Three-axis Stabilized Satellite Back-stepping Adaptive Control

Aihua Zhang, Haitao Meng, Xing Huo

China College of Engineering, BHU, Jinzhou, China
E-mail: Jsxinxi_zah@163.com

Abstract: The reaction wheels (RWS) play an important role in satellite attitude control, redundant wheel can improve the reliability of system and control accuracy. Aiming at the attitude stability of satellite from unknown disturbance, a controller has been designed based on the back-stepping method and using Lyapunov methods to analyze the stability of the system. Simulation result presents the reliability and effectiveness of proposed method.

Key Words: Reaction Wheel, Adaptive Back-stepping, Stability

1 INTRODUCTION

Deep space environment satellite effect by different kinds of disturbance torque, there are some uncertain factors lead to satellite inertia and time-varying. Many scholars have studied and common sliding mode control, optimal control, adaptive control, and their mutual combination method. Literature [1] put forward for the adaptive control for the moment of inertia of the unknown, do not consider interference effects. Literature [2] to control the position of gravity gradient interference but do not have general this interference. In literature [4] a disturbance compensator was introduced however, the mathematical model for linear after may not be suitable for the original system. Literature [5] proposed a robust control based on neural network, but the controller designs accuracy is not high. Literature [10] the adaptive variable structure combined with intelligent control technology, neural network control to compensate the input saturation and nonlinear, achieve good control effect. In literature [7] the back-stepping is used in the design of satellite attitude control, but the design of the controller needs to satellite inertia and precise values of outside interference. Literature [9], According to environmental disturbance torque of small satellite attitude control problem, the adaptive estimation disturbance torque and two continuous function in the design on the basis of combining the back-stepping attitude controller is designed, the interference has certain robustness. Satellite attitude control is a kind of nonlinear coupling of multiple inputs multiple output control system, the back-stepping control is a very effective control method, ensure the stability of the whole system.

2 MATHEMATICAL MODELS

2.1 Satellite Attitude Dynamics Satellite

Because of quaternion describing satellite attitude there is no strange phenomenon, therefore this article attitude quaternion method is used to describe the attitude of the satellite attitude kinematics equation can be expressed as

$$\dot{\boldsymbol{q}} = \frac{1}{2}(\boldsymbol{q}^{\times} + q_0 \boldsymbol{I}_3)\boldsymbol{\omega} \tag{1}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\omega} \tag{2}$$

We take $\boldsymbol{\omega} \in R^3$ as the angular velocity in this system of the satellite relative to the inertial system in the projection, $\boldsymbol{Q} = \begin{bmatrix} q_0 & \boldsymbol{q}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in R^4$ is the satellite body coordinate system and inertial attitude quaternion, and meet the $q_0^2 + \boldsymbol{q}^{\mathrm{T}}\boldsymbol{q} = 1$ .In addition order $\boldsymbol{I}_3$ for 3 unit matrix, and define $\boldsymbol{a}^{\times}$ is

$$\boldsymbol{a}^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{3}$$

In view of the rigid satellite, consider its three reaction flywheel is adopted to improve the attitude control, and three reaction wheels are respectively installed on the satellite three axis of ontology, is at this point can be concluded as follows the satellite attitude dynamics:

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} = \boldsymbol{u} + \boldsymbol{d}(t) \tag{4}$$

Where $\boldsymbol{J} \in R^{3\times3}$ is suitable for satellite inertia matrix, $\boldsymbol{u} \in R^3$ for three axis of ontology reaction flywheel effect on satellite control torque, $d(t)$ is the outside disturbance torque.

$$(q_d)^{\mathrm{T}} q_d + (q_{do})^2 = 1 \tag{5}$$

This work is supported by National Nature Science Foundation under Grant (Project No.61304149, 61573071), the National Natural Science Foundation under Liaoning (Project No.2015020042) and the Excellent Talents to Support Projects in Liaoning Province (Project No. LJQ2015003).

2.2 The Tracking Error Equation

Set the desired body coordinate system relative to the inertial system expected unit quaternion to $q_{dv} = [q_{do}\ q_d]$, attitude error quaternion said the expectations in body coordinate system relative to the body coordinate system of the unit quaternion..

$$e = q_{do}q - q_o q_d + q^{\times} q_d \tag{6}$$

$$e_o = q_o q_{do} + q^{\mathrm{T}} q_d \tag{7}$$

Where e_v meets the constraint

$$e^{\mathrm{T}} e + (e_o)^2 = 1 \tag{8}$$

To consider the error of angular velocity ω_e as angular velocity in body coordinate system

$$\omega_e = \omega - R\omega_d \tag{9}$$

Where ω_d regarded as expected body coordinate system which relatives to the angular velocity of inertial system; R is the desired body coordinate system which relatives to the body coordinate system rotation matrix

$$R = \left[(e_o)^2 - e^{\mathrm{T}} e \right] I_3 + 2ee^{\mathrm{T}} - 2e_o e^{\times} \tag{10}$$

From equations above, satellite attitude tracking error equation:

$$\dot{e} = \tfrac{1}{2}\left(e^{\times} + e_o I_3\right)\omega_e \tag{11}$$

$$\dot{e}_o = -\tfrac{1}{2} e^{\mathrm{T}} \omega_e \tag{12}$$

$$\begin{aligned} J\dot{\omega}_e = &-\left(\omega_e + R\omega_d\right)^{\times} J\left(\omega_e + R\omega_d\right) \\ &+ J\left(\left(\omega_e\right)^{\times} R\omega_d - R\dot{\omega}_d\right) + u + d \end{aligned} \tag{13}$$

For convenience of following controller design, we first give the following assumptions:

Assumption 1: the rotational inertia J of the satellite is the unknown and long value of the positive definite symmetric matrix, there exist constant $J_{\min}$ $J_{\max}$ to establish

$0 < J_{\min} \le \|J\| \le J_{\max} < \infty$

Assumption 2: the interference torque is unknown but bounded.

The goal of the controller designed in this paper can be expressed as: in view of the existence of interference and unknown moment of inertia in control satellite control system, we design the control law of satellite attitude control is that when $t \to \infty$, $e \to 0$ or $\omega \to 0$ or e and ω arbitrary converge to a small area which contains.

3 THE ATTITUDE CONTROL SYSTEM DESIGN

The back-stepping control aimed at a class of strict feedback system, a series of virtual control values calculated by recursive computation to get the final control input. For the existence of unknown inertia matrix and interference torque of the satellite attitude tracking problem, this paper proposes a step adaptive inverse controller design method based on Lyapunov stability theory, in this way, it can ensure consistent asymptotic stability of the closed-loop system.

In order to facilitate the design of the control law, we define a new variable

$$x_1 = e_1 \tag{14}$$

$$x_2 = \omega_e - \alpha \tag{15}$$

I*n* E*q. (1*5), x_1 x_2 are the new state variables, α is calm the function

Step 1: define Lyapunov function based Eq. (11) and Eq. (12)

$$V_1 = (x_1)^{\mathrm{T}} x_1 + (1 - e_o)^2 \tag{16}$$

The derivative of V_1

$$\dot{V}_1 = (x_1)^{\mathrm{T}} (x_2 + \alpha) \tag{17}$$

We serve ω_e as a virtual control value, ε is constant. Eq. (20) substituted into Eq. (17)

$$\alpha = -\varepsilon x_1 \tag{18}$$

$$\dot{V}_1 = (x_1)^{\mathrm{T}} x_2 - \varepsilon (x_1)^{\mathrm{T}} x_1 \tag{19}$$

Obviously, when $x_1 \neq 0$ $x_2 = 0$ $\dot{V}_1 < 0$, asymptotically x_1 is stabled

Step 2: $\dot{x}_2 = \dot{\omega}_e - \dot{\alpha}$ (20) can be get from Eq. (15).

Where $\theta = \left[J_{11}\ J_{22}\ J_{33}\ J_{12}\ J_{13}\ J_{23}\right]^{\mathrm{T}}$ can be defined as the inertia parameters of satellite describing vector, its corresponding estimates value is

$\tilde{\theta} = \left[\tilde{J}_{11}\ \tilde{J}_{22}\ \tilde{J}_{33}\ \tilde{J}_{12}\ \tilde{J}_{13}\ \tilde{J}_{23}\right]^{\mathrm{T}}$

The estimated error is $\bar{\theta} = \theta - \tilde{\theta}$

At the same time, Eq. (20) left by J and substitute into Eq. (13):

$$\begin{aligned} J\dot{x}_2 = &-\left(\omega_e + R\omega_d\right)^{\times} J\left(\omega_e + R\omega_d\right) \\ &- J\dot{\alpha} + J\left(\left(\omega_e\right)^{\times} R\omega_d - R\dot{\omega}_d\right) \\ &+ u + d \end{aligned} \tag{21}$$

Theorem: aiming at the existence of unknown moment of inertia and torque wheel control satellite attitude tracking control system, under the limit of assumption 1 and 2, control Eq. (22) and the adaptive law Eq. (23) can ensure the angular velocity to bound and the closed-loop system is asymptotically stable. Then

$$u = -x_1 - kx_2 - G\tilde{\theta} \tag{22}$$

Where
$$\begin{aligned} G = &-\left(\omega_e + R\omega_d\right)^{\times} L\left(\omega_e + R\omega_d\right) \\ &+ L\left(\varepsilon\dot{e}\right) + J\left(\left(\omega_e\right)^{\times} R\omega_d - R\dot{\omega}_d\right) \end{aligned}$$

Define the operator $L(\xi)=\begin{bmatrix} \xi_1\,0\,0\,\xi_2\,\xi_3\,0 \\ 0\,\xi_2\,0\,\xi_1\,0\,\xi_1 \\ 0\,0\,\xi_3\,0\,\xi_1\,\xi_2 \end{bmatrix}$ and for any ξ, there exists $J\xi = L(\xi)\theta$

$$\dot{\tilde{\theta}} = proj\left(\tilde{\theta}, Y^{-1}G^{\mathrm{T}}x_2\right) \tag{23}$$

Where Y is suitable for any positive diagonal matrix, the projection operator $proj(\tilde{\theta}, Y^{-1}G^{\mathrm{T}}x_2)$

$$\begin{cases} \rho_1 < \varepsilon \\ \rho_2 + \frac{1}{4\gamma^2} < \varepsilon \end{cases} \tag{24}$$

At this time, k is the control parameter

Prove the Stability

Proof:

$$V = (x_1)^{\mathrm{T}} x_1 + (1-e_o)^2 + \tfrac{1}{2}(x_2)^{\mathrm{T}} J x_2 + \tfrac{1}{2}\overline{\theta}^{\mathrm{T}} Y \overline{\theta}$$

The derivative of V is

$$\begin{aligned} \dot{V} = &-\varepsilon\|x_1\|^2 + (x_2)^{\mathrm{T}}\left(x_1 + G\tilde{\theta} + u + d\right) \\ &+\overline{\theta}^{\mathrm{T}}G^{\mathrm{T}}x_2 + \left(\tilde{\theta}-\theta\right)^{\mathrm{T}} Y\dot{\tilde{\theta}} \end{aligned} \tag{25}$$

Define a function

$$H = \dot{V} + \|z\|^2 - \gamma^2\|d\|^2 \tag{26}$$

Among them, $z = \left[\sqrt{\rho_1}x_1 \ \sqrt{\rho_2}x_2\right]^{\mathrm{T}}$ ρ_1, ρ_2 is the weighted coefficient; Eq. (22) and Eq. (23) substitute into Eq. (26)

$$\begin{aligned} H = &-(\varepsilon-\rho_1)\|x_1\|^2 - \left[k-\rho_2-\tfrac{1}{4\gamma^2}\right]\|x_2\|^2 \\ &+\left(\tilde{\theta}-\theta\right)^{\mathrm{T}} Yproj(\tilde{\theta}, Y^{-1}G^{\mathrm{T}}x_2) \\ &+\overline{\theta}^{\mathrm{T}}G^{\mathrm{T}}x_2 - \left\|\tfrac{1}{2\gamma}x_2 - \gamma d\right\|^2 \\ &\le -(\varepsilon-\rho_1)\|x_1\|^2 - \left[k-\rho_2-\tfrac{1}{4\gamma^2}\right]\|x_2\|^2 \\ &-\left\|\tfrac{1}{2\gamma}x_2-\gamma d\right\|^2 + \left(\tilde{\theta}-\theta\right)^{\mathrm{T}} Yx_2 + \overline{\theta}^{\mathrm{T}}G^{\mathrm{T}}x_2 \\ &\le -(\varepsilon-\rho_1)\|x_1\|^2 - \left[k-\rho_2-\tfrac{1}{4\gamma^2}\right]\|x_2\|^2 \\ &\le -\beta\|x\|^2 \end{aligned} \tag{27}$$

Where

$$\beta = \min\left\{(\varepsilon-\rho_1), \left[k-\rho_2-\tfrac{1}{4\gamma^2}\right]\right\} \tag{28}$$

Then

$$\begin{aligned} \dot{V} &\le -\beta\|x\|^2 - \|z\|^2 + \gamma^2\|d\|^2 \\ &\le \gamma^2\|d\|^2 - \|z\|^2 \end{aligned} \tag{29}$$

And then

$V(x(t)) - V(x(0)) \le \gamma^2\int_0^{\mathrm{T}}\|d\|^2 dt - \int_0^{\mathrm{T}}\|z\|^2 dt$, the system stability

4 THE SIMULATION

The rotational inertia of the satellite is

$$J = \begin{bmatrix} 55.3 & 0.22 & 0.43 \\ 0.22 & 50.5 & -0.27 \\ 0.43 & -0.27 & 41.2 \end{bmatrix} kg \bullet m^2$$

$$d = \begin{bmatrix} 1.7\cos(0.01t) + \sin(0.02t) - 1 \\ 2\cos(0.01t) + \sin(0.02t) + 3 \\ 1.7\cos(0.01t) + \sin(0.02t) - 2 \end{bmatrix} \times 10^{-3} N \bullet m$$

$$\omega_1 = \begin{bmatrix} \sin(0.05t) + 8 \\ 2\cos(0.05t) \\ -\sin(0.05t) - 8 \end{bmatrix} \times 10^{-3} rad/s$$

Take the initial value of spacecraft $\omega(0) = [0.01 \ 0.02 \ -0.01]^{\mathrm{T}} rad/s$

$q_v(0) = [0.94 \ 0.28 \ -0.13 \ 0.13]^T$

Take $\gamma = 0.9$, $\rho_1 = 1$ $\rho_2 = 1$ $\varepsilon = 3$ $k = 5$

The simulation results show that the system has better interference suppression performance

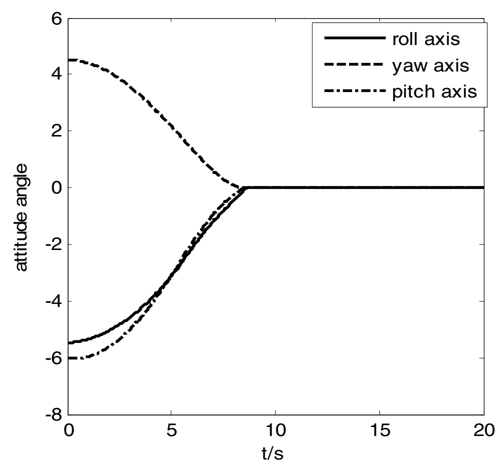

Fig.1 attitude quaternion response curve

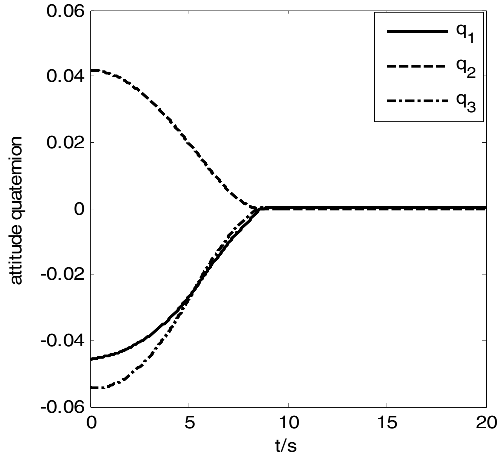

Fig.2 attitude quaternion response curve

5 FORMATTING INSTRUCTIONS

Focusing on the unknown moment of inertia and under the condition of unknown disturbance torque, adaptive back-stepping method adopted in controller design to make the satellite quickly achieve a desired posture, meanwhile, the interference has played a very good inhibitory effective. Data driven technology [12-16] applied to this field is my next work.

REFERENCES

[1] Wodesenbet E, Kidanes. Su S P. Optimization for buckling loads of grid stiffened composite panels [J]. Composite Struvture, 2003, 60: 159-169.

[2] aunky N, Knight N F, Amburdr. Optimal design of general stiffened composite circular cylinders for global buckling with strength constraints [J].Composite Structures.1998,32: 197-20.

[3] C.T. Zhang, T. Zhou, stiffness of the composite materials and other network cylinder structure optimization design [J], high-tech fibers and applications, 2008 (3) : 10 -13.

[4] L. Zhou, C.S. Jiang and Y. L. Du based on the footwork of robust adaptive terminal sliding mode control[J]. control theory and application of 26, no. 6 (2009) : 678-682.

[5] Wu Q, Saif M. Robust fault diagnosis for a satellite large angle attitude system using an iterative neuron PID observer[C]//Proceedings of the American control Conference,2006.

[6] G. T. Yan and Bin Song. based on the robust adaptive inverse spacecraft attitude tracking control of gait[J]. Shanghai aerospace 28, no. 3 (2011): 32-36.

[7] Jiang Y, Hu Q L, Ma G F .Adaptive back-stepping fault tolerant control for flexible spacecraft with unknown bounded disturbances and actuator failures [J]. ISA Transactions, 2010,31(5):1066-1073.

[8] X. M. Zhao. "an adaptive sliding mode variable structure robust fault-tolerant control of small satellite attitude[J]. The computer engineering and application of 2012,48(19): 220-224.

[9] G. f. Ma, Y. Jiang Q. L.Hu. Satellite delay step the fault-tolerant control [J]. Journal of aviation, 2010, 31 (5): 1066-1073.

[10] Y. Jiang Q. L.Hu, G. F. Ma. Control input saturation are the one of spacecraft attitude maneuver intelligent robust control [J]. Journal of aerospace, 2009, 30 (1): 188-192.

[11] Q. L. Hu. Robust adaptive sliding mode attitude maneuvering and vibration damping of three-axis-stabilized flexible spaceraft with actuator saturation limits[J].Nonlinear Dynamics,2009,55:301-321.

[12] S. Yin, X. Li, H. Gao, O. Kaynak, Data-based techniques focused on modern industry: An overview[J]. IEEE Transactions on Industrial Electronics, 2015，62(1):657-667.

[13] S. Yin, Z. Huang, Performance monitoring for vehicle suspension system via fuzzy positivistic C-means clustering based on accelerometer measurements[J]. IEEE/ASME Transactions on Mechatronics, 2015, 20(5):2613-2620.

[14] S. Yin, X. Zhu, O. Kaynak, Improved PLS focused on key performance indictor related fault diagnosis[J]. IEEE Transactions on Industrial Electronics, 2015,62(3):1651-1658.

[15] S. Yin, X. Zhu, Intelligent particle filter and its application on fault detection of nonlinear system[J]. IEEE Transactions on Industrial Electronics, 2015,62(6):3852-3861.

[16] S. Yin, O. Kaynak, Big data for modern industry: challenges and trends[J]. Proceedings of the IEEE, 2015,102(3):143-146.