A novel SPSA-based IMC-PID Data-driven Control Method

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Abstract: This paper proposes a new data-driven control (DDC) based on SPSA(simultaneous perturbation stochastic approximation). Inheriting the essence of PID controller: simple and practical, an IMC-PID controller is conducted as Function Approximator in SPSA tuning. All the parameters in this polynomial controller are tuned online simultaneously directly via system I/O data, rather than only the filter coefficients tuned off-line.IMC concept introduced into DDC structure help provide initial value, the range of controller parameters and the random disturbance vectors of SPSA. This effort improves the SPSA stochastic approximation performance and every IMC-PID controller parameter has physical meaning. Compared with the traditional SPSA DDC, this method is more simple and practical and is expected to be applied in nonlinear, time-varying and large time delay process.

Key Words: Data-driven control, SPSA, IMC-PID, online tuning

1 INTRODUCTION

Data-driven control (DDC) methods have attracted a great deal of attentions among researchers and practitioners alike, for their unique ability to overcome the process uncertainties and for their lack of dependence on an accurate process model. DDC as a design philosophy encompasses all control theories and methods where the controller is designed by directly using on-line or off-line I/O data of the controlled system or the information embedded in such data, but not any explicit knowledge in the form of mathematical model of the controlled process [1-5].

SPSA-based(simultaneous perturbation stochastic approximation) DDC methods is a kind of classical DDC method[6-8]. The method assumes that the nonlinear dynamics of the controlled plant are unknown. The controller structure also called Function Approximator(FA), is fixed and its parameters are tuned by SPSA method. The approximator often is neural network(NN) structure. With the NN structure controller, SPSA-based DDC can process a class of nonlinear objects.

But in actual situations, the parameters of the neural network controller are complex and some related initial values are lack of physical guidance, sometimes the stability of the algorithm is not satisfactory.

Relative to the neural network control structure, the polynomial approximation structure is simple and has clear physical meanings, such as commonly used PID controller. PID is a wildly used DDC method based on off-line data and utilized in 95% of practical industrial process. The dominance of the PID in industry teaches us that the controller design should be simple, generic and model-independent [11-13]. That is the key point in DDC.

Inheriting the essence of PID controller : simple and practical, this paper proposes a new SPSA-based DDC, in which IMC-PID controller is conducted as function approximator. All the parameters in this polynomial controller are tuned online simultaneously directly via system I/O data, rather than only the filter coefficients tuned off-line as traditional IMC does.

IMC strategy here just provides initial value, the range of controller parameters and random disturbance vectors, which will help improve the SPSA stochastic approximation performance. Every IMC-PID controller parameters all have actual physical meaning. Compared with the traditional SPSA-NN DDC controller, this method is more simple and practical and is expected to be applied in nonlinear, time-varying and large time delay process.

The rest of this paper is organized as follows. A brief overview of basic SPSA and SPSA-based DDC strategy is discribed in Section 2. The SPSA-based polynomial DDC is presented in Section 3 where an IMC-PID function approximator is proposed. Simulation validation for a system of water turbidity time-delay process is given in Section 4, followed by the concluding remarks in Section 5.

2 SPSA-based DDC algorithm

2.1 Basic SPSA algorithm

Consider the problem of minimizing a continuously differentiable loss function $L(\theta)$, where θ is a p-dimention vector. $\{\Delta_{ki}, i=1,...,p, k=1,2,...\}$ is a random vector sequence, usually Δ_{ki} is with independent bounded symmetric distribution. c_k is a scale coefficient, usually

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being considered as a constant or sequence approaching zero and a_k is gain sequence, satisfying

$$a_k = a / (k + A)^{\alpha} \tag{1}$$

$$c_k = c / k^{\gamma} \quad \text{(k=1,2,...)} \tag{2}$$

Here the nonnegative coefficient a, c, A, α and γ are selected according to ref. [6]-[8].

The measurement of L is as follows where the parameter is $\theta_k \pm c_k \Delta_k$ respectively:

$$L_k^{(+)} = L(\theta_k + c_k \Delta_k) + \xi_k^{(+)} \tag{3}$$

$$L_k^{(-)} = L(\theta_k - c_k \Delta_k) + \xi_k^{(-)} \tag{4}$$

$\xi_k^{(+)}$ and $\xi_k^{(-)}$ represent measurement noise term satisfying certain conditions.

Then the estimations of the gradient can be expressed as follows :

$$\hat{g}_k(\hat{\theta}_k) = \begin{bmatrix} \dfrac{L_k^{(+)} - L_k^{(-)}}{2c_k \Delta_{k1}} \\ \dfrac{L_k^{(+)} - L_k^{(-)}}{2c_k \Delta_{k2}} \\ \vdots \\ \dfrac{L_k^{(+)} - L_k^{(-)}}{2c_k \Delta_{kp}} \end{bmatrix} = \frac{L_k^{(+)} - L_k^{(-)}}{2c_k} \begin{bmatrix} \dfrac{1}{\Delta_{k1}} \\ \dfrac{1}{\Delta_{k2}} \\ \vdots \\ \dfrac{1}{\Delta_{kp}} \end{bmatrix} \tag{5}$$

In SPSA method, the recursive formula is used to estimate the parameter sequence as follows :

$$\theta_{k+1} = \theta_k - a_k \hat{g}_k(\hat{\theta}_k) \tag{6}$$

From the description above, we know that only two measurement of L are needed in one iteration since all the elements of θ_k randomly perturbed together to obtain these two measurements $L_k^{(+)}$ and $L_k^{(-)}$.

2.2 SPSA-based DDC algorithm

SPSA-based DDC method uses only closed-loop measurement to tune the parameters of the controller rather than based on a mathematical model of the controlled plant, and the control diagram is depicted as Fig. 1.

The SPSA-based control method assumes that the nonlinear dynamics of the controlled plant are unknown. The controller serves as a function approximator（FA）. Its structure is fixed and the parameters are tunable. The FA is neural network (NN), but also can be polynomial, or other type of approximator. For example, if a multilayer feed-forward NN is selected as the controller, the number of layers and nodes are determined and then the tunable connected weighted coefficients are the parameter of the controller θ.

The control inputs of the NN are the control signals $u(k\text{-}1)$, $u(k\text{-}1)$, $u(k\text{-}3)$……, system outputs $y(k)$, $y(k\text{-}1)$, $y(k\text{-}2)$……within a fixed time window before the current instant k.

The aim of the controller design is, at each time instant k, to find an optimal controller parameter θ_k^*, minimizing the control performance index as follows:

$$L_k = E[(y_{k+1} - r_{k+1})^T A_k (y_{k+1} - r_{k+1}) + u_k^T B_k u_k]$$

Where y_k and u_k are the output and input of the controlled plant at instant k, r_{k+1} is the desired output of the controlled plant at instant $k + 1$. A_k and B_k as positive semidefinite weight matrix, respectively reflects the weight of the tracking error and control energy.

The mathematical model of a controlled plant is unknown, so the traditional optimal techniques cannot be used here due to lack of gradient knowledge of $\partial L_k / \partial u_k$ .This leads to SPSA show superiority to solve this control optimal problem.

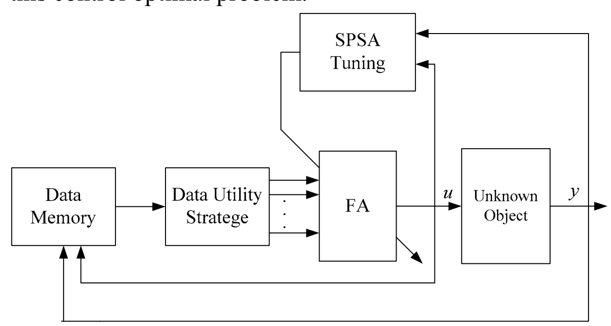

Fig 1. Diagram of SPSA data-driven control

However in SPSA-based control algorithm, there is a drawback: the stochastic perturbation to the parameter may have some randomness since the parameters having no practical meaning and then lead to wasted product if it is used in practice. At the same time too much parameters in NN-based SPSA structure may lead to instability.

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3 SPSA-Based IMC polynomial DDC strategy

3.1 The overall control structure

A polynomial FA structure is more simple and efficient than NN in practical situations. The input of Polynomial FA is error series $\{e(k), e(k-1), e(k-2)\ldots\}$, control series $\{u(k-1), u(k-2)\ldots\}$, the output is $u(k)$. Its structure shows as follows:

$$\frac{u(z^{-1})}{e(z^{-1})} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}} \tag{7}$$

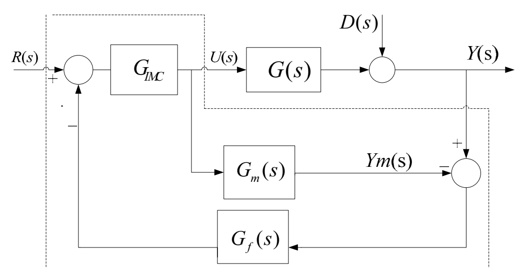
where, $a_0 \sim a_n$, $b_1 \sim b_n$ are tuned coefficients. We can see that the controller is constructed by the linear combination of error sequence and control signal sequence.

With the increase of system order n, more parameters need to be tuned at one SPSA process, but system dynamic characteristic cannot be improved obviously. So n is not expected too big.

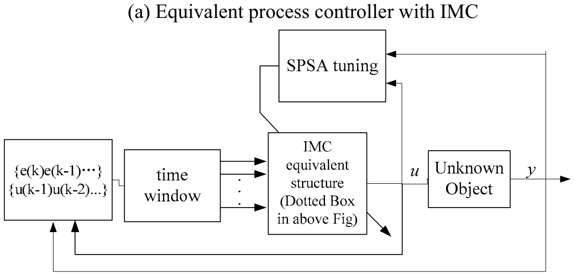
When the polynomial FA structure is fixed, it is important to make every parameter' physical meaning clear to avoid failure of optimization process. When object properties are complex, especially with large time delay, we should make the adjustment of the polynomial parameters clear and effective enough to improve the control quality of system.

This paper gives internal model controller (IMC) concept into the SPSA-based DDC structure, giving every parameter' physical meaning related to IMC. But different from traditional IMC, all the components are included in the IMC equivalent composite structure, as shown in Figure 2. We will tune all the parameters of the equivalent polynomial together with SPSA method. The initial values of polynomial parameters and SPSA random perturbation vector are selected according to rough information IMC which can be inaccurate since the SPSA tuning is going on effectively. By this way, the internal model part is incorporated into the equivalent polynomial, and the parameters are tuned in DDC way. We call this SPSA-based IMC-PID DDC controller. So the model-based IMC control is developed into a DDC method while the controller does not need to rely too much on plant model and then improves the performance of the controller.

The overall structure diagram shows in figure 2. Figure 2(a) shows the equivalent process controller with IMC, figure2(b) shows how to use SPSA-based DDC to tune the controller.



(a) Equivalent process controller with IMC



(b) SPSA-based DDC structure

Fig 2. SPSA-Based IMC polynomial DDC strategy

3.2 SPSA algorithm IMC-PID

According to IMC process in Figure2(a), the plant model G_m is decomposed into two parts: G_{m-} and G_{m+}, G_{m-} stands for the minimum phase part; G_{m+} stands for the unstable part and delay. G_{IMC} is internal model controller. For simplicity the conventional process object is considered as a first order plus time delay process as follows

$$G_m(s) = \frac{K_m}{T_m s+1} e^{-\tau_m s} \tag{8}$$

According to IMC principle G_{IMC} can be expressed as follows

$$G_{IMC}(s) = G_{m-}^{-1}(s) f(s)$$

The equivalent feedback controller in the dotted box can be written as follows :

$$\begin{aligned} G_C(s) &= \frac{G_{-}^{-1}(s) f(s)}{1-G_m(s) G_{m-}^{-1}(s) f(s)} \\ &= \frac{G_{+}^{-1}(s) f(s)}{G(s)(1-G_{+}^{-1}(s) f(s))} \end{aligned} \tag{9}$$

Let filter $f(s) = \frac{1}{\lambda s+1}$, and make the first order Pade Approximation for the delay part $e^{-\tau s} \approx \frac{1-0.5\tau s}{1+0.5\tau s}$

Thus we have

$$G_c(s) = \frac{0.5 T_m \tau_m s^2 + (T_m + 0.5\tau_m)s + 1}{K_m(\lambda+\tau_m)s(\frac{0.5\lambda\tau_m}{\lambda+\tau_m}s+1)} \tag{10}$$

Considering a common partly differential PID structure

$$\begin{aligned} G_c(s) &= K_c(1+\frac{1}{T_i s}+\frac{T_d s}{T_f s+1}) \\ &= \frac{K_c[T_i(T_f+T_d)s^2+(T_i+T_f)s+1]}{T_i s(T_f s+1)} \end{aligned}$$

Comparing the parameters in (10) and(11)respectively, we get：

$$T_f = \frac{0.5\lambda\tau_m}{\lambda+\tau_m} \tag{12}$$

$$K_c = \frac{T_m+0.5\tau_m-T_f}{K_m(\lambda+\tau_m)} \tag{13}$$

$$T_i = T_m + 0.5\tau_m - T_f \tag{14}$$

$$T_d = \frac{0.5 T_m \tau_m}{T_i} - T_f \tag{15}$$

By this way, the internal model part is incorporated into the equivalent polynomial, subsequently we can tune the parameters in SPSA-based DDC way. We get the following discrete expressions of (10) or (11)

$$\frac{u(z^{-1})}{e(z^{-1})} = K_c\left[1+\frac{T_s}{T_i}\frac{1}{1-z^{-1}}+\frac{T_d(1-\eta)}{T_s}\frac{1-z^{-1}}{1-\eta z^{-1}}\right] \quad (16)$$

Here, T_s is the sampling time, note

$$\eta = T_f/(T_s+T_f).$$

Further we have

$$\Delta u(k) = K_c[e(k)-e(k-1)]+\frac{K_cT_s}{T_i}e(k)+\frac{K_cT_d(1-\eta)}{T_s}\times [e(k)-2e(k-1)+e(k-2)]+\eta[u(k-1)-u(k-2)] \quad (17)$$

The vector form is rewritten as follows

$$\Delta u(k) = \theta^T X \quad (18)$$

Here

$$\theta = \begin{bmatrix} K_c \\ K_cT_s/T_i \\ K_cT_d(1-\eta)/T_s \\ \alpha \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} \quad (19)$$

$$X = \begin{bmatrix} e(k)-e(k-1) \\ e(k) \\ e(k)-2e(k-1)+e(k-2) \\ u(k-1)-u(k-2) \end{bmatrix} \quad (20)$$

Now we get the controller parameter θ, and the controller input vector X in SPSA tuning. Parameters K_m,T_m and τ_m are the estimate values of the plant, so can be used to compute initials of θ_0 for SPSA according to formula(12)-(15),note $\theta_0 = [k_{10} \quad k_{20} \quad k_{30} \quad k_{40}]^T$

Model information will directly influence the following SPSA tuning, however it can give range guidance for SPSA since every parameter element has physical meaning. The four part of θ_0 represent Proportional, differential, integral and filter constant time respectively. The corresponding perturbation parameter c should be chosen as a vector according to parameter's physical meaning and related to θ_0 , not just considered as a scalar. In order to make the simultaneous perturbation effectively, we need to make sure that quantitative value between each element parameter theta. The gain parameter a should also be regarded as a vector related to every parameter's convergence rate.

Note that

$$c_0 = [c_1 \quad c_2 \quad c_3 \quad c_4]^T \quad (21),$$

$$a_0 = [a_1 \quad a_2 \quad a_3 \quad a_4]^T \quad (22)$$

Combined with formula (1)and(2),the estimations of the gradient formula(6) will be rewritten as

$$\hat{g}_k(\hat{\theta}_k) = \left[\frac{L_k^{(+)}-L_k^{(-)}}{2c_{k1}\Delta_{k1}} \; \frac{L_k^{(+)}-L_k^{(-)}}{2c_{k2}\Delta_{k2}} \; \frac{L_k^{(+)}-L_k^{(-)}}{2c_{k3}\Delta_{k3}} \; \frac{L_k^{(+)}-L_k^{(-)}}{2c_{k4}\Delta_{k4}}\right] \quad (23)$$

And the optimization equation is expressed as follows

$$\theta_{k+1} = \theta_k - \begin{bmatrix} a_{k1} & 0 & 0 & 0 \\ 0 & a_{k2} & 0 & 0 \\ 0 & 0 & a_{k3} & 0 \\ 0 & 0 & 0 & a_{k4} \end{bmatrix} \hat{g}_k(\hat{\theta}_k) \quad (24)$$

Considering parameter's physical meaning, we maintain parameter disturbance in an effective range to ensure SPSA DDC's stability, i.e. let $\theta_k \in [\theta_{\min}, \theta_{\max}]$.

We found that when applying SPSA, no matter the controller is NN or polynomial, it is necessary to smooth the control output signal generated by SPSA tuning to ensure system stability. Here we take the smoothing factor β in u_k to get the real output u_{k_final}, that is

$$u_{k\_final} = (1-\beta)u_k + \beta u_{k-1} \quad (25)$$

As for plant's parameters varying quickly, some improvements in the convergence rate of SPSA can be found in [9][10][15].We once proposed an efficient one-sided SPSA method which can improve convergence rate and accuracy of optimization procedure, only using 1/3 less measurement of the cost function than conventional SPSA[15].

4 Simulations

Consider the following first order plus time-delay system

$$G_0(s) = \frac{0.902}{50s+1}e^{-26s} \quad (26)$$

The plant above just use to generate I/O data. The time unit is minute. This is a real measured model from our project in a coagulant dosage process in water plant. The process input is dosage inverter' output frequency, the process output is turbidity. This is a time-varying big delay system, the delay time is approaching 30min. We once applied IMC-PID control strategy into this big delay turbidity control system.

In simulation, we just demonstrate even if we have very little model information, the proposed DDC will control over it. The simulation platform is matlab 2008, sampling time T_s is 2 min.The reference input is unit step signal. Gain coefficient values ***a*** and ***c*** in optimization process are given according to the θ_0**.** Other parameters are fixed as : A=50，α=0.602 ， γ=0.101 ， β=0.2. Δ_{ki} obeys independent Bernoulli distribution.

Case 1 Suppose the rough estimate of plant is

$$G_m(s) = 1.2\mathrm{e}^{-30s} / (35s+1)$$

We get the value of θ_0=[0.3704 0.0185 0.0965 0.8333]. So a and c are selected as below: a=[0.2;0.1;0.1;0.2], c=[0.02;0.01;0.01;0.02]**.** Fig 3 shows the four parameters tuning curve. Fig 4-5 shows system control input (i.e. the controller output) and the system output response curve respectively.

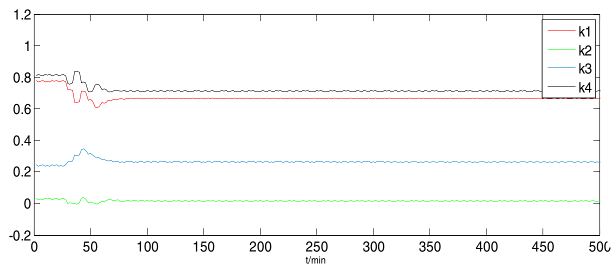


Fig 3. Parameters online tuning curve of θ in Case 1

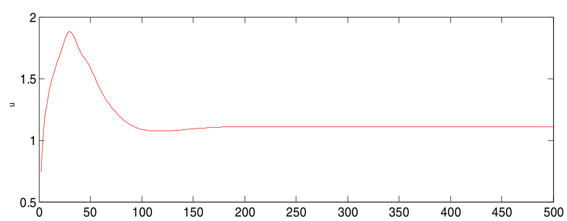


Fig 4. system control input(controller output) curve in Case 1

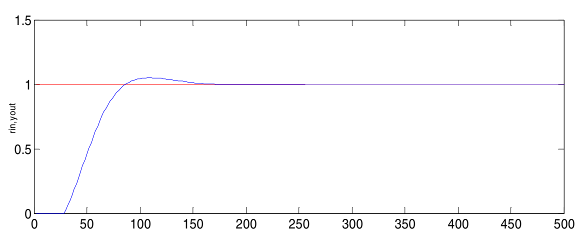


Fig 5. system output response curve in Case 1

Case 2 Suppose the rough estimate of plant is

$$G_m(s) = 2\mathrm{e}^{-15s} / (20s+1)$$

Add noise $\xi(k)$ into the output of the plant (26), given $\xi \in N(0, 0.01^2)$. Fig 6 shows the four parameters online tuning curve. Fig7 and Fig 8 show system control input (i.e. the controller output) and the system output response curve respectively. The results demonstrate that SPSA-based IMC-PID DDC method has good ability to handle model unknown or inaccuracy, time-delay and noisy situation due to controller's excellent stochastic approximation ability and incorporated meaningful parameters. Experiments also show that even with more complicated plant than plant (26), linear or even nonlinear object, this method still exhibits good performance. That is because learning the dynamic linearization information of the plant from I/O data is the kernel tool of DDC methods.

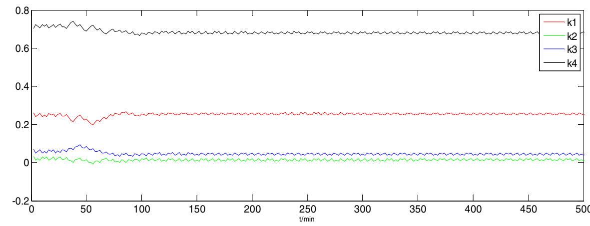


Fig 6. Parameters online tuning curve of θ in Case 2

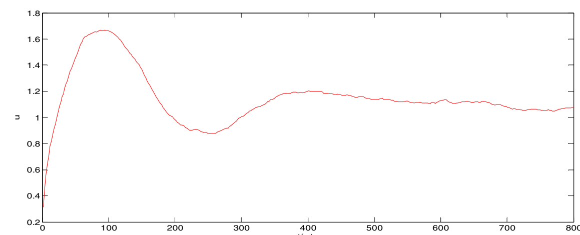


Fig7. system control input(controller output) curve in Case 2

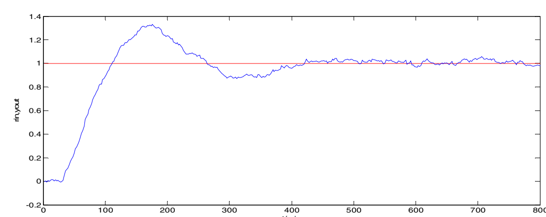


Fig 8. system output response in Case 2

5 conclusion

This Paper combines the dual advantages of model-based control and data-driven control. The traditional IMC is a kind of model-based control method by adjusting the filter coefficient to overcome inaccurate model information. To some extend it is simple and practical especially for some big-delay plant like water coagulant dosage process we conducted in water plant. In this proposed SPSA-based DDC， IMC-PID controller is conducted as Function Approximator. All the parameters in this polynomial controller are tuned online simultaneously directly via system I/O data, rather than only the filter coefficients tuned off-line.IMC information help provide initial value and range of controller parameters and random disturbance vectors of SPSA. This effort improves the SPSA stochastic approximation performance. Compared with the traditional SPSA neural network DDC controller, this method is more simple and practical and is expected to be applied in time-varying ,uncertainty, and large time delay process.

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