developed to estimate $\Phi(k)$

$$\hat{\Phi}(k)=\hat{\Phi}(k-1)+\frac{\eta\Delta\mathbf{u}(k-1)\left(\Delta\mathbf{x}(k)-\hat{\Phi}(k-1)\Delta\mathbf{u}(k-1)\right)}{\mu+\left\|\Delta\mathbf{u}(k-1)\right\|^2}, \tag{17}$$

where μ and η are the weighting factor and step size factor, respectively.

$\Phi(k+1)\cdots,\Phi(k+n_u-1)$ should be predicted according to estimated sequence $\hat{\Phi}(1)\cdots,\hat{\Phi}(k)$. In this paper, the multi-level hierarchical forecasting method [13] is applied here to predict PJM parameters.

At time k, $\hat{\Phi}(1)\cdots,\hat{\Phi}(k)$ have been calculated by (17). Using these estimated values, an auto regressive model for prediction is established as follows

$$\begin{aligned}\hat{\Phi}(k+1)&=\Upsilon_1(k)\hat{\Phi}(k)+\Upsilon_2(k)\hat{\Phi}(k-1)\\ &\quad+\cdots+\Upsilon_{n_p}(k)\hat{\Phi}(k-n_p+1),\end{aligned} \tag{18}$$

where Υ_i, $i=1,\cdots,n_p$ is coefficient matrix, and n_p is the fixed model order.

Using (18), prediction equation becomes

$$\begin{aligned}\hat{\Phi}(k+j)&=\Upsilon_1(k)\hat{\Phi}(k+j-1)+\Upsilon_2(k)\hat{\Phi}(k+j-2)\\ &\quad+\cdots+\Upsilon_{n_p}(k)\hat{\Phi}(k+j-n_p),\end{aligned} \tag{19}$$

where $j=1,\cdots,N_u-1$.

Let $\bar{\Upsilon}(k)=\left[\Upsilon_1(k),\cdots,\Upsilon_{n_p}(k)\right]$, $\hat{\bar{\Phi}}(k)=\begin{bmatrix}\hat{\Phi}(k)\\ \vdots\\ \hat{\Phi}(k-n_p+1)\end{bmatrix}$.

The unknown coefficient matrix $\bar{\Upsilon}$ is calculated by

$$\bar{\Upsilon}(k)=\bar{\Upsilon}(k-1)+\frac{\hat{\bar{\Phi}}(k-1)}{\delta+\left\|\hat{\bar{\Phi}}(k-1)\right\|^2}\left[\hat{\Phi}(k)-\bar{\Upsilon}(k-1)\hat{\bar{\Phi}}(k-1)\right], \tag{20}$$

where δ is a positive constant.

3.3 Control Scheme

Integrating control algorithm (15), parameter estimation algorithm (17), and the prediction algorithm (19)-(20), Model-free adaptive predictive control (MFAPC) scheme is designed as follows

$$\hat{\Phi}(k)=\hat{\Phi}(k-1)+\frac{\eta\Delta\mathbf{u}(k-1)\left(\Delta\mathbf{x}(k)-\hat{\Phi}(k-1)\Delta\mathbf{u}(k-1)\right)}{\mu+\left\|\Delta\mathbf{u}(k-1)\right\|^{2}},\tag{21}$$

$$\bar{\Upsilon}(k)=\bar{\Upsilon}(k-1)+\frac{\hat{\bar{\Phi}}(k-1)}{\delta+\left\|\hat{\bar{\Phi}}(k-1)\right\|^{2}}\left[\hat{\Phi}(k)-\bar{\Upsilon}(k-1)\hat{\bar{\Phi}}(k-1)\right],\tag{22}$$

$$\bar{\Upsilon}(k)=\bar{\Upsilon}(1)\text{ , if }\left\|\bar{\Upsilon}(k)\right\|\geq\gamma_{\max},\tag{23}$$

$$\begin{aligned}\hat{\Phi}(k+j)&=\Upsilon_{1}(k)\hat{\Phi}(k+j-1)+\Upsilon_{2}(k)\hat{\Phi}(k+j-2)\\&+\cdots+\Upsilon_{n_p}(k)\hat{\Phi}(k+j-n_p),\ j=1,2,\cdots,n_u-1,\end{aligned}\tag{24}$$

$$\text{If } sign\left(\hat{\Phi}(k+j)\right)\neq sign\left(\hat{\Phi}(1)\right)\ ,\ \hat{\Phi}(k+j)=\hat{\Phi}(1)\ ,\quad j=0,2,\cdots,n_u-1,\tag{25}$$

$$\Delta\bar{\mathbf{u}}(k)=-\left[\hat{\mathbf{A}}^{T}(k)\hat{\mathbf{A}}(k)+\lambda\mathbf{I}\right]^{-1}\hat{\mathbf{A}}^{T}(k)\mathbf{M}\mathbf{x}(k).\tag{26}$$

where ε , M λ , μ , η and δ are positive constants, $\hat{\mathbf{A}}(k)$ and $\hat{\Phi}(k+j)$ are the estimations of $\mathbf{A}(k)$ and $\hat{\Phi}(k+j)$, $j=1,\cdots,(n_u-1)$, respectively.

4. SIMULATION RESULTS

In this section, an isolated junction with 4 phases is simulated on MATLAB platform to show the effectiveness of the proposed MFAPC scheme for phase splits. The initial number of vehicles of the junction is $\mathbf{x}(0)=[6,5,5,6]^{T}$. $t_i(k)$, $d_i(k)$, $S_i(k)$ are randomly varying within interval $[0.05,0.1]$, $[100,200]$ and $[900,1000]$, respectively. Sampling time and cycle time are set to be 120 seconds. Traffic demands of four links are shown in Fig. 2.

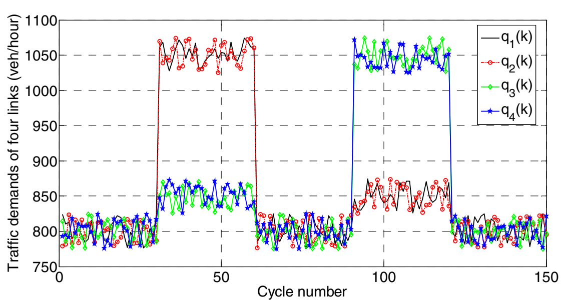

Fig.2 Traffic demands of four links

The parameters of MFAPC scheme are $n=3$, $n_u=1$, $\varepsilon=10^{-5}$, $\delta=1$, $\eta=1$, $\gamma_{\max}=10$. The initial value of PJM is set to be $\hat{\Phi}(1)=\begin{bmatrix}-1&-1&0&0\\-1&-1&0&0\\0&0&-1&-1\\0&0&-1&-1\end{bmatrix}$. Fixed time controller is set to be $u_i(1)=30, i\in\{1,2,3,4\}$ according to the initial traffic demands of four links. The total time spend $TTS=T\sum_{k=1}^{150}\sum_{i=1}^{4}x_i(k)$ is select as the evaluation criteria.

Simulation results by using the proposed MFAPC and fixed time controller are shown in Figures 3-4. The total time spends are $TTS_{MFAPC}=19074$ and $TTS_{FT}=53082$. It shows that the number of vehicles of four links reduced by the proposed MFAPC scheme effectively despite the unknown time varying parameters and demands.

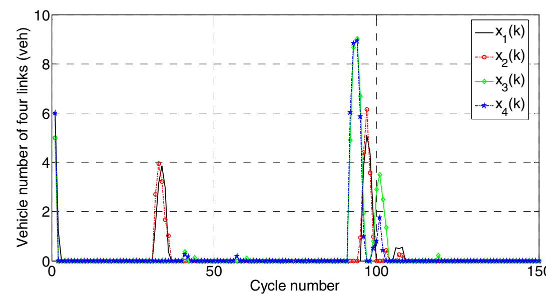

Fig.3 Vehicle number of four links (MFAPC)

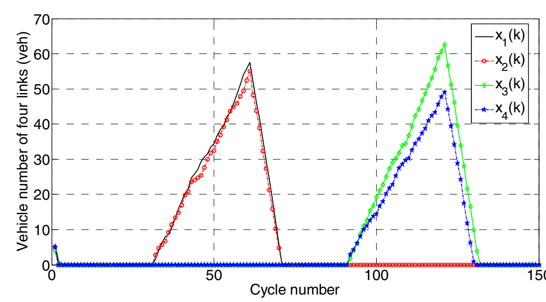

Fig.4 Vehicle number of four links (Fixed time control)

5. CONCLUSIONS

In this paper, a model free adaptive predictive control based phase splits strategy is proposed for the urban traffic network. It does not require an accurate model of the network for controller design. In fact, the phase splits problem is the control problem of MIMO discrete-time systems. Although the accurate model of the network can not be obtained, the unknown network is transformed into an equivalent simplified data model. Then, MFAPC scheme is developed based on the data model. Simulation results show the robustness of the proposed phase splits strategy to time-varying exit rates, saturation flows and traffic demands.

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