A Closed-loop PD-type Iterative Learning Algorithm for Discrete Singular Systems

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Abstract: Based on a dynamic decomposition standard form of discrete singular systems, we propose a closed-loop PD-type iterative learning control algorithm to study the problem of the state tracking in this paper. Furthermore, we put forward the convergence conditions of the algorithm and prove the convergence of the algorithm theoretically. The effectiveness of the closed-loop PD-type iterative learning control algorithm is proved by the illustrative example.

Key Words: Iterative learning control, Discrete singular systems, Closed-loop PD-type, Convergence

1 Introduction

Iterative learning control (ILC) is a significant branch of learning control which was proposed formally by Arimoto[1] in 1984. To improve the control quantity, ILC applies repeatedly the former tested information to get the control input which can track the desired output. Compared with traditional control algorithm, ILC can solve the dynamic systems with rather high uncertainty in a simple way, and ILC needs less former tested knowledge and calculation. In addition, it has strong adaptability and can be verified easily. More importantly, ILC doesn't depend on the explicit mathematical model of the dynamic systems, in other words, it is an algorithm using the input signal produced by iterative algorithm to make the output of the systems tracking the desired output. The researches of ILC are of great significance to complex models and to those dynamic systems which are uncertainty and nonlinear coupling[2-3] .

The singular systems[4] are the more generalized dynamic systems with widely practical background. Besides, the singular systems are a natural representation of the objective systems, which are used extensively in the field of circuit theory, large scale systems, and economics[5-7]. At present, scholars at home and abroad have paid attention to the researches of ILC of singular systems and have achieved certain accomplishments. In paper [8], a new ILC algorithm is proposed to study the state tracking problem based on singular value decomposition and the convergence of the algorithm is analyzed. The paper [9] proves the convergence of the P-type ILC algorithm for fast sub-system of linear singular systems under certain conditions. The paper [10] discusses the conditions of robust convergence for ILC under the action of closed-loop PD-type ILC algorithm. In paper [11], according to the decomposition form of singular systems, a mixture of PD-type ILC algorithm is proposed to study the state tracking problem and the convergence of the algorithm is completely analyzed. The paper [13] studies the state tracking problem of the singular systems with time-delay and proves that the iterative learning algorithm is convergent under certain conditions. In paper [14], the convergence conditions for open-loop and closed-loop ILC algorithm are discussed separately. An open-closed-loop PD-type ILC algorithm for a class of nonlinear systems is proposed and the convergence of the algorithm is proved theoretically.

By using ILC algorithm, we can study the state tracking problem of the singular systems. This paper aims at the state tracking problem of the discrete singular systems, using a closed-loop PD-type ILC algorithm. Because of using the present information of the systems, the closed-loop algorithm has better convergence and stronger robustness[12] compared with the open-loop algorithm. By using the proposed convergence conditions of PD-type ILC algorithm, we prove the effectiveness of the algorithm theoretically. And the illustrative example which shows the effectiveness of the algorithm is presented at the end of the paper.

2 Description of the discrete singular systems and the algorithm

Repeatable discrete linear time-invariant singular systems are described as follows

$$Ex_k(i+1) = Ax_k(i) + Bu_k(i) \quad (1)$$

where $E, A \in R^{n\times n}, B \in R^{n\times m}$ are constant matrices. E is a singular matrix and

$$rank(E) = q < n$$

i is the time variable and $i \in [1,2,3,...,T]$. k is the iteration times.

We decompose system (1) into the dynamic decomposition standard form[12] and the systems could be expressed as

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$$\begin{cases} x_k^{(1)}(i+1) = A_{11}x_k^{(1)}(i) + A_{12}x_k^{(2)}(i) + B_1u_k(i) \\ 0 = A_{21}x_k^{(1)}(i) + A_{22}x_k^{(2)}(i) + B_2u_k(i) \end{cases} \quad (2)$$

where $x_k^{(1)}(i) \in R^q, x_k^{(2)}(i) \in R^{(n-q)}$,

$$x_k(i) = [x_k^{(1)}(i), x_k^{(2)}(i)]^T.$$

In this paper, we discuss the state tracking problem of discrete singular systems. Therefore we only consider the system (2). We assume that the discrete singular systems satisfy the following conditions:

I) Discrete singular systems are regular, controllable and observable. Besides, the matrix A_{22} is reversible.

II) System (2) meets the following initial condition,

$$x_k(0) = x_d(0), k = 0,1,2,3...,$$

in which k represents the iteration times.

III) For a given desired trajectory $x_d(i)$, there always exists a corresponding control input $u_d(i)$ to make the following correct on finite interval $[0,T]$

$$\begin{cases} x_d^{(1)}(i+1) = A_{11}x_d^{(1)}(i) + A_{12}x_d^{(2)}(i) + B_1u_d(i) \\ 0 = A_{21}x_d^{(1)}(i) + A_{22}x_d^{(2)}(i) + B_2u_d(i) \end{cases} \quad (3)$$

For the decomposition form of discrete singular systems in (2), we use PD-type ILC algorithm as follows

$$u_{k+1}(i) = u_k(i) + \Gamma_1 e_{k+1}^{(1)}(i+1) + \Gamma_2 e_{k+1}^{(2)}(i) \quad (4)$$

in which k represents the iteration times.

$$e_{k+1}^{(1)}(i+1) = x_d^{(1)}(i+1) - x_{k+1}^{(1)}(i+1),$$
$$e_{k+1}^{(2)}(i) = x_d^{(2)}(i) - x_{k+1}^{(2)}(i).$$

where $\Gamma_1 \in R^{m\times q}, \Gamma_2 \in R^{m\times(n-q)}$ are iterative learning gain matrices.

3 Convergence analysis of the closed-loop PD-type ILC algorithm

Definition 1 The λ norm of the discrete time vector

$$h: \{0,1,2...,T\} \to R^n$$

is defined as follows

$$\|h\|_\lambda = \sup_{0\le i\le T}\{\lambda^i \|h(i)\|\}, (0<\lambda<1)$$

In the formula, $\|\cdot\|$ is a vector norm in R^n.

For $\lambda^T \le \lambda^i \le \lambda^0$ ($0<\lambda<1$), then we can deduce following properties

$$\|h\|_\lambda \le \sup_{0\le i\le T}\|h(i)\| \le \lambda^{-T}\|h\|_\lambda, h:\{0,1,2...,T\} \to R^n$$

Theorem 1 For dynamic decomposition of discrete singular systems in (2), we assume that it satisfies the given conditions I)-III), then if the condition $\|H\| \le \rho < 1$ holds,

where $H = G_1^{-1}, G_1 = I + \Gamma_1\hat{B}_1 - \Gamma_2\hat{B}_2$

$$\hat{B}_1 = B_1 - A_{12}\hat{B}_2, \hat{B}_2 = A_{22}^{-1}B_2$$

we can derive that the closed-loop PD-type iterative learning control algorithm in (4), which the algorithm is uniformly convergent and the state $x_k(i)$ in (2) is uniformly convergent to the desired trajectory $x_d(i)$ when the $k \to \infty$, which means

$$\lim_{k\to\infty} x_k(i) = x_d(i), i \in [0,1,2,...,T].$$

Proof: Because

$$0 = A_{21}x_{k+1}^{(1)}(i) + A_{22}x_{k+1}^{(2)}(i) + B_2u_{k+1}(i)$$

and the matrix A_{22} is reversible, then

$$x_{k+1}^{(2)}(i) = -A_{22}^{-1}A_{21}x_{k+1}^{(1)}(i) - A_{22}^{-1}B_2u_{k+1}(i) \quad (5)$$

The formula (5) is substituted in the formula

$$x_k^{(1)}(i+1) = A_{11}x_k^{(1)}(i) + A_{12}x_k^{(2)}(i) + B_1u_k(i)$$

then we can get

$$x_{k+1}^{(1)}(i+1) = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_{k+1}^{(1)}(i) + (B_1 - A_{12}A_{22}^{-1}B_2)u_{k+1}(i) \quad (6)$$

letting $\hat{A}_{11} = A_{11} - A_{12}A_{22}^{-1}A_{21}$, $\hat{B}_1 = B_1 - A_{12}A_{22}^{-1}B_2$, then the formula (6) can be written as

$$x_{k+1}^{(1)}(i+1) = \hat{A}_{11}x_{k+1}^{(1)}(i) + \hat{B}_1u_{k+1}(i) \quad (7)$$

By the formula

$$\Delta u_{k+1}(i) = u_d(i) - u_{k+1}(i)$$

and ILC algorithm, we can get

$$\Delta u_{k+1}(i) = \Delta u_k(i) - (\Gamma_1 e_{k+1}^{(1)}(i+1) + \Gamma_2 e_{k+1}^{(2)}(i)) \quad (8)$$

By the formula (7), we can get

$$e_{k+1}^{(1)}(i+1) = \hat{A}_{11}e_{k+1}^{(1)}(i) + \hat{B}_1\Delta u_{k+1}(i) \quad (9)$$

Since $e_{k+1}^{(2)}(i) = x_d^{(2)}(i) - x_{k+1}^{(2)}(i)$

$$= -A_{22}^{-1}A_{21}e_{k+1}^{(1)}(i) - A_{22}^{-1}B_2\Delta u_{k+1}(i)$$
$$= -\hat{A}_{21}e_{k+1}^{(1)}(i) - \hat{B}_2\Delta u_{k+1}(i) \quad (10)$$

where $\hat{A}_{21} = A_{22}^{-1}A_{21}, \hat{B}_2 = A_{22}^{-1}B_2$.

The formula (9) and (10) are substituted in the formula (8), we can get

$$\Delta u_{k+1}(i) = \Delta u_k(i) - (\Gamma_1\hat{A}_{11} - \Gamma_2\hat{A}_{21})e_{k+1}^{(1)}(i) - (\Gamma_1\hat{B}_1 - \Gamma_2\hat{B}_2)\Delta u_{k+1}(i) \quad (11)$$

The formula (11) is rewritten as follows

$$(I + \Gamma_1\hat{B}_1 - \Gamma_2\hat{B}_2)\Delta u_{k+1}(i) = \Delta u_k(i) - (\Gamma_1\hat{A}_{11} - \Gamma_2\hat{A}_{21})e_{k+1}^{(1)}(i) \quad (12)$$

Letting $G_1 = I + \Gamma_1\hat{B}_1 - \Gamma_2\hat{B}_2$, we can get that the matrix G_1 is reversible through assumptions. By the formula (12), we can get the equation as follows

$$\Delta u_{k+1}(i) = G_1^{-1}\Delta u_k(i) - G_1^{-1}(\Gamma_1\hat{A}_{11} - \Gamma_2\hat{A}_{21})e_{k+1}^{(1)}(i) \quad (13)$$

letting $H = G_1^{-1}$, $G = G_1^{-1}(\Gamma_1\hat{A}_{11} - \Gamma_2\hat{A}_{21})$

we can get

$$\Delta u_{k+1}(i) = H\Delta u_k(i) - Ge_{k+1}^{(1)}(i) \tag{14}$$

By the following initial conditions and the formula (9),

$$x_k(0) = x_d(0), k = 0,1,2,3...$$

we can get

$$e_{k+1}^{(1)}(i) = \hat{A}_{11}{}^{i} e_{k+1}^{(1)}(0) + \sum_{j=0}^{i-1} \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \Delta u_{k+1}(j)$$

$$= \sum_{j=0}^{i-1} \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \Delta u_{k+1}(j) \tag{15}$$

Therefore, the formula (14) can be written as follows

$$\Delta u_{k+1}(i) = H\Delta u_k(i) - G\sum_{j=0}^{i-1} \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \Delta u_{k+1}(j) \tag{16}$$

By taking norms on both sides of the formula (16) and have the formula multiplied by λ^i, then we can derive following formula

$$\lambda^i \|\Delta u_{k+1}(i)\| \le \lambda^i \|H\| \|\Delta u_k(i)\| +$$

$$\lambda^i \|G\| \sum_{j=0}^{i-1} \left\| \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \right\| \|\Delta u_{k+1}(j)\| \tag{17}$$

Letting $g = \|G\|$, $b = \sup_{\substack{1\le i\le T \\ 0\le j\le i\text{-}1}} \left\| \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \right\|$, by the formula (17), then

$$\lambda^i \|\Delta u_{k+1}(i)\| \le \|H\| \|\Delta u_k\|_\lambda + \lambda^i g \sum_{j=0}^{i-1} b \|\Delta u_{k+1}(j)\|$$

$$\le \|H\| \|\Delta u_k\|_\lambda + gb\frac{\lambda}{1-\lambda} \|\Delta u_{k+1}\|_\lambda \tag{18}$$

By the formula (18), we can get

$$\|\Delta u_{k+1}\|_\lambda \le \|H\| \frac{1-\lambda}{1-(1+gb)\lambda} \|\Delta u_k\|_\lambda \tag{19}$$

Because $\|H\| \le \rho < 1$, so we can find a λ which is small enough to meet the condition

$$\|H\| \frac{1-\lambda}{1-(1+gb)\lambda} < 1 \tag{20}$$

By the formula (19) and (20), we can get

$$\lim_{k\to\infty} \|\Delta u_k\|_\lambda = 0 \tag{21}$$

By the properties of norm of definition one, then

$$\sup_{1\le i\le T} \|\Delta u_k(i)\| \le \lambda^{-T} \|\Delta u_k\|_\lambda$$

so we can get

$$\lim_{k\to\infty} \sup_{1\le i\le T} \|\Delta u_k(i)\| = 0 \tag{22}$$

That means the iterative learning control algorithm in formula (4) is uniformly convergent.

For λ satisfies the conditions above, we take norms on both sides of the formula (15) and have the formula multiplied by λ^i, so we can get

$$\lambda^i \left\| e_{k+1}^{(1)}(i) \right\| \le \sum_{j=0}^{i-1} \left\| \hat{A}_{11}{}^{i-j-1} \hat{B}_1 \right\| \lambda^{i-j} \|\Delta u_{k+1}\|_\lambda$$

$$\le b\frac{\lambda}{1-\lambda} \|\Delta u_{k+1}\|_\lambda \tag{23}$$

then we can get

$$\left\| e_{k+1}^{(1)} \right\|_\lambda \le b\frac{\lambda}{1-\lambda} \|\Delta u_{k+1}\|_\lambda \tag{24}$$

By the formula (22), we can deduce

$$\lim_{k\to\infty} \sup_{1\le i\le T} \left\| e_k^{(1)}(i) \right\| = 0 \tag{25}$$

Similarly, we can deduce

$$\lim_{k\to\infty} \sup_{1\le i\le T} \left\| e_k^{(2)}(i) \right\| = 0 \tag{26}$$

Since

$$x_d(i) - x_k(i) = \begin{pmatrix} x_d^{(1)}(i) \\ x_d^{(2)}(i) \end{pmatrix} - \begin{pmatrix} x_k^{(1)}(i) \\ x_k^{(2)}(i) \end{pmatrix} = \begin{pmatrix} e_k^{(1)}(i) \\ e_k^{(2)}(i) \end{pmatrix}$$

so $\lim_{k\to\infty} x_k(i) = x_d(i)$ is correct on finite interval $[0, T]$.

4 Simulation of the algorithm

Assuming that system matrices E, A, B are as follows

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & \text{-}1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

According to the PD-type algorithm in (4), let the gain matrix be

$$\Gamma = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

The matrices satisfy the convergence conditions. And the desired trajectory:

$$x_d(i) = [x_d{}^{(1)}(i), x_d{}^{(2)}(i)]^T = [5\sin(0.4i), i^2]^T$$

letting system (1) initial learning state and initial input be

$$x(0) = [0,0]^T, u(0) = [0,0]^T.$$

then the simulation results are shown in Fig.1- Fig.4.

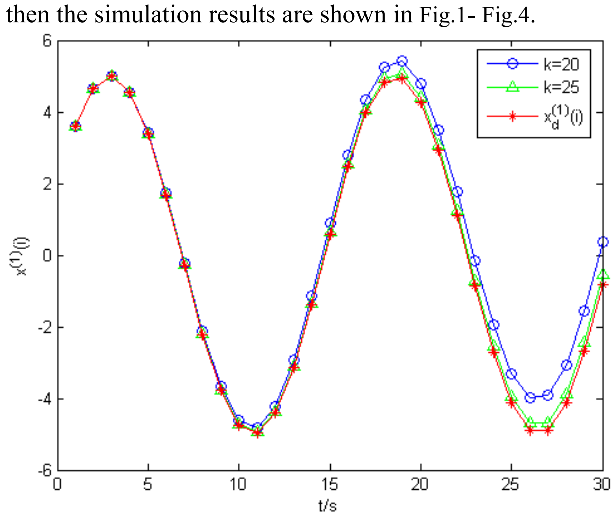

Fig.1: The state tracking of $x_d^{(1)}(i)$

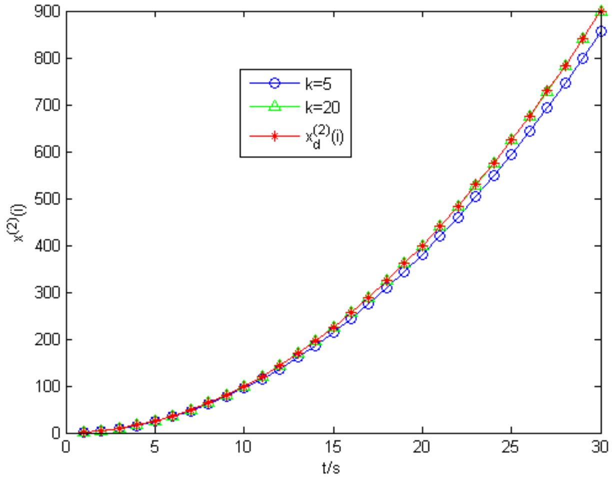

Fig.2: The state tracking of $x_d^{(2)}(i)$

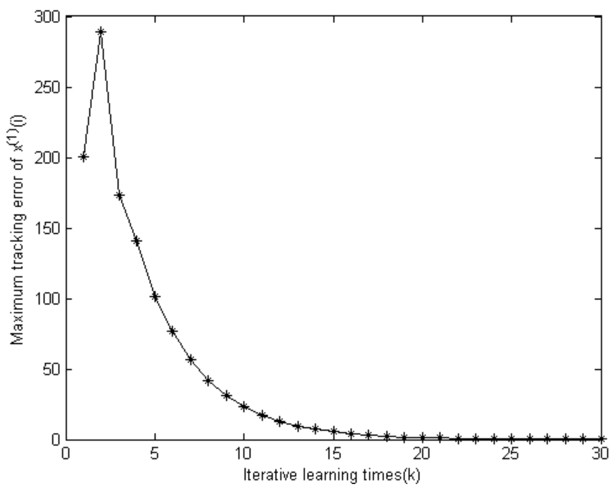

Fig.3: Maximum tracking error of $x_d^{(1)}(i)$

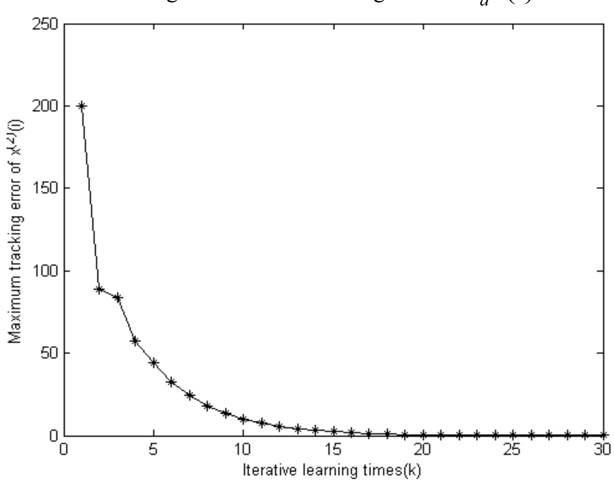

Fig.4: Maximum tracking error of $x_d^{(2)}(i)$

Fig.1 shows the tracking of the desired trajectory $x_d^{(1)}(i)$ by using ILC algorithm (4) at 20th and 25th iterations. The algorithm can track the desired trajectory when the k is 25. Fig.2 shows the tracking of the desired trajectory $x_d^{(2)}(i)$ by using ILC algorithm (4) at 5th and 20th iterations. The algorithm can track the desired trajectory when the k is 20. Fig.3 and Fig.4 show the maximum tracking error. The maximum tracking error is close to 0 with the increasing of the iteration times. The simulation results indicate the effectiveness of the closed-loop PD-type ILC algorithm for the discrete singular systems.

5 Summary

This paper aims at a class of discrete singular systems by using the dynamic decomposition standard, and it proposes ILC algorithm (4) and gives the convergence conditions of the algorithm. Finally, it proves that the closed-loop PD-type ILC algorithm can guarantee the complete tracking of the desired trajectory on finite interval.

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