Closed-Loop Subspace Identification with Prior Information

Yali Zhao, Jinxu Cheng, Youqing Wang*

College of Information Science and Technology, Beijing University of Chemical Technology, Beijing, 100029 China

*E-mail: wang.youqing@ieee.org

Abstract: Adding prior information to open-loop subspace identification has been proven to be effective in obtaining state-space models with improved accuracy. The purpose of this study is to improve the accuracy of close-loop subspace identification by using prior information. The proposed method uses a parsimonious model formulation to remove the correlation between future input and past innovation, which is the significant challenge in closed-loop subspace identification. In addition, every row is considered as an optimal multi-step ahead predictor and the constrained least square (CLS) approach is used to incorporate prior information. The mathematical analysis and simulation results verify the feasibility and effectiveness of the proposed method.

Key Words: Closed-loop Identification, Subspace Identification, Parsimonious Models, Constrained Least Square (CLS), Prior Information

1 INTRODUCTION

Subspace identification has drawn much attention in the past three decades. Canonical variate analysis (CVA) [1, 2], N4SID [3], and MOESP [4] are typical schemes for subspace identification, which mainly focus on open-loop identification. However, in the real application, closed-loop identification is required due to system safety, control quality, physical constraints, and so on. In recent fifteen years, closed-loop subspace identification has attracted much interests from academia and industry. A closed-loop subspace identification approach based on principal component (SIMPCA) was proposed by Wang and Qin [5]. Innovation estimation method (IEM) [6] and whitening filter approach (WFA) [7] were proposed to deal with the closed-loop date. An orthogonal projection approach was proposed to handle closed-loop subspace when there is white noise [8].

Due to numerical simplicity and stability, the subspace identification method (SIM) has been significantly developed in the recent decades. Furthermore, the state space description is very convenient for estimation, filtering, prediction and control. However, for SIM, the date quality significantly influences the identification performance. Usually, measurement noises will pollute the important information and insufficient input excitation will lead to information-absent; hence, these practical factors make the system identification more difficult. On the other hand, prior information could be adopted to improve the identification performance. Evidently, it is convenient to exploit the prior information on time constant and input-output no direct relation [9].

The existing studies have endeavored a number of ways to corporate prior information into subspace identification. Lyzell et al. proposed a method to handle certain structure information in subspace identification [10]. Trnka and Havlena applied prior information such as DC gains and transient response smoothness to subspace identification based on Bayesian approach [11]. Alenany et al. proposed scheme incorporates the prior information using the constrained least square (CLS) algorithm that includes an equality constraint [12]. However, all these above-mentioned studies are just for open-loop system.

In this study, a new method was proposed to include the prior information to closed-loop subspace identification. The rest of this paper is organized as follows. Section 2 introduces the notation and overview of subspace identification with parsimonious subspace models. Section 3 presents the equality constraints are used to describe the prior information and apply it to every row of subspace identification. Section 4 shows the prior information translating into equality constraints. In Section 5, two simulation examples are shown to illustrate the applicability of the proposed approaches. Section 6 concludes the paper.

2 SUBSPACE IDENTIFICATION

2.1 Conventional Subspace Models

In this paper, an innovation model formulation is used to describe the system as follows:

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{1}$$

$$y_k = Cx_k + Du_k + e_k \tag{2}$$

where $u_k \in \Re^m$, $x_k \in \Re^n$, $y \in \Re^l$, $e_k \in \Re^l$ are the system input, state, output and innovation, respectively. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{l\times n}$, $D \in \Re^{l\times m}$, $K \in \Re^{n\times l}$ are system matrices with appropriate dimensions.

Based on the innovation form, an extended state space model can be formulated as follows:

This work is supported by the National Natural Science Foundation of China under Grant 61374099, the Program for New Century Excellent Talents in University under Grant NCET-13-0652, and Beijing Higher Education Young Elite Teacher Project under Grant YETP0505.

$$Y_f = \Gamma_f X_f + H_f U_f + G_f E_f$$
$$Y_p = \Gamma_p X_p + H_p U_p + G_p E_p \quad (3)$$

The input block-Hankel matrices are formulated as

$$U_p \triangleq \begin{bmatrix} u_{k-p} & u_{k-p+1} & \cdots & u_{k-p+j-1} \\ u_{k-p+1} & u_{k-p+2} & \cdots & u_{k+j} \\ \vdots & \vdots & \cdots & \vdots \\ u_{k-1} & u_k & \cdots & u_{k+j-2} \end{bmatrix}$$
$$U_f \triangleq \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+j-1} \\ u_{k+1} & u_{k+2} & \cdots & u_{k+j} \\ \vdots & \vdots & \cdots & \vdots \\ u_{k+f-1} & u_{k+f} & \cdots & u_{k+f-2} \end{bmatrix} \quad (4)$$

Matrices Y_p, Y_f, E_p, E_f can be defined in a similar way. Index f denotes the future and p denotes the past. Define Eq. (5) to respect the past output and input date as

$$Z_p \triangleq \begin{bmatrix} Y_p \\ U_p \end{bmatrix} \in \Re^{(lp+mp)\times j} \quad (5)$$

The state sequences are defined as:

$$X_p \triangleq \begin{bmatrix} x_{k-p} & x_{k-p+1} & \cdots & x_{k-1} \end{bmatrix};$$
$$X_f \triangleq \begin{bmatrix} x_k & x_{k+1} & \cdots & x_{k+f-1} \end{bmatrix}. \quad (6)$$

The extend observability matrix and the lower block triangular Toeplitz matrices are given as

$$\Gamma_f \triangleq \begin{bmatrix} C^{\mathrm{T}} & (CA)^{\mathrm{T}} & \cdots & \left(CA^{f-1}\right)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \Re^{lf\times n}$$
$$H_f \triangleq \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \cdots & D \end{bmatrix} \in \Re^{lf\times mf} \quad (7)$$
$$G_f \triangleq \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}BK & CA^{f-3}K & \cdots & I \end{bmatrix} \in \Re^{lf\times nf}$$

2.2 Parsimonious Subspace Models

In the algorithm, U_f need to be excluded those non-causal terms to guarantee that output are independent of future input in every row, so that H_f and G_f are lower block triangular Toeplitz matrices. On the other hand, in order to remove the correlation of future input and past innovation, one can partition the extended state space model row-wise as follows:

$$Y_f \triangleq \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{ff} \end{bmatrix}; Y_i \triangleq \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{fi} \end{bmatrix}; i = 1,2,3\ldots,f \quad (8)$$

Subscript fi respects the *i-th* row block matrix.

Partition U_f and E_f in the same way with Y_f to define U_{fi}, U_i, E_{fi}, E_i, for $i = 1,2\ldots f$. Further denote

$$H_{fi} \triangleq \begin{bmatrix} CA^{i-2}B & \cdots & CB & D \end{bmatrix} = \begin{bmatrix} g_{i-1} & \cdots & g_1 & g_0 \end{bmatrix}$$
$$G_{fi} \triangleq \begin{bmatrix} CA^{i-2}K & \cdots & CK & I \end{bmatrix} = \begin{bmatrix} G_{i-1} & \cdots & G_1 & G_0 \end{bmatrix} \quad (9)$$

where g_i and G_i are the Markov parameters for the deterministic input and innovation sequence, respectively. Furthermore, it should be noticed that g_i corresponds to impulse response coefficients.

The following partitioned equations can be obtained

$$Y_{fi} = \Gamma_{fi} X_f + H_{fi} U_i + G_{fi} E_i, \forall 1,2,\ldots,f \quad (10)$$

$e(k)$ in the innovation model can be eliminated by substituting Eq. (2) to Eq. (1), then the following relation can be obtained

$$x_{k+1} = Ax_k + Bu_k + K(y_k - Cx_k + Du_k) \quad (11)$$

Iterating Eq. (11) and substituting Y_p and U_p with Z_p, the Eq. (12) can be obtained [13]

$$X_f = L_z Z_p + A_K^p X_p \quad (12)$$

where

$$L_z \triangleq \begin{bmatrix} \Delta_p(A_K, K) & \Delta_P(A_K, B_K) \end{bmatrix}$$
$$\Delta_P(A_K, B_K) \triangleq \begin{bmatrix} A^{p-1}B & \cdots & AB & B \end{bmatrix}$$
$$A_K \triangleq A - KC$$
$$B_K \triangleq B - KD$$

Substituting Eq. (12) into Eq. (10), one can obtain

$$Y_{fi} = \Gamma_{fi} L_z Z_p + \Gamma_{fi} A_K^p X_p + H_{fi} U_i + G_{fi} E_i, \forall 1,2,\ldots,f \quad (13)$$

The second term in the RHS of Eq. (13) tends to zero when parameter p tends to infinity, as a result, it can express as the following estimates

$$Y_{fi} = \Gamma_{fi} L_z Z_p + H_{fi} U_i + G_{fi} E_i, \forall 1,2,\ldots,f \quad (14)$$

In the closed-loop system, it should be noted that the future input is not independent of the past innovation and the solution for this problem will be shown in section 3.

3 SUBSPACE IDENTIFICATION WITH CLS

3.1. Parsimonious with Innovation Estimation

Setting $i = 1$, Eq. (14) can be expressed as

$$Y_{f1} = \begin{bmatrix} \Gamma_{f1} L_z & H_{f1} \end{bmatrix} \begin{bmatrix} Z_p \\ U_1 \end{bmatrix} + E_1 \quad (15)$$

Therefore, a least squares estimate of the innovation process is

$$\hat{E}_1 = Y_{f1} - \begin{bmatrix} \hat{L}_{w1} & \hat{H}_{f1} \end{bmatrix} \begin{bmatrix} Z_p \\ U_1 \end{bmatrix} \quad (16)$$

For a general $i = 1,2\ldots f$. Noticing that

$$E_i = \begin{bmatrix} E_{f1} \\ E_{f2} \\ \vdots \\ E_{fi} \end{bmatrix} = \begin{bmatrix} E_{i-1} \\ E_{fi} \end{bmatrix} \tag{17}$$

and replacing E_{i-1} with $\hat{E}_{i-1}$, hence Eq. (14) can be changed as

$$Y_{fi} = \begin{bmatrix} \Gamma_{fi} L_z & H_{fi} & G_{fi}^{-} \end{bmatrix} \begin{bmatrix} Z_p \\ U_i \\ \hat{E}_{i-1} \end{bmatrix} + E_{fi} \tag{18}$$

where $G_{fi}^{-} \triangleq \begin{bmatrix} CA^{i-2}K & CA^{i-3}K & \cdots & CK \end{bmatrix}$.

Subspace identification can be replaced by solving the following optimal multi-step ahead prediction for Y_{fi} [14].

$$\hat{L}_{wi}, \hat{H}_{fi}, \hat{G}_{fi}^{-} = \underset{L_{wi},\ H_{fi},\ G_{fi}^{-}}{\arg\min} \left\| Y_{fi} - \begin{bmatrix} L_{wi} & H_{fi} & G_{fi}^{-} \end{bmatrix} \begin{bmatrix} Z_p \\ U_i \\ \hat{E}_{i-1} \end{bmatrix} \right\| \tag{19}$$

$$\hat{Y}_{fi} = \begin{bmatrix} \hat{L}_{wi} & \hat{H}_{fi} & \hat{G}_{fi}^{-} \end{bmatrix} \begin{bmatrix} Z_p \\ U_i \\ \hat{E}_{i-1} \end{bmatrix} \tag{20}$$

There is no assumption that future input to be uncorrelated with past innovation in Eq. (19), because past innovation has already been estimated. It only need that future innovation to be independent of past input, which is satisfied in every row. The innovation date can be calculated recursively with the following equation

$$\hat{E}_i = \begin{bmatrix} \hat{E}_{i-1} \\ \hat{E}_{fi} \end{bmatrix} \tag{21}$$

when matrices $\hat{L}_{wi}, \hat{H}_{fi}, \hat{G}_{fi}^{-}$ are known, the system matrices A, B, C, D can be estimated.

3.2. CLS Approach

According to Eq. (20), one can get the optimal multi-step ahead prediction for Y_{fi} .

In order to use prior information, a long vector and Kronecker product will be used in the sequel.

$$vec(\mathrm{ABC}) = (\mathrm{C}^{\mathrm{T}} \otimes A) vec(\mathrm{B}) \tag{22}$$

where vec stands for the operation of forming a long vector from a matrix by stacking its columns one under another and the operator $\otimes$ indicates the Kronecker product.

Using Eq. (22), Eq. (20) can be rewritten as follows:

$$vec(\hat{Y}_{fi}) = \left(\begin{bmatrix} Z_p^{\mathrm{T}} & U_i^{\mathrm{T}} & \hat{E}_{i-1}^{\mathrm{T}} \end{bmatrix} \otimes I_{l\times l} \right) vec\left(\begin{bmatrix} \hat{L}_{wi} & \hat{H}_{fi} & \hat{G}_{fi}^{-} \end{bmatrix} \right) \tag{23}$$

In order to explain the process clearly, the long vector is shown as follows:

$$vec\left(\begin{bmatrix} \hat{L}_{wi} & \hat{H}_{fi} & \hat{G}_{fi}^{-} \end{bmatrix} \right) = \begin{bmatrix} \theta_l \\ \theta_{gi} \\ \theta_{ki} \end{bmatrix} \tag{24}$$

where $\theta_l = vec\left(\hat{L}_{w1}\right)$, $\theta_{gi} = vec\left(\begin{bmatrix} g_{i-1} & \dots & g_0 \end{bmatrix}\right)$, $\theta_{ki} = vec\left(\hat{G}_{fi}^{-}\right)$.

Substituting Eq. (24) to Eq. (23) yields

$$\underbrace{vec(\hat{Y}_{fi})}_{\hat{y}_i} = \underbrace{\left(\begin{bmatrix} Z_p^{\mathrm{T}} & U_i^{\mathrm{T}} & \hat{E}_{i-1}^{\mathrm{T}} \end{bmatrix} \otimes I_{l\times l} \right)}_{z_i} \underbrace{\begin{bmatrix} \theta_l \\ \theta_{gi} \\ \theta_{ki} \end{bmatrix}}_{\theta_i} \tag{25}$$

Eq. (25) is used to incorporate with prior information in subspace identification. The advantage is that Eq. (25) gives estimates of the impulse response coefficients $\theta_{gi} = vec\left(\begin{bmatrix} g_{i-1} & \dots & g_0 \end{bmatrix}\right) \in \Re^{lmi}$, which are related to the physical characteristics of the system. In this section, it is proposed to solve Eq. (25) in a least squares sense with added equality constraints representing the prior knowledge. This leads to the new problem

$$\min_{\theta} \left\| \hat{y}_i - Z_i \theta_i \right\|_2^2 \tag{26}$$

Subject to the following equality constraints

$$A_{eq} \theta_i = b_{eq} \tag{27}$$

Transform the prior information to a set of equality constraints $\{A_{eq}, b_{eq}\}$, and choose specific part $\{A_{eq}, b_{eq}\}$ as i changing.

Solving Eq. (26) and Eq. (27) to get $\hat{\theta}_i$, the Markov parameters can be reconstructed with $\hat{\theta}_i$ as follows:

$$\begin{bmatrix} \tilde{L}_{wi} & \tilde{H}_{fi} & \tilde{G}_{fi}^{-} \end{bmatrix} = \begin{bmatrix} \hat{\theta}_i(1:l,:) & \hat{\theta}_i(l+1:2l,:) & \cdots & \hat{\theta}_i(end-l+1:\mathrm{end},:) \end{bmatrix} \tag{28}$$

Thus, it is easy to get $\begin{bmatrix} \tilde{L}_{wi} & \tilde{H}_{fi} & \tilde{G}_{fi}^{-} \end{bmatrix}$ which are the estimation of $\begin{bmatrix} L_{wi} & H_{fi} & G_{fi}^{-} \end{bmatrix}$, then

$$\tilde{E}_{fi} = Y_{fi} - \begin{bmatrix} \tilde{L}_{wi} & \tilde{H}_{fi} & \tilde{G}_{fi}^{-} \end{bmatrix} \begin{bmatrix} Z_p \\ U_i \\ \tilde{E}_{i-1} \end{bmatrix} \tag{29}$$

$$\tilde{E}_i = \begin{bmatrix} \tilde{E}_{i-1} \\ \tilde{E}_{fi} \end{bmatrix} \tag{30}$$

when matrices $\begin{bmatrix} \tilde{L}_{wi} & \tilde{H}_{fi} & \tilde{G}_{fi}^{-} \end{bmatrix}, i = 1, 2 \dots f$ are known, the system matrices A, B, C, D can be estimated.

Please notice that parameters with "$\sim$" represent the estimated parameters with prior information and parameters with "^" represent the estimation without prior information.

In summary, in order to ensure the prior information were included, CLS approach is adopted to substitute the simple least squares estimate. Turning into algorithm improvement is that Eq. (23) to Eq. (30) replace Eq. (20) to Eq. (21), recount the estimation of L_w, H_f, G_f and E, so that the more accurate system matrices which contain the prior information can be gotten.

3.3. Summary of Closed-Loop Subspace Identification with Prior Information

- Form the matrices U_f, Y_f according to their definitions in Section 2, and partition the Hankel matrix by row to calculate $\hat{y}_i, z_i$ in Eq. (25).
- Transform prior information to a set of equality constraints $\{A_{eq}, b_{eq}\}$ and choose specific part of them as i changing.
- Solving the least squares optimization problem Eq. (26) with constraint Eq. (27), and compute an estimated $\tilde{\theta}_i$ for the impulse response parameters that is $\tilde{\theta}_{gi}$ in $\tilde{\theta}_i$.
- Structure the following Hankel matrix T from $\{\tilde{g}_0 \ \ \cdots \ \ \tilde{g}_{f-1}\}$.

$$T = \begin{pmatrix} \tilde{g}_1 & \tilde{g}_2 & \cdots & \tilde{g}_{f/2} \\ \tilde{g}_2 & \tilde{g}_3 & \cdots & \tilde{g}_{f/2+1} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{g}_{f/2} & \tilde{g}_{f/2+1} & \cdots & \tilde{g}_{f-1} \end{pmatrix} \quad (31)$$

- Apply Kung's realization algorithm [15], factorize T by using SVD as

$$T = USV^{\mathrm{T}} \quad (32)$$

- Determine the order n of the system by using the significant singular values of S. Then obtain estimates of the observability Γ and controllability Δ matrices as following

$$\Gamma = U(:,1:n)\mathrm{S}^{1/2}, \Delta = S^{1/2}V(:,1:n)^{\mathrm{T}} \quad (33)$$

- Finally, one can compute the system matrices as $A = \underline{\Gamma}^{\dagger}\overline{\Gamma}$, $\mathrm{B} = \Delta(:,1:m)$, $\mathrm{C} = \Gamma(1:l,:)$, $D = \tilde{g}_0$ (Denting $\underline{\Gamma} = \Gamma(1:l(f/2-1),:)$ and $\overline{\Gamma} = \Gamma(\mathrm{l}+1:l(f/2),:)$).

4 PRIOR INFORMATION

4.1 Known Time Constant

According to paper [12], for first-order SISO systems, there is the following relationship between the time constant and the impulse response coefficients:

$$g_i = \delta g_{i-1}, i \geq 2 \quad (34)$$

where $\delta = e^{-T/\tau}$, and T is the sampling period and τ is the time constant. Hence, A_{eq}, b_{eq} corresponding to a known time constant can be given as

$$A_{eq} = \begin{bmatrix} & 0 & \cdots & 0 & -1 & \delta & 0 & \\ & 0 & \cdots & -1 & \delta & 0 & 0 & \\ 0_1 & & \iddots & \iddots & \iddots & \iddots & & 0_2 \\ & -1 & \delta & \cdots & 0 & 0 & 0 & \end{bmatrix} \quad (35)$$

$$b_{eq} = 0_{(f-2)\times 1} \quad (36)$$

where $0_1 = 0_{(f-2)\times l(lf+mf)}$, $0_2 = 0_{(f-2)\times ll(i-1)}$, when applied to each row, Eq. (27) should be adjusted to

$$A_{eqi}\theta_i = \mathrm{b}_{eqi}, i > 2 \quad (37)$$

where

$$A_{eqi} = A_{eq}(1:(i-2),:)$$

$$\mathrm{b}_{eqi} = \mathrm{b}_{eq}(\mathrm{i}:(\mathrm{i}-2),:)$$

when $i \leq 2$, there is no constraint equation.

4.2 Known Input-Output No direct Relation

In some systems, inputs do not affect outputs directly and hence the input-output direct coefficient is zero. However, the zero coefficient in specific situation cannot be obtained by identification from noisy date. There is a nature idea to pre-specify the coefficient in identification and that can be very important prior information.

If there is a zero coefficient of the association between the input and output, the impulse response coefficients without time delay at this channel is zero, that is

$$g_0 = 0 \quad (38)$$

While there is no need to impose restrictions on other impulse response coefficients. Taking *vec* operation for both sides of Eq.(38) and all the impulse response coefficients yield the relation as

$$\begin{bmatrix} & I_{lm\times lm} \\ 0_{flm\times lm(f-1)} & I_{lm\times lm} \\ & \vdots \\ & I_{lm\times lm} \end{bmatrix} \times \underbrace{\begin{bmatrix} vec(g_{f-1}) \\ vec(g_{f-2}) \\ \vdots \\ vec(g_{k+1}) \\ vec(g_k) \\ vec(g_{k-1}) \\ \vdots \\ vec(g_0) \end{bmatrix}}_{\theta_{gf}} = 0_{flm\times 1} \quad (39)$$

In term of θ, Eq. (39) can be written as

$$\underbrace{\begin{bmatrix} 0_{flm\times(lf(l+m)+lm(f-1))} & 1_{flm\times 1} & 0_{flm\times ll(i-1)} \end{bmatrix}}_{A_{eq}}\theta = \underbrace{0_{flm\times 1}}_{b_{eq}} \quad (40)$$

If zero coefficient of input and output are known, it is possible to incorporate them with corresponding constraints. One should adjust the row-wise calculation with constraints as follows

$$Aeq(1:i,:)\theta_i = beq(1:\mathrm{i},:) \quad (41)$$

5 SIMULATIONS

Parsimonious with innovation estimation is proposed by Lin et al. [6] to deal with closed-loop subspace identification, which is generally abbreviated as PARSIM. The current study is mainly based on PARSIM, so it can be termed as PARSIM-Prior. In the simulation section, the two methods are compared to validate that the proposed algorithm have more advantages.

5.1 Example 1: Known Time Constant

In this paper, simulate the process proposed in [6] as follows

$$y_k + ay_{k-1} = bu_{k-1} + e_k + ce_{k-1} \quad (42)$$

with a feedback controller

$$u_k = -Ky_k + r_k \qquad (43)$$

where $a = -0.9$, $b = 1$, $c = 0.9$ and $K = 0.6$, the standard deviation for e_k is 0.25 and that for r_k is two; both of the signals are Gaussian white noise. Twenty Monte-Carlo simulations are performed.

In this case that time constant is known, the closed-loop subspace identification Figure 1 shows the pole estimates from PARSIM and PARSIM-Prior for closed-loop date. The PARSIM-Prior gives the more accurate mean value and small variance.

From Table 1, it can be seen that the mean value of the estimated a and b from PARSIM-Prior are better than the PARSIM method. Furthermore, the variance of every parameter form PARSIM-Prior shows good result, that is to say PARSIM-prior have the more stable performance.

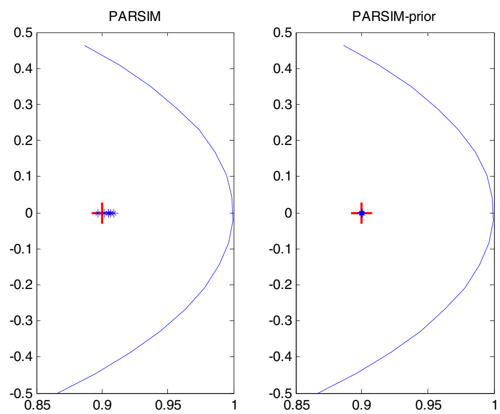

Fig 1. Pole estimates for the simulation example 1: known time constant. The left is pole estimates of PARSIM; the red cross are the true value and the blue stars are the estimated values from 20 runs. The right is pole estimates of PARSIM-Prior.

Table 1. Mean and variance of parameter estimates for example 1 (the variance in the bracket)

	Rule value	PARSIM	PARSIM-Prior
a	-0.9	-0.9549(0.0629)	-0.8996(2.7552e-005)
b	1	0.901(0.6398)	0.9876(0.04)

5.2 Example 2: Known D=0

In this case, it is assumed that the output is not affected by the input directly, i.e., D=**0**. The process and the feedback controller are the same with example 1.

$$y_k + ay_{k-1} = bu_{k-1} + e_k + ce_{k-1} \qquad (44)$$

$$u_k = -Ky_k + r_k \qquad (45)$$

Figure 2 shows the pole estimates from PARSIM and PARSIM-Prior for closed-loop date, there are no significant difference in the pole estimates and both estimated value have the satisfied performance. Table 2 show the mean value of the estimated a and b for PARSIM-Prior and the PARSIM method, but there another parameter in PARSIM method because of zero estimates biased. The process will change into as follow

$$y_k + ay_{k-1} = b_1u_k + bu_{k-1} + e_k + ce_{k-1} \qquad (46)$$

Both the two methods have good performance. In terms of the estimation accuracy of parameters a and b, PARSIM shows some weak superiority; however, it introduces prominent bias for parameter b_1 in frequency domain.

The estimates of frequency response for the closed-loop simulations are shown in Fig 3. We can see that the estimated frequency responses for PARSIM-Prior gives the better estimation. PARSIM method have bad phase estimates and bad magnitude estimation at high frequency, which may bring non-ignorable error in real application. PARSIM has an angle bias of 360 degrees in the phase estimation, due to existence of parameter b_1. The bias presents in every run of twenty Monte-Carlo simulations, and only one simulation result was shown the phenomenon as an example.

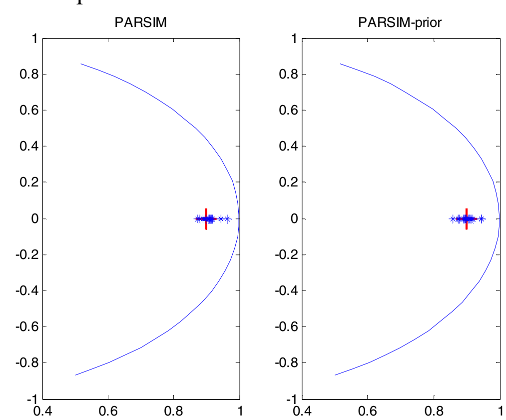

Fig 2. Pole estimates for the simulation example 2: known input-output zero coefficient. The left is pole estimates of PARSIM; the red cross denotes the true value and the blue stars denotes the estimated values from 20 runs. The right is pole estimates of PARSIM-Prior.

Table 2. Mean and variance of parameter estimates for example 2 (the variance in the bracket)

	Rule value	PARSIM	PARSIM-Prior
a	-0.9	-0.9030(0.0038)	-0.9070(0.0076)
b_1	0	-0.0029(2.5344e-004)	0
b	1	0.9641(0.0881)	0.9652(0.0938)

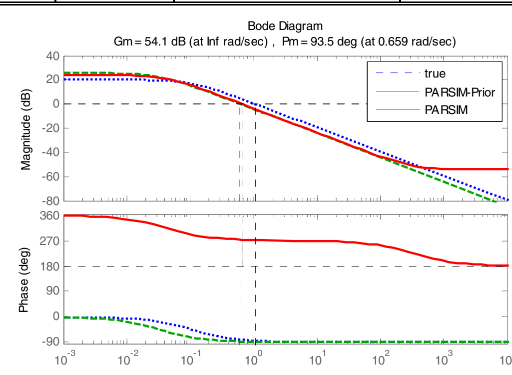

Fig 3. The Bode plot for the simulation example 2: known D=0

6 CONCLUSIONS

The accuracy of subspace identification is highly depend on the input and output date, which are easily to be influenced by the measurement noises and insufficient input excitation. In order to improve the accuracy of the identified model, the prior information, such as time constant and input-output no direct relation, has been used in the proposed SIM method. Particularly, the proposed method can be used in the closed-loop control system. The simulation results demonstrate that the proposed method has improved estimation accuracy and can generate unbiased frequency response with satisfactory variances. Furthermore, various prior information applied to closed-loop subspace identification deserve future investigation.

REFERENCES

[1] W. E. Larimore, "System identification, reduced-order filtering and modeling via canonical variate analysis," in American Control Conference, pp. 445-451, 1983.

[2] W. E. Larimore, "Canonical variate analysis in identification, filtering, and adaptive control," in Proceeding of the 29th IEEE Conference on Decision and Control, pp. 596-604, 1990.

[3] P. V. Overschee and B. D. Moor, "N4SID: subspace algorithms for the identification of combined deterministic stochastic systems," Automatica, vol. 30, pp. 75–93, 1994.

[4] M. Verhaegen and P. Dewilde, "Subspace model identification part 1. The output-error state-space model identification class of algorithms," International Journal of Control, vol. 56, pp. 1187-1210, 1992.

[5] J Wang and S. J. Qin, "A new subspace identification approach based on principal component analysis," Journal of Process Control, vol. 12, pp. 841–855, 2002.

[6] W. Lin, S. J. Qin, and L. Ljung, "On consistency of closed-loop subspace identification with innovation estimation," in Proceedings of the 43rd IEEE Conference on Decision and Control, vol.30, pp. 2195-2200, 2004.

[7] A. Chiuso and G. Picci, "Consistency analysis of some closed-loop subspace identification methods," Automatica, vol. 41, pp. 377–391, 2005.

[8] T. Katayama and H. Tanaka, "An approach to closed-loop subspace identification by orthogonal decomposition," Automatica, vol. 43, pp. 1623-1630, 2007.

[9] A. Alenany, H. Shang, M. Soliman, and I. Ziedan, "Subspace identification with prior steady-state information," in 2010 International Conference on Computer Engineering and Systems (ICCES), pp. 103 – 108,2010.

[10] C. Lyzell, M. Enqvist, and L. Ljung, "Handling certain structure information in subspace identification," in Proceedings of IFAC Symposium on System Identification, vol. 168, pp. 353 - 358, 2009.

[11] P. Trnka and V. Havlena, "Subspace like identification incorporating prior information," Automatica, vol. 45, pp. 1086–1091, 2009.

[12] A. Alenany, H. Shang, M. Soliman, and I. Ziedan, "Improved subspace identification with prior information using constrained least squares," IE Control Theory & Applications, vol. 5, pp. 1568-1576, 2011.

[13] T. Knudsen, "Consistency analysis of subspace identification methods based on a linear regression approach," Automatica, vol. 37, pp. 81–89, 2001.

[14] P. V. Overschee and B. D. Moor, "Subspace identification for linear systems, theory, implementation, applications," Springer Us, 1996.

[15] S. Y. Kung, K. S. Arun, D. V. B. Rao, S. Y. Kung, K. S. Arun, and D. V. B. Rao, "A new identification and model reduction algorithm via singular value decomposition," in Proceedings of the 12th Asilomar Conference on Circuits Systems Computers, pp705-714, 1978.