A Novel Quadratic Programming Based Model-free Adaptive Control for I/O Constrained Nonlinear Systems

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Abstract: A novel quadratic programming (QP) based model-free adaptive control approach is proposed in this work to deal with constrained nonlinear discrete-time systems. Three types of constraints are considered together in this work, i.e., the constraints on the boundary of control input, the boundary of system output, as well as the change rate of control input between two time instants. All these constraints are transferred into a unified linear matrix inequality (LMI). Then, a control input index function is designed with respect to control errors and control input changes in a quadratic form. The control law is attained by minimizing the index function subjected to the LMI via quadratic programming (QP). The designed approach is data-based only and does not need an exactly linear model. Both theoretical and simulative results verify that the proposed approach is effective to applications. the effectiveness of the proposed approach.

Key Words: Model-free adaptive control, Input and output constrained, Nonlinear systems, Quadratic Programming

1 Introduction

Iterative learning control (ILC) [1-3] is most suitable for repetitive dynamics operating over a fixed time interval. In general, there are three major structures of ILC system: Pure feedforward (PFF) ILC, Pure feedback (PFB) ILC, and feedback – feedforward (FBFF) ILC.

Nowadays, the complex processes are most common in real industries with equipments in a large scale [1-3]. Apparently, it becomes more difficult to model such large processes using first principles or identification techniques. As a direct result, traditional model based control approaches encounter some difficulties when applied into these complex processes. The unmodeled dynamics is inevitable in the modeling process, which is the major factor degrading the robustness [2, 3].

On the other hand, so much measurement data appears in the operating processes of many industries that the some useful control knowledge of the process operation can be obtained from the measurements. Hence, how to design a control method using the system measurements only, without requiring any model information, becomes urgent for complex practical systems.

Data-driven control approaches imply that the controller design and analysis only need the process I/O measurements [2, 3]. Nowadays, many data-driven control methods have been developed, such as, model-free adaptive control [3-6], iterative learning control [7-10], unfalsified control [11], virtual reference feedback tuning [12], iterative feedback tuning [13], and so on.

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Comparatively, the theoretical results of model-free adaptive control (MFAC) are guaranteed with rigorous mathematical analysis. Model-free adaptive control approach is proposed for a class of nonlinear systems [3-6]. The key innovation of MFAC is that a new completely equivalent dynamical linearization approach is developed by introducing a new concept of "pseudo-partial derivative (PPD)". Moreover, the time-varying PPD parameter can be estimated by using the I/O data. Different from Neural networks based control approaches; neither external testing signals nor training process is required for MFAC.

Recently, model-free adaptive control has attracted many attentions due to its simple structure, small computational cost, easy implementation, and strong robustness. In [14], a predictive adaptive control is proposed by using the basic idea of MFAC. A higher order control law of MFAC is proposed in [15] to enhance the control performance by utilizing much more control knowledge obtained from previous time instants. In [5], the theoretical conclusions of PFDL-based MFAC were proved rigorously. Further, the results of MFAC were extended from SISO nonlinear systems to MIMO nonlinear systems with mathematical analysis in [6]. An adaptive observer based MFAC is further proposed in [16] and the stability and convergence is analyzed through Lyapunov method. Now, MFAC has been applied into many real processes, such as the chemical industry [17, 18], linear motor control [19], injection modeling process [20], pH value control [21], and so on.

In addition to uncertainty and complexity, most real processes to be controlled are generally subjected to various types of physical constraints, such as input saturation and state constraints due to physical limitations of the actuators or the safety requirements. For example, the automatic train operation system is constrained not only by the input force due to the mechanical features, but also by the operation

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speed to meet the safety requirement. Despite the fact that a well established theory for model-free adaptive control has been developed, there are few results on MFAC of constrained systems. Further research in MFAC of nonlinear systems subject to constraints could broaden the range of the practical applications in which MFAC may be used.

It is well known that optimization-based control [22-24] is a systematic tool of dealing with system constraints. However, the optimal control approaches require an accurate linear model without fully considering the effect of disturbances. Model predictive control (MPC) also is a powerful technique for controlling constrained systems [25-26]. However, some initial knowledge on the plant is also required for the MPC approaches.

In this work, a new model-free adaptive control approach is proposed for nonlinear systems with both input and output constraints. Three constraints are considered in this work, i.e., the constraints on system input, system output, as well as the change rate of the input signals between two consecutive instants. Then, the quadratic programming (QP) is utilized to obtain the control law is by optimizing a control input objective function subjected to all the above constraints. Different from traditional constraint optimal control and model predictive control, the learning gain of the QP-based optimal control law can be estimated by an updating law. In addition, the proposed approach is a data-driven control strategy without using any model information. The simulation study confirms the applicability of the proposed approach.

This paper is organized as follows. Section 2 is the problem formulation. Section 3 designs the QP-based MFAC approach. The tracking error convergence is proved in Section 4. Some simulation results are given in Section 5. The conclusion is attained in Section 6.

2 Problem Formulation

Consider a discrete-time nonlinear system represented in the following NARX model,

$$y(k+1) = f(y(k),\cdots,y(k-n_y),u(k),\cdots,u(k-n_u)) \tag{1}$$

where $y(k)\in R$ and $u(k)\in R$ are the system output and input, respectively; n_y and n_u are two positive integers denoting system orders, which are unknown; $f(\cdot)$ is an unknown nonlinear function.

The proposed novel QP-based model-free adaptive control is based on a completely equivalent linearization of nonlinear system (1). Before preceding the linearization technique, two assumptions are introduced as follows.

Assumption 1 [3]: The partial derivative of $f(\cdot)$ with respect to the control input $u(k)$ is continuous.

Assumption 2 [3]: The system (1) is generalized Lipschitz, that is, $|y(k_1+1)-y(k_2+1)|\le b|u(k_1)-u(k_2)|$ for $\forall k_1\ne k_2$, $k_1,k_2\ge 0$ and $u(k_1)\ne u(k_2)$, where $y(k_i+1) = f(y(k_i),\cdots,y(k_i-n_y),u(k_i),\cdots,u(k_i-n_u))$, $i=1,2$; $b>0$ is a constant.

Theorem 1 [3]: For the nonlinear system (1), satisfied with the assumptions 1 and 2, there must exist a parameter $\phi(k)$, called PPD, such that the nonlinear system (1) can be transformed into the following CFDL description when $\Delta u(k)\ne 0$,

$$\Delta y(k+1) = \phi(k)\Delta u(k) \tag{2}$$

and $|\phi(k)|\le b$.

where $\Delta y(k+1) = y(k+1)-y(k)$, $\Delta u(k) = u(k)-u(k-1)$.

In this work, three forms of constraints are considered as follows.

(1) Constraint on the boundaries of the control input signals

$$u^{low}\le u(k)\le u^{hi}. \tag{3}$$

(2) Constraints on the rate of input changes between two consecutive time instants,

$$\Delta u^{low}\le \Delta u(k)\le \Delta u^{hi}. \tag{4}$$

(3) Constraints on the upper and lower bounds of the system outputs,

$$y^{low}\le y(k+1)\le y^{hi}. \tag{5}$$

The control objective is to find an optimal control input subjected to the above constraints (3) and (4) such that the output of system (1) can track the desired output $y^*(k)$ and be constrained by (5).

3 Controller Design via Quadratic Programming

3.1 Constraint Conditions Transformation

To facilitate the subsequent controller design and convergence analysis, all the above constraints (3) – (5) can be rewritten as linear inequalities with respect to $\Delta u(k)$.

Subtracting $u(k)$ from both sides of (3), one obtains

$$u^{low}-u(k-1)\le \Delta u(k)\le u^{hi}-u(k-1). \tag{6}$$

Subtracting $y(k)$ from both sides of (5), yields,

$$y^{low}-y(k)\le \Delta y(k+1)\le y^{hi}-y(k) \tag{7}$$

According to (2), inequality (7) becomes,

$$y^{low}-y(k)\le \phi(k)\Delta u(k)\le y^{hi}-y(k) \tag{8}$$

Combing (4), (6) and (8) into a whole linear inequality, one obtains,

$$\overline{\boldsymbol{A}}(k)\Delta u(k)\le \overline{\boldsymbol{b}}(k) \tag{9}$$

where $\overline{\boldsymbol{A}}(k)=\begin{bmatrix}-1\\1\\1\\-1\\-\phi(k)\\\phi(k)\end{bmatrix}$, $\overline{\boldsymbol{b}}(k)=\begin{bmatrix}-\Delta u^{low}\\\Delta u^{hi}\\u^{hi}-u(k-1)\\-u^{low}+u(k-1)\\y(k)-y^{low}\\y^{hi}-y(k)\end{bmatrix}$.

Denote $\Delta u^{low*} = \max(u^{low} - u(k-1), \Delta u^{low})$ and $\Delta u^{hi*} = \min(u^{hi} - u(k-1), \Delta u^{hi})$. Then inequality (9) is rewritten as

$$\boldsymbol{A}(k)\Delta u(k) \le \boldsymbol{b}(k) \quad (10)$$

where $\boldsymbol{A}(k) = \begin{bmatrix} -1 \\ 1 \\ -\phi(k) \\ \phi(k) \end{bmatrix}$ and $\boldsymbol{b}(k) = \begin{bmatrix} -\Delta u^{low*} \\ \Delta u^{hi*} \\ y(k) - y^{low} \\ y^{hi} - y(k) \end{bmatrix}$.

3.2 Constrained MFAC Design

Consider the following objective function,

$$J(e(k+1), u(k)) = |e(k+1)|^2 + \lambda |u(k) - u(k-1)|^2 \quad (11)$$

where $e(k+1) = y^*(k+1) - y(k+1)$.

According to Eq. (2),

$$e(k+1) = y^*(k+1) - y(k+1) = y^*(k+1) - y(k) - \phi(k)\Delta u(k) \quad (12)$$

Substituting (12) into (11), one obtains

$$J(e(k+1), u(k)) = |y^*(k+1) - y(k) - \phi(k)\Delta u(k)|^2 + \lambda |u(k) - u(k-1)|^2 \quad (13)$$

Minimizing the above objective function subjected to the above inequality (10), that is,

$$\begin{cases} \min\limits_{u(k)} J(e(k+1), u(k)) \\ s.t.\ A(k)\Delta u(k) \le b(k) \end{cases} \quad (14)$$

one can get the control law.

Note that in (14), the objective function together with the feasible region defined by the constraint inequality constitutes a quadratic programming problem. To get the control input $u(k)$, we should solve this QP problem.

Note that $\phi(k)$ is unknown in (14), therefore an adaptive updating law for the parameter $\phi(k)$ is provided as follows [3],

$$\hat{\phi}(k) = \hat{\phi}(k-1) + \frac{\eta \Delta u(k-1)}{\mu + \Delta u(k-1)^2} (\Delta y(k) - \hat{\phi}(k-1)\Delta u(k-1)) \quad (15)$$

$$\hat{\phi}(k) = \hat{\phi}(0), \text{if } |\hat{\phi}(k)| \le \varepsilon, \text{or } \operatorname{sgn}(\hat{\phi}(k)) \ne \operatorname{sgn}(\hat{\phi}(1)), \text{ or } |\Delta u(k-1) \le \varepsilon| \quad (16)$$

where $\mu > 0$ is a weighting factor, $0 < \eta < 2$ is a step-size constant series, which can make the parameter estimate algorithm Eq.(15) more general. Moreover, the initial value $\hat{\phi}(0)$ is selected as $\hat{\phi}(0) > \varepsilon > 0$. (16) is a reset algorithm such that the estimate algorithm (15) can track time varying PPD parameter $\phi(k)$ more effectively [3].

Combining (15) and (16), the previous QP problem (14) becomes

$$\begin{cases} \min\limits_{u(k)} \left\{ |y^*(k+1) - y(k) - \hat{\phi}(k)\Delta u(k)|^2 + \lambda |u(k) - u(k-1)|^2 \right\} \\ s.t.\ \hat{\boldsymbol{A}}(k)\Delta u(k) \le \boldsymbol{b}(k) \end{cases} \quad (17)$$

where $\hat{\boldsymbol{A}}(k) = \begin{bmatrix} -1 \\ 1 \\ -\hat{\phi}(k) \\ \hat{\phi}(k) \end{bmatrix}$.

For the QP problem (17), one can use many well-known iterative techniques such as interior-point-type method to obtain the control law at the *k*-th time instant.

4 Convergence Analysis

Theorem 2: For the nonlinear constrained systems (1) and (2)-(4), under assumptions 1-2, the proposed QP-MFAC (15) – (17) guarantees that (a) the parameter estimation value $\hat{\phi}(k)$ is bounded; and (b) both the input change rate and the tracking error converges to zero as time instant *t* approaches infinity.

Proof. The boundedness of parameter estimation $\hat{\phi}(k)$ has been proved in [3].

Since $(e(k+1), u(k))$ is the optimal solution of index function (11) and the corresponding system output and input values are satisfied with the constraints defined in (10). When $y^*(k+1) = y_{sp}$ is a constant, according to (12),

$$e(k+1) = e(k) - \phi(k)\Delta u(k) \quad (18)$$

When $\Delta u_{k+1} = 0$ is fixed, we can get $e(k+1) = e(k)$. Thus, $(e(k), 0)$ is a feasible point. It's obvious that the following equation is satisfied,

$$J(e(k+1), u(k)) \le J(e(k), 0) = J(e(k), u(k)) - \lambda |u(k) - u(k-1)|^2 \quad (19)$$

Then,

$$J(e(k+1), u(k)) \le J(e(0), u(0)) - \lambda \sum_{i=1}^{k} |u(k) - u(k-1)|^2 \quad (20)$$

that is,

$$0 \le J(e(k+1), u(k)) + \lambda \sum_{i=1}^{k} |u(k) - u(k-1)|^2 \le J(e(0), u(0)) < \infty \quad (21)$$

which implies that $\lim\limits_{k \to \infty} \Delta u(k) = 0$.

Assume that there exists an optimal control input u^* such that the system output achieves the desired value exactly.

$$\Gamma = [-e(k+1), u^* - \Delta u(k)]^T$$

Then

$$\begin{aligned} \nabla J(e(k+1), u(k))\big|_{(e(k+1), \Delta u(k))} \cdot \Gamma &= [e(k+1), \Delta u(k)] \cdot \begin{bmatrix} -e(k+1) \\ u^* - \Delta u(k) \end{bmatrix} \\ &= -e^2(k+1) + \Delta u(k) u^* - \Delta u^2(k) \end{aligned} \quad (22)$$

which implies that

$$0 \le e^2(k+1) + \Delta u^2(k) \le \Delta u(k) u^*$$

Hence, $\lim\limits_{k \to \infty} e(k+1) = 0$ since $\lim\limits_{k \to \infty} \Delta u(k) = 0$.

5 Simulations

Consider a nonlinear discrete-time system as follows [3],

$$y(k+1)=\begin{cases}\dfrac{y(k)}{1+y^2(k)}+u^3(k) & k\le 500\\ \dfrac{y(k)y(k-1)y(k-2)u(k-1)(y(k-2)-1)+a(k)u(k)}{1+y^2(k-1)+y^2(k-2)} & k>500\end{cases}$$

where $a(k)=1+round(k/500)$ is a time-varying parameter.

This nonlinear system consists of two nonlinear subsystems in series. It is clear that this controlled nonlinear system is with time-varying structure, time-varying parameter and time-varying orders.

The desired output is given by

$$y^*(k+1)=\begin{cases}0.5\times(-1)^{round(k/300)}, k\le 300\\ 0.5\sin(k\pi/100)+0.3\cos(k\pi/50), 300<k\le 700\\ 0.5\times(-1)^{round(k/300)}, k>700\end{cases}$$

The system input and output constraints are given as $-0.8\le u(k)\le 0.8$, $-0.5\le \Delta u(k)\le 0.5$, and $-1\le y(k)\le 1$.

In the simulation, the controller parameters are set as $\eta=1$, $\mu=0.01$, and $\lambda=0.3$. The system initial states are selected as $y(0)=0$, $u(0)=0$, $\Delta u(0)=0$. Applying the proposed constrained MFAC scheme (15) – (17), the simulation results are shown as the blue lines in figures 1 – 3.

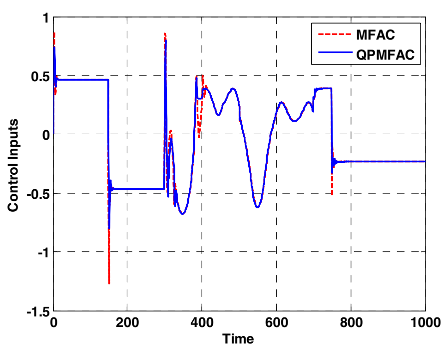

Fig.1 The profile of control inputs

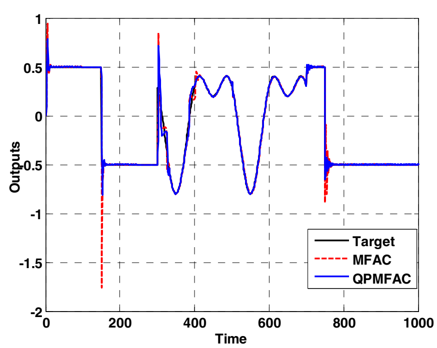

Fig.2 The profile of system outputs

Fig. 1 is the profile of control input signals. It is seen that the control input signals (blue line) obtained by applying the proposed constrained MFAC approach varies within the limits for all time instants. Fig. 2 is the profile of system output. It is obvious that the output profile (blue line) is also varying within its limits. Fig. 3 is the tracking error profile, from which the convergence of the proposed constraint MFAC scheme is confirmed.

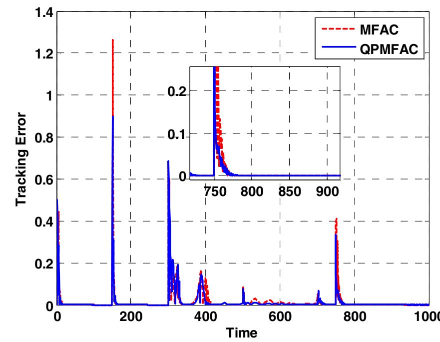

Fig.3 The profile of tracking errors

For the purposed of comparison, the traditional CFDL based model-free adaptive control scheme [3] is also applied to the above system, where only the QP-based control law (17) is replaced by

$$u(k)=u(k-1)+\frac{\rho\hat{\phi}(k)}{\lambda+\left|\hat{\phi}(k)\right|^2}(y_d(k+1)-y(k))$$

For comparison, the simulation is conducted under the same control background, that is, selecting the same controller parameters and the same initial values, the simulation results are shown as the red lines in figures 1 – 3.

From Fig. 1, one can see that the control input given by the traditional MFAC is beyond its lower and upper bounds. Similarly, the system output when applying traditional MFAC also exceeds its lower and upper bounds. In addition, from Fig. 3, we can conclude that the proposed constrained MFAC converges faster than the traditional one.

6 Conclusions

A novel QP-MFAC approach is proposed for nonlinear discrete-time systems with both input and output constraints, where the upper and lower bounds of control input and system output, and the change rate of control input are considered as the system constraints. The control law is gained by minimizing a quadratic objective function under a linear matrix inequality via quadratic programming. The proposed approach is data-driven without requiring an exact model of the controlled process.

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