Point-to-Point Tracking of Integrated Predictive Iterative Learning Control By Using Updating-Reference and CARIMA Model

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Abstract: For point-to-point tracking control problem of batch process, a novel design method based on Controlled Auto-regressive Integrated Moving Average (CARIMA) model and updating-reference is proposed in this paper. In the proposed approach, integrated predictive iterative learning control (IPILC) is used for the trajectory tracking control. Comparing with other point-to-point tracking control algorithms, the proposed control scheme performs better in robustness, and reduces the computation load which occurs in those algorithms based on the lifted model for non-Lyapunov-stable systems. Furthermore, updating-reference relaxes the constraints for system outputs and leads to faster convergence than the fixed-reference control algorithms. Simulation results on typical systems show the effectiveness of the proposed algorithm.

Key Words: Iterative Learning Control, Point-to-point Tracking, Updating Reference, Controlled Auto-Regressive Integrated Moving Average Model

1 INTRODUCTION

Iterative learning control (ILC) is an effective technique for controlling systems that execute the same task repeatedly. Because this control scheme can take advantage of the information of previous executions (batches, iterations, trials, or cycles), the control performance is improved from batch to batch. Since introduced by Arimoto [1] in 1984, ILC has been widely used in many practical industrial processes [2], including robotics, manufacturing, and batch reaction etc.

It is well known that the goal of the standard ILC is to track an entire static reference trajectory [3]. However, in many practical applications, the system outputs at certain time instants are critical rather than other instants in the whole motion profile. Examples include production line automation, robotic arm, industrial manipulator, and so on [4]. Thus, point-to-point ILC has been introduced as an effective technique for such tasks and leads to increasingly growing interest [5,6].

Initially, a solution of point-to-point ILC is to generate a suitable motion profile which passes through the prescribed points in advance and then to design a standard ILC controller to track it [5]. The typical approach to generate such a predetermined reference is Input Shaping [6]. However, this strategy may fail to exploit extra freedom gained by removing the unnecessary constraints.

In recent years, Freeman etc. [7, 8, 9] put forward a novel design framework, which combines point-to-point tracking with a general class of additional objectives, and makes use of the available extra freedom to improve control performance. The method is based on a lifted model and suitable for Lyapunov-stable systems. It can perform perfect for time invariant system in a noise-free scenario and obtain fast convergence and high precision. But there are still some difficulties for computation load of the strategy due to the lifted model, and its robustness for practical systems involving model perturbations and disturbances should be studied further.

Considering those problems above, more applicable and effective solutions for practical industrial processes should be studied. Shi etc. [10] proposed a design method for indirect iterative learning control based on CARIMA model and generalized predictive control (GPC). The method realized an integrated optimization procedure for the parameters of the existing feedback controller and designed a simple iterative learning controller for a static reference tracking.

Based on the CARIMA model as used in [10], this paper proposes a point-to-point tracking control of the integrated predictive iterative learning control (IPILC) algorithm by using updating-reference trajectory. Comparing with those methods based on the lifted model, the proposed algorithm based on CARIMA model can be applied to some non-Lyapunov-stable systems. Updating reference can equivalently relax the constraints for outputs and lead to faster convergence. In the proposed strategy, model predictive control is integrated in the IPILC method as a feedback part to enhance the robustness for point-to-point tracking control in a noisy environment.

The paper is organized as follows: in section 2, the problem description is presented and some basic concepts are introduced; in section 3, the design algorithm of IPILC based on CARIMA model and updating-reference is described in detail; in section 4, the proposed method is

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applied to numerical simulations to demonstrate the effectiveness of the proposed algorithm; finally, conclusions are given.

2 PROBLEM FORMULATION

For simplicity in this paper, a single-input and single-output (SISO) process, which operates repetitively over a finite duration, is depicted by the Controlled Auto-Regressive Integrated Moving Average (CARIMA) model as follows: [10]

$$A(q_t^{-1})y(t,k)=B(q_t^{-1})u(t,k)+w(t,k) \quad t=0,1,2,\cdots,T \quad k=1,2,\cdots \tag{1}$$

where t and k represent the discrete-time and iteration indices, q_t^{-1} indicates the unit backward shifting operator along the time index t, T is a constant time duration of each iteration, $y(t,k)$, $u(t,k)$, and$w(t,k)$ denote output, input and disturbance at t in the kth iteration, respectively. $A(q_t^{-1})$, $B(q_t^{-1})$ are both operator polynomials, which are formulated by

$$A(q_t^{-1})=1+a_1q_t^{-1}+a_2q_t^{-2}+\cdots+a_nq_t^{-n} \tag{2}$$

$$B(q_t^{-1})=b_1q_t^{-1}+b_2q_t^{-2}+\cdots+b_mq_t^{-m} \tag{3}$$

where n and m are the orders of input and output in the process model, and $\{a_i;\, i=1,2,\cdots,n\}$, $\{b_i; i=1,2,\cdots,m\}$ are the parameters of $A(q_t^{-1})$ and $B(q_t^{-1})$.

In the point-to-point tracking control problem, the reference sequence is usually described by

$$Y_r=[y_r(t_1),y_r(t_2),\cdots,y_r(t_s)]^T\in R^s \tag{4}$$

where t_i indicates a certain prescribed time instants.

The task of the point-to-point ILC algorithm is to obtain the input sequence $\boldsymbol{U}(k)=[u(1,k),u(2,k),\cdots,\ u(T,k)]$ in the kth iteration such that

$$\lim_{k\to\infty}\|e(t_i,k)\|=0 \tag{5}$$

where $e(t_i,k)=\ y_r(t_i)-y(t_i,k)$, $y(t_i,k)$ and $e(t_i,k)$ are the output value and tracking error at the t_i time sample instant in the kth iteration, respectively. In addition, for the practical system involving perturbation and disturbance, it is necessary to take robustness into account.

3 DESIGN ALGORITHM

Consider the model (1) operating repeatedly over the iteration duration $[0,T]$. The principle for updating-reference is given that

$$r_u(t_i,k+1)=r_u(t_i,k)=\cdots=y_r(t_i) \tag{6}$$

where $r_u(t_i,k)$ denotes the reference value at the t_i time instant in the kth iteration.

According to the above principle (6), the whole reference trajectory is introduced along the kth iteration by

$$r_u(t,k+1)=r_u(t,k)+\lambda(t)(r_u(t,k)-y(t,k)) \quad t=1,2,\cdots,T \tag{7}$$

where $\lambda(t)$ is a time-varying proportional coefficient and the specific description is

$$\lambda(t)=\begin{cases}0, & t\in\{t_1,t_2,\cdots,t_s\}\\ \lambda_0(\lambda_0\neq 0), & else\end{cases} \tag{8}$$

For the processes described by (1), ILC law is depicted as

$$\left.\begin{aligned}u(t,k)&=u(t,k-1)+v(t,k)\\ u(t,0)&=0,\quad t=0,1,2,\cdots,T\end{aligned}\right\} \tag{9}$$

where $v(t,k)$ is the update law to be designed, and $u(t,0)$ denotes the input value at time instant t and the initial iteration, and it is supposed to be zero for the initial iteration.

According to the analysis in [10], the similar design method can be taken in the following design procedure for point-to-point tracking. And the ultimate design result is presented below.

Let Δ_k indicate the iteration-wise backward difference operator [10]. Then

$$v(t,k)=\Delta_k u(t,k)=u(t,k)-u(t,k-1) \tag{10}$$

Substituting (10) into the process model (1), one has [10]

$$A(q_t^{-1})y(t,k)=B(q_t^{-1})v(t,k)+A(q_t^{-1})y(t,k-1)+\Delta_k w(t,k) \tag{11}$$

To facilitate the analysis, the process information at any time t in the kth iteration is divided into known and unknown components, which are satisfied [10]

$$(\boldsymbol{A}_0\ \boldsymbol{A}_1)\begin{pmatrix}\boldsymbol{y}(_t^{t-n+1},k)\\ \boldsymbol{y}(_{t+N-1}^{t+1},k)\end{pmatrix}=(\boldsymbol{B}_0\ \boldsymbol{B}_1)\begin{pmatrix}\boldsymbol{v}(_{t-1}^{t-m},k)\\ \boldsymbol{v}(_{t+N-1}^{t},k)\end{pmatrix}+(\boldsymbol{A}_0\ \boldsymbol{A}_1)\begin{pmatrix}\boldsymbol{y}(_t^{t-n+1},k-1)\\ \boldsymbol{y}(_{t+N-1}^{t+1},k-1)\end{pmatrix}+\Delta_k\boldsymbol{w}(_{t+N}^{t+1},k) \tag{12}$$

where N and M are referred as the time-wise prediction horizon and control horizon respectively, and additionally,

$$\boldsymbol{f}(_{t_2}^{t_1},k)=[f(t_1,k)\quad f(t_1+1,k)\cdots f(t_2,k)]^T \tag{13}$$

$\boldsymbol{f}\in\{\boldsymbol{y},\boldsymbol{e},\boldsymbol{v},\boldsymbol{w}\}$, and matrices $\boldsymbol{A}_0$, $\boldsymbol{A}_1$, $\boldsymbol{B}_0$, $\boldsymbol{B}_1$ are defined in the equations (14) and (15) [10].

It is obvious that $\boldsymbol{A}_1$ is a non-singular matrix, then we have [10]

$$\begin{aligned}\boldsymbol{y}(_{t+N}^{t+1},k)&=\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\boldsymbol{v}(_{t+N-1}^{t},k)-\boldsymbol{A}_1^{-1}\boldsymbol{A}_0\Delta_k(\boldsymbol{y}(_t^{t-n+1},k))\\ &+\boldsymbol{A}_1^{-1}\boldsymbol{B}_0\boldsymbol{v}(_{t-1}^{t-m+1},k)+\boldsymbol{y}(_{t+N}^{t+1},k-1)+\boldsymbol{A}_1^{-1}\Delta_k(\boldsymbol{w}(_{t+N}^{t+1},k))\end{aligned} \tag{16}$$

Then the predictive value of output $\boldsymbol{y}(_{t+N}^{t+1},k)$ is [10]

$$\hat{\boldsymbol{y}}(_{t+N}^{t+1}|\,t,k\,|\,k)=\boldsymbol{G}\boldsymbol{v}(_{t+N-1}^{t},k)+\boldsymbol{F}(t,k)+\boldsymbol{H}(t,k) \tag{17}$$

where

$$(\boldsymbol{A}_0 \ \vdots \ \boldsymbol{A}_1) = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & \cdots & a_1 & | & 1 & 0 & \cdots & 0 & 0 \\ 0 & a_n & a_{n-1} & \cdots & a_2 & | & a_1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & a_n & \cdots & a_3 & | & a_2 & a_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & | & * & * & \cdots & a_1 & 1 \end{pmatrix}_{N\times(N+n)} \tag{14}$$

$$(\boldsymbol{B}_0 \ \vdots \ \boldsymbol{B}_1) = \begin{pmatrix} b_m & b_{m-1} & b_{m-2} & \cdots & b_2 & | & b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_m & b_{m-1} & \cdots & b_3 & | & b_2 & b_1 & \cdots & 0 & 0 \\ 0 & 0 & b_m & \cdots & b_4 & | & b_3 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & * & | & * & * & \cdots & b_2 & b_1 \end{pmatrix}_{N\times(N+m-1)} \tag{15}$$

$$\boldsymbol{G} = \boldsymbol{A}_1^{-1}\boldsymbol{B}_1 \tag{18}$$

$$\boldsymbol{F}(t,k) = \boldsymbol{A}_1^{-1}\boldsymbol{B}_0\boldsymbol{v}(_{t-1}^{t-m+1},k) - \boldsymbol{A}_1^{-1}\boldsymbol{A}_0\Delta_k(\boldsymbol{y}(_t^{t-n+1},k)) \tag{19}$$

$$\boldsymbol{H}(t,k) = \boldsymbol{y}(_{t+N}^{t+1},k-1) \tag{20}$$

In order to design an ILC law for the model (1) under the framework of model predictive control, an objective function is formulated in matrix form as [11]

$$J(t,k,\boldsymbol{v}(_{t+M-1}^{t},k)) = \frac{1}{2}\boldsymbol{e}^T(_{t+N}^{t+1},k)\boldsymbol{Q}\boldsymbol{e}(_{t+N}^{t+1},k) + \frac{1}{2}\boldsymbol{v}^T(_{t+M-1}^{t},k)\boldsymbol{S}\boldsymbol{v}(_{t+M-1}^{t},k) \tag{21}$$

where $\boldsymbol{Q} \in \Re^{N\times N}$ and $\boldsymbol{S} \in \Re^{N\times N}$ are weighting matrices. After straightforward computation, the update law is obtained [10]

$$\boldsymbol{v}(_{t+M-1}^{t},k) = (\boldsymbol{G}^T\boldsymbol{Q}\boldsymbol{G}+\boldsymbol{S})^{-1}\boldsymbol{G}^T\boldsymbol{Q}[\boldsymbol{r}_u(_{t+N}^{t+1},k) - \boldsymbol{F}(t,k) - \boldsymbol{H}(t,k)] \tag{22}$$

Substituting (22) into (9), the control law is

$$\begin{aligned} u(t,k) &= u(t,k-1) + \mathbf{1}_{Mt}\boldsymbol{v}(_{t+M-1}^{t},k) \\ &= u(t,k-1) + \mathbf{1}_{Mt}\boldsymbol{K}[\boldsymbol{r}_u(_{t+N}^{t+1},k) - \boldsymbol{F}(t,k) - \boldsymbol{H}(t,k)] \end{aligned} \tag{23}$$

where $\boldsymbol{K} = (\boldsymbol{G}^T\boldsymbol{Q}\boldsymbol{G}+\boldsymbol{S})^{-1}\boldsymbol{G}^T\boldsymbol{Q}$ and $\mathbf{1}_{Mt} = [1\ 0\ \cdots\ 0]_{1\times M}$.

According to the analysis above, it can be noted that matrix inversion is executed twice just in (18) and (22), i.e. $\boldsymbol{A}_1^{-1} \in \Re^{N\times N}$ and $(\boldsymbol{G}^T\boldsymbol{Q}\boldsymbol{G}+\boldsymbol{S})^{-1} \in \Re^{M\times M}$. Owing to N and M selected in (12), equations (18) and (22) are relatively easy to be calculated even for some non- Lyapunov-stable systems. On the contrary, for the algorithms based on the lifted model [11], the matrix inversion of the lifted model $\boldsymbol{G}_0 \in \Re^{T_s\times T_s}$ is hard to compute when the sampling time T_s is set to be very large. In addition, for non-Lyapunov-stable systems, the element $g_{T_s,T_s} = \boldsymbol{C}\boldsymbol{A}^{Ts-1}\boldsymbol{B}$ increases exponentially with T_s and leads to large computation load.

On account of these defects above, algorithms based on the lifted model are not suitable for non-Lyapunov-stable systems and high frequency sampling systems. In contrast, the proposed algorithm can overcome these difficulties. These merits will be illustrated by the simulation results in the next section.

4 NUMERICAL ILLUSTRATION

To illustrate the applicability and effectiveness of the proposed algorithm, numerical simulations are conducted in this section.

4.1 Case 1: unstable system

Considering the SISO system model as follows:

$$\begin{aligned} &y(t,k) + 2y(t-1,k) + 2y(t-2,k) \\ &= u(t-1,k) + 0.5u(t-2,k) + w(t,k) \end{aligned} \tag{24}$$

where $t\in[0,100], k\in\Re^+$, $y(t,0)=0$, $u(t,0)=0$, and $w(t,0)=0$ denotes the external disturbance.

For the point-to-point tracking control problem, the critical points are set as $\{y(10,k)=3, y(19,k)=5.7, y(22,k)=6.6, y(37,k)=11.1\}$. And the tracking error is defined below

$$\begin{aligned} &\|\boldsymbol{E}_p(k)\| = \sum_{i=1}^{s}[y_r(t_i) - y(t_i,k)]^2 \\ &t_i \in \{10,19,22,37\}, i = 1,2,\cdots,s \end{aligned} \tag{25}$$

It is noted that the system model (24) is unstable. And it can be verified that the point-to-point ILC control algorithms based on the lifted model cannot be applied to this system. However, the proposed algorithm can overcome the difficulties.

For the CARIMA-IPILC algorithm, the corresponding parameters are selected as follows: N=4, M=3, $\lambda_0 = -0.8$, $\boldsymbol{Q} = 5\boldsymbol{I}_N$, $\boldsymbol{S} = \boldsymbol{I}_M$, $w(t,k) = 0$. Fig.1 demonstrates that the proposed algorithm can be applied for non-Lyapnuov-stable system, in which the algorithm (gradient point-to -point ILC) based on the lifted model fails to settle.

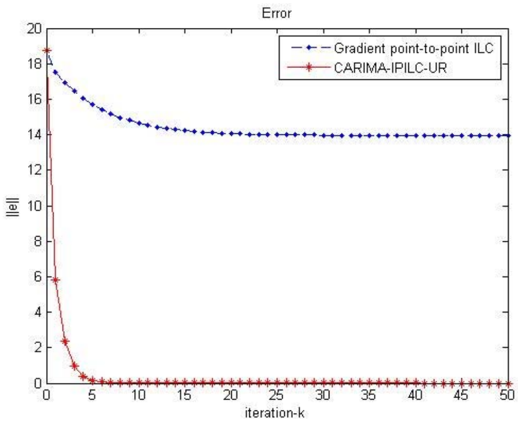

Fig.1 Convergence comparison of tracking error for the unstable system

4.2 Case 2: stable system

Consider a stable system as follows:

$$\begin{aligned} &y(t,k)-1.5y(t-1,k)+0.7y(t-2,k) \\ &=u(t-1,k)+0.5u(t-2,k)+w(t,k) \end{aligned} \tag{26}$$

where $t\in[0,100], k\in\Re^{+}$, $y(t,0)$=0, $u(t,0)$=0, and $w(t,0)$=0. And the critical points are set as $\{$ $y(32,k)$=6, $y(40,k)$=8, $y(65,k)$=−5, $y(92,k)$ =9 $\}$.

To investigate the performance of the proposed method using updating-reference, noted as CARIMA-IPILC-UR, the static reference for the fixed reference algorithm [11] is also studied, noted as CARIMA-IPILC-FR, in which the reference trajectory is set properly and passes through those critical points too.

Simulation results are shown in Fig.2, Fig.3, Fig.4 and Fig.5. As shown in Fig. 2, it can be noticed that the proposed method (CARIMA-IPILC-UR) is obviously superior to the fixed reference algorithm (CARIMA-IPILC-FR) and converge faster. In addition, the reference, output and input profiles in the 1st, 3rd, 5th and 30th iterations are presented in Fig. 3-5. It can be seen that the reference, output and input profiles are all convergent accordingly in the proposed method.

In order to compare the robustness of the proposed approach with the optimal point-to-point tracking algorithm [9], the disturbance is added to the system and assumed as

$$w(t,k)=3\times10^{12}\sin(\lceil 8rand(k)\rceil \pi t) \tag{27}$$

Then the simulation results are presented in Fig.6. Because MPC part is integrated in the control law [11], the proposed algorithm (CARIMA-IPILC-UR) obtains better robustness than the optimal point-to-point algorithm (gradient point-to-point ILC) [9]. This merit is quite meaningful for the practical systems characterizing with model uncertainty and disturbance.

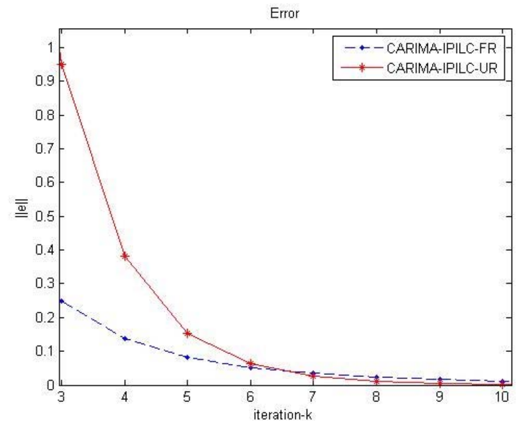

Fig.2 Comparison of tracking performance for the stable system

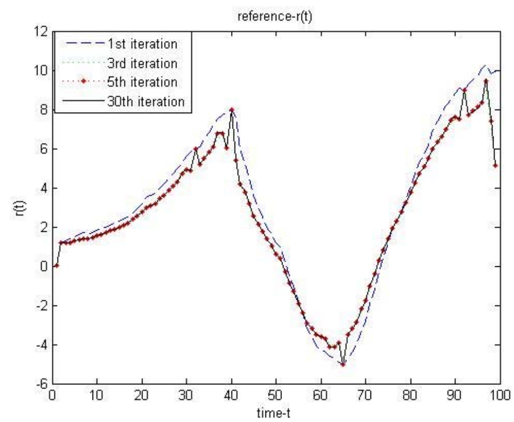

Fig.3 The updating-reference at the 1st, 3rd, 5th an 30th iterations

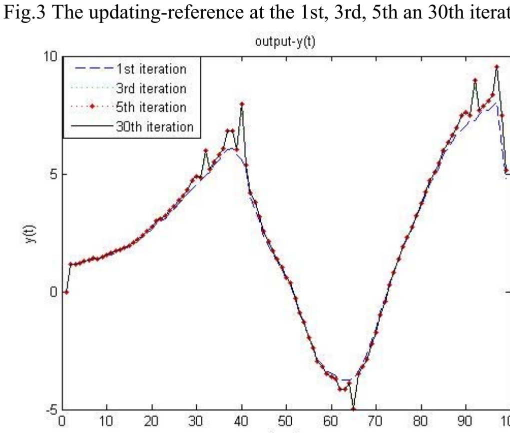

Fig.4 Output profiles at the 1st, 3rd, 5th an 30th iterations

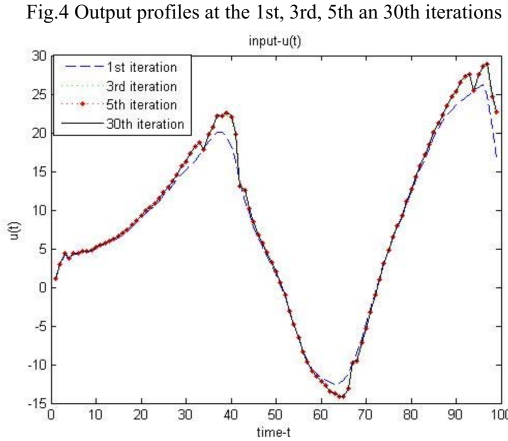

Fig.5 Input profiles at the 1st, 3rd, 5th an 30th iterations

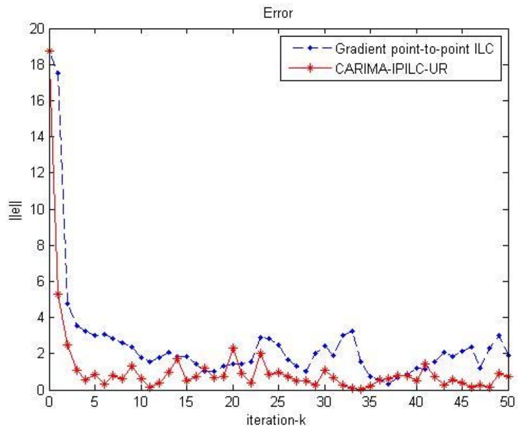

Fig.6 Comparison of tracking performance under the disturbance

5 CONCLUSION

For the point-to-point tacking problem of batch process, this paper proposes a novel design approach based on CARIMA model and updating-reference. The proposed approach combines model predictive control with iterative learning control. Comparing with other point-to-point tracking control algorithms, this control scheme is superior to those algorithms based on lifted model especially for non-Lyapunov-stable systems. In addition, updating-reference relaxes the constraints for system outputs and leads to faster convergence than the fixed-reference control methods. Moreover, to analyze in the two-dimensional perspective, the approach generates a feedback part to obtain better robustness within the iteration. Simulation results on typical systems show the effectiveness of the proposed algorithm. Future work will try to further explore the convergence properties of the IPILC algorithm based on CARIMA model and updating-reference theoretically.

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