A Class of Second Order Strong Hyperbolic Distributed Parameter Systems for Iterative Learning Control

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Abstract: The work is connected with the development of stability theory methods for a class of second order strong hyperbolic distributed parameter systems for iterative learning control. This research works out the specific P-type control law for the system and proves its robustness and convergence via mapping and semi group method. The system state mild solution is built. The paper used mapping method with the P-type learning law, thus can guarantee the output tracking errors on L2 space converge along the iteration axis. An example has been shown to verify the effectiveness of the new proposed algorithm.

Key words: Iterative learning control; Second order strong hyperbolic distributed parameter system; Semi group theory

1 INTRODUCTION

During the last three decades, Iterative Learning Control(ILC) has developed greatly. It is suitable for dynamic system with repetitive trajectory. The main idea of ILC is to adjust input signal with error which is produced by expected output minus current output, and to produce a new input signal for next iterative cycle and repeat the whole process to make practical output convergence to expected output.

Iterative Learning Control was first proposed by Arimoto in 1984 in Ref.[1]. Since then, ILC become a very important issue of control field. A Lot of achievements have been published as Ref.[2-4]. Many of the discussed systems described by ordinary differential equations. However, there are so many systems can be modeled by partial differential equations but papers about this field are rare. On the other hand, distributed parameter system can not be approached by ordinary differential equations. So ILC of distributed parameter system is a very important research field.

Several approaches for updating the control law with repeated trials on identical tasks, making use of stored preceding inputs and output errors, have been proposed and analyzed. Optimal iterative learning algorithm refers to [2-5], higher-order learning algorithm refers to [6], learning algorithm based on 2-D system theory refers to [7], iterative learning algorithm with forgetting factor refers to [8], learning algorithm by high order refers to [9-11], and initial error learning for learning algorithm refers to [12-16]. Several algorithms assume that the initial condition is fixed; People have also proposed many methods to improve the iterative learning algorithm.

Although these algorithms have their own characteristics and applied fields, they have not introduced the significance of iterative learning algorithm for in distributed parameter system. Ref.[17-20] introduced parabolic control issue using semigroup method for iterative learning control, but he didn't give reader learning condition and convergence proof. The paper presents a novel approach for second order strong hyperbolic distributed parameter systems for iterative learning algorithm and strict proof the learning algorithm sufficient conditions in distributed parameter system. According to the method, we can guarantee the output tracking errors on L^2 space converge along the iteration axis. They were not delicate enough.

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2 Problem Description and Convergence Analysis

Consider a class of second order strong hyperbolic distributed parameter system as follows:

$$\begin{cases} \dfrac{\partial^2 Q(x,t)}{\partial t^2} = D\Delta Q(x,t) + f\big(t, Q(x,t), u(x,t)\big) \\ y(x,t) = g\big(t, Q(x,t)\big) + h(t)u(x,t) \end{cases}$$

$$(x,t) \in \Omega \times [0,T] \qquad (1)$$

Here $Q(x,t) \in R^m, u(x,t) \in R^m, y(x,t) \in R^l$ are the state of system, control input and output. Where $D = \mathrm{diag}\big(d_1, d_2, \cdots, d_n\big), 0 < p_i \le d_i < +\infty, p_i$ is known constant, Ω is an open bounded domain, $\partial\Omega$ is smooth, Δ is Laplace operator. For convenience, we consider SISO system for system (1) . Then the system can transfer system (2).

$$\begin{cases} \dfrac{\partial^2 Q(x,t)}{\partial t^2} = D\dfrac{\partial^2 Q(x,t)}{\partial x^2} + f\big(t, Q(x,t), u(x,t)\big) \\ y(x,t) = g\big(t, Q(x,t)\big) + h(t)u(x,t) \end{cases}$$

$$(x,t) \in (a,b) \times [0,T] \qquad (2)$$

Where $u(x,t) \in R, y(x,t) \in R.$

For system(2), we give the following suppose.

Suppose 1. For the given initial value $Q(0,t), \frac{\partial Q(x,t)}{\partial t}|_{t=0}$, the boundary value $Q(a,t), Q(b,t)$ and control input $u(x,t)$, the solution $Q(x,t), y(x,t)$ of system(2) is unique within $(a,b)\times[0,T]$.

Suppose 2. For h(t), we have $0<\alpha_1\le h(t)\le\alpha_1, \forall t\in[0,T]$, mark $E\triangleq[0,T]\times R^n\times R$. nonlinear function $f(t,Q(x,t),u(x,t))$ is Lipschitz continues.

$$\|f(t,Q_1,u_1)-f(t,Q_2,u_2)\| \le L_f(\|Q_1-Q_2\|+\|u_1-u_2\|)$$

We mark $\hat{E}\triangleq[0,T]\times R^n, \forall(t,Q)\in\hat{E}, \left\|(\frac{\partial g}{\partial Q})^T\right\|\le L_g$, where L_f and L_g are unknown constant.

Suppose 3. There exists unique control input $u_r(x,t)$ for given ideal trajectory $y_r(x,t)$.

$$\begin{cases}\frac{\partial^2 Q_r(x,t)}{\partial t^2}=D\frac{\partial^2 Q_r(x,t)}{\partial x^2}+f(t,Q_r(x,t),u_r(x,t))\\ y_r(x,t)=g(t,Q_r(x,t))+h(t)u_r(x,t)\end{cases}$$

$(x,t)\in(a,b)\times[0,T]$.

If the system(2) is repeatable interval $t\in[0,T]$, then rewrite the system(2).

$$\begin{cases}\frac{\partial^2 Q_k(x,t)}{\partial t^2}=D\frac{\partial^2 Q_k(x,t)}{\partial x^2}+f(t,Q_k(x,t),u_k(x,t))\\ y_k(x,t)=g(t,Q_k(x,t))+h(t)u_k(x,t)\end{cases}$$

$$(x,t)\in(a,b)\times[0,T] \qquad (3)$$

We find suitable learning law to make the iterative learning sequence $y_k(x,t)$ convergence to ideal output $y_r(x,t)$.

$$\lim_{k\to\infty}\|e_k(x,t)\|_{L^2,s}=0$$

where $e_k(x,t)=y_r(x,t)-y_k(x,t)$.

We take a positive number ε that satisfied (4) in suppose 2.

$$\frac{\alpha_2}{\alpha_1}<\frac{\sqrt{1+\varepsilon}+1}{\sqrt{1+\varepsilon}-1} \qquad (4)$$

For system(2), we use P-type learning law(5).

$$u_{k+1}(x,t)=u_k(x,t)+qe_k(x,t) \qquad (5)$$

Lemma 1. The suppose 1-4 is right, if we have

$$\rho=\max_{t\in[0,T]}|1-qh(t)|<\frac{1}{\sqrt{1+\varepsilon}} \qquad (6)$$

then the system(2) is convergence in P-type(5), namely $\lim_{k\to\infty}\|e_k(x,t)\|_{L^2,s}=0$.

Proof. Let $\delta Q_k(x,t)=Q_{k+1}(x,t)-Q_k(x,t), \delta u_k(x,t)=u_{k+1}(x,t)-u_k(x,t)$, for convenience, mark $Q_k+\theta_k\delta Q_k=\xi_k$, where $0\le\theta_k\le1$. Using Taylor formula, according to (3) and (5) we have

$$\begin{aligned}y_{k+1}(x,t)&=g(t,Q_{k+1})+h(t)u_{k+1}(x,t)\\&=g(t,Q_k+\theta_k\delta Q_k)+h(t)(u_k+\delta u_k)\\&=g(t,Q_k)+\frac{\partial g}{\partial Q}|_{\xi_k}\delta Q_k+h(t)u_k\\&\quad+h(t)\delta u_k\\&=y_k(x,t)+\frac{\partial g}{\partial Q}|_{\xi_k}\delta Q_k+qh(t)e_k(x,t)\end{aligned}$$

So

$$\begin{aligned}E_{k+1}(x,t)&=e_k(x,t)+y_k(x,t)-y_{k+1}(x,t)\\&=e_k(x,t)\frac{\partial g}{\partial Q}|_{\xi_k}\delta Q_k-qh(t)e_k(x,t)\\&=[1-qh(t)]e_k(x,t)-\frac{\partial g}{\partial Q}|_{\xi_k}\delta Q_k \qquad (7)\end{aligned}$$

Take (7) norm, according to suppose 2 and (6), we have

$$|e_{k+1}(x,t)|\le\rho|e_k(x,t)|+L_g\|\delta Q_k\|$$

$$|e_{k+1}(x,t)|^2\le$$

$$(1+\varepsilon)\rho^2|e_k(x,t)|^2+\left(1+\tfrac{1}{\varepsilon}\right)L_g\|\delta Q_k\|^2 \qquad (8)$$

Integral x of (8) from a to b, we have

$$\|e_{k+1}(x,t)\|_{L^2}^2\le(1+\varepsilon)\rho^2\|e_k(x,t)\|_{L^2}^2+(1+\tfrac{1}{\varepsilon})L_g\|\delta Q_k\|_{L^2}^2 \qquad (9)$$

So

$$\begin{aligned}\|e_{k+1}(x,t)\|_{L^2,\lambda}&=\max_{t\in[0,T]}e^{-\lambda t}\|e_{k+1}(x,t)\|_{L^2}^2\\&=(1+\varepsilon)\rho^2\|e_{k+1}(x,t)\|_{L^2,\lambda}\\&\quad+\left(1+\tfrac{1}{\varepsilon}\right)L_g{}^2\|\delta Q_k\|_{L^2,\lambda} \qquad (10)\end{aligned}$$

By system(3)

$$\frac{\partial^2(\delta Q_k(x,t))}{\partial t^2}=D\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}+f(t,Q_{k+1}(x,t),u_{k+1}(x,t))-f(t,Q_k(x,t),u_k(x,t)) \qquad (11)$$

Using $\frac{\partial(\delta Q_k(x,t))}{\partial t}$ as inner product, we have

$$\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T\frac{\partial^2(\delta Q_k(x,t))}{\partial t^2}=\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T D\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}+\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T f(t,Q_{k+1}(x,t),u_{k+1}(x,t))-f(t,Q_k(x,t),u_k(x,t)) \qquad (12)$$

While

$$\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T \frac{\partial^2(\delta Q_k(x,t))}{\partial t^2} = \frac{1}{2}\frac{\partial}{\partial t}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|^2 \quad (13)$$

$$\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T D \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} = \sum_{i=1}^{n} d_i \left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i \quad (14)$$

According to suppose 2 and (5), we have

$$\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)^T \left(f(t, Q_{k+1}(x,t), u_{k+1}(x,t)) - f(t, Q_k(x,t), u_k(x,t))\right) \le L_f\{\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|^2 + \frac{1}{2}\|\delta Q_k(x,t)\|^2 + \frac{1}{2}q^2|e_k(x,t)|^2 \quad (15)$$

Substitute (13)(14)(15) in (12),

$$\frac{\partial}{\partial t}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|^2 \le \sum_{i=1}^{n} 2d_i \left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i + L_f\{2\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|^2 + \|\delta Q_k(x,t)\|^2 + q^2|e_k(x,t)|^2 \quad (16)$$

Integral x of (16) from a to b, we have

$$\frac{d}{\partial t}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 \le \sum_{i=1}^{n} 2d_i \int_a^b\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\}dx + L_f\{2\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 + \|\delta Q_k(x,t)\|^2 + q^2\|e_k(x,t)\|_{L^2}^2\} \quad (17)$$

Integration by parts (17), we have

$$\int_a^b\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\}dx = \left\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\right\}\Big|_{x=b}^{x=a} - \int_a^b\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\}dx \quad (18)$$

According to condition of suppose (4), we have

$$\left\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\right\}\Big|_{x=b}^{x=a} = 0, \text{ so}$$

$$\int_a^b\{\left(\frac{\partial(\delta Q_k(x,t))}{\partial t}\right)_i \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)_i\}dx = -\frac{d}{dt}\left\{\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx\right\} \quad (19)$$

Substitute (19) in (17), we have

$$\frac{d}{dt}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial x}\right\|_{L^2}^2 \le -\frac{d}{dt}\left\{\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx\right\} + L_f\{2\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 + \|\delta Q_k(x,t)\|_{L^2}^2 + q^2\|e_k(x,t)\|_{L^2}^2\}$$

Namely

$$\frac{d}{dt}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial x}\right\|_{L^2}^2 + \left\{\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx\right\} \le L_f\{2\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 + \|\delta Q_k(x,t)\|_{L^2}^2 + q^2\|e_k(x,t)\|_{L^2}^2\}$$

As D is positive definite matrix, so

$$\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx \ge 0 \quad (20)$$

So

$$\frac{d}{dt}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial x}\right\|_{L^2}^2 + \left\{\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx\right\} \le L_f\{2\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 + 2\int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx + \|\delta Q_k(x,t)\|_{L^2}^2 + q^2\|e_k(x,t)\|_{L^2}^2\}$$

Using suppose 4 and Gronwall lemma, we have

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 + \int_a^b \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right)^T D \left(\frac{\partial(\delta Q_k(x,t))}{\partial x}\right) dx \le \int_0^t e^{2L_f(t-\tau)}\{L_f\|\delta Q_k(x,\tau)\|_{L^2}^2 + L_f q^2\|e_k(x,\tau)\|_{L^2}^2\}d\tau.$$

According to (20), we have

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 = L_f e^{2L_f T}\frac{e^{\lambda t}-1}{\lambda}\{\|\delta Q_k(x,\tau)\|_{L^2,\lambda} + q^2\|e_k(x,\tau)\|_{L^2,\lambda}\}$$

So

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} = \max_{t\in[0,T]} e^{-\lambda t}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2}^2 \le L_f e^{2L_f T}\frac{1-e^{\lambda t}}{\lambda}\left\{\|\delta Q_k(x,\tau)\|_{L^2,\lambda} + q^2\|e_k(x,\tau)\|_{L^2,\lambda}\right\} \quad (21).$$

According to initial value of suppose 4, we have

$$\|\delta Q_k(x,t)\|_{L^2}^2 = \int_0^t \frac{d}{d\eta}\|\delta Q_k(x,\eta)\|_{L^2}^2 d\eta$$

$$= \int_0^t \frac{d}{d\eta}\{\int_a^b \{(\delta Q_k(x,\eta))^T \delta Q_k(x,\eta)\}dx\}d\eta$$

$$= \int_0^t \int_a^b \{(\delta Q_k(x,\eta))^T \frac{\partial}{\partial\eta}\{\delta Q_k(x,\eta)\}dxd\eta$$

By Cauchy-Schawrz inequality, we have

$$\|\delta Q_k(x,t)\|_{L^2}^2$$

$$\le 2\int_0^t \{\sqrt{\int_a^b \|\delta Q_k(x,\eta)\|^2 dx}\sqrt{\int_a^b \left\|\frac{\partial}{\partial\eta}\{\delta Q_k(x,\eta)\}\right\|^2 dx}\}d\eta$$

$$= 2\int_0^t \{\|\delta Q_k(x,\eta)\|_{L^2}\left\|\frac{\partial(\delta Q_k(x,\eta))}{\partial\eta}\right\|_{L^2}\}d\eta$$

$$\le 2\int_0^t \{\|\delta Q_k(x,\eta)\|^2{}_{L^2}\left\|\frac{\partial(\delta Q_k(x,\eta))}{\partial\eta}\right\|_{L^2}^2\}d\eta$$

Using Gronwall lemma, we get

$$\|\delta Q_k(x,\eta)\|_{L^2}^2 \le e^t\int_0^t e^{(\lambda-1)\eta}d\eta\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} = \frac{e^{\lambda t}-e^t}{\lambda-1}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda}.$$

We take $\lambda > 1$, then

$$\|\delta Q_k(x,t)\|_{L^2,\lambda} = \max_{t\in[0,T]} e^{-\lambda t}\|\delta Q_k(x,t)\|_{L^2}^2 \le \max_{t\in[0,T]}\frac{1-e^{-(\lambda-1)t}}{\lambda-1}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda}$$

$$= \frac{1-e^{-(\lambda-1)t}}{\lambda-1}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} \qquad (22)$$

substitute (22) in (21), then we have

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} \le L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}\{\frac{1-e^{-(\lambda-1)T}}{\lambda-1}\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} + q^2\|e_k(x,\tau)\|_{L^2,\lambda}\},$$

Let λ be large enough, then $L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}\frac{1-e^{-(\lambda-1)T}}{\lambda-1} < 1$,

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda}$$

$$\le \frac{1-e^{-(\lambda-1)T}}{\lambda-1}\;\frac{L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}q^2}{1-L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}\frac{1-e^{-(\lambda-1)T}}{\lambda-1}}\|e_k(x,\tau)\|_{L^2,\lambda}$$

We remark $O(\lambda^{-2})\frac{1-e^{-(\lambda-1)T}}{\lambda-1}\frac{L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}q^2}{1-L_f e^{2L_fT}\frac{1-e^{\lambda T}}{\lambda}\frac{1-e^{-(\lambda-1)T}}{\lambda-1}}$, then get

$$\left\|\frac{\partial(\delta Q_k(x,t))}{\partial t}\right\|_{L^2,\lambda} \le O(\lambda^{-2})\|e_k(x,\tau)\|_{L^2,\lambda} \qquad (23)$$

Obviously, when λ is large enough, the $O(\lambda^{-2})$ is arbitrary small.

Substitute (23) in (10), we have

$$\|e_{k+1}(x,t)\|_{L^2,\lambda} \le \{(1+\varepsilon)\rho^2 + (1+\frac{1}{\varepsilon})L_g^2 O(\lambda^{-2})\}\|e_k(x,\tau)\|_{L^2,\lambda}$$

By (6), we have $(1+\varepsilon)\rho^2 < 1$, so when λ is large enough, then

$$(1+\varepsilon)\rho^2 + \left(1+\frac{1}{\varepsilon}\right)L_g^2 O(\lambda^{-2}) < 1$$

So according to contracting mapping principle, we know

$$\lim_{k\to\infty}\|e_k(x,t)\|_{L^2,\lambda} = 0$$

So $\lim_{k\to\infty}\|e_k(x,t)\|_{L^2,s} = 0$

QED.

3 SIMULATION NUMERICAL EXAMALE

For system (2), we remark

$$Q(x,t) = [Q_1, Q_2, \cdots Q_n]^T,$$

take $D = I_n, h(t) = 2.5sint + 3.5$,

$$g(t,Q(x,t)) = t\sum_{i=1}^{n}\cos Q_i, f(t,Q(x,t),u(x,t)) = [\cos Q_q + t\sin u\cos Q_2 + t\sin u \cdots \cos Q_n + t\sin u]^T,$$

then $L_g = L_f = \sqrt{n}T, \alpha_1 = 1, \alpha_2 = 6$. Take $\varepsilon = \frac{7}{9}$,

we have $6 = \frac{\alpha_2}{\alpha_1} < \frac{\sqrt{1+\varepsilon}+1}{\sqrt{1+\varepsilon}-1} = 7$, then we can proof (4) is right.

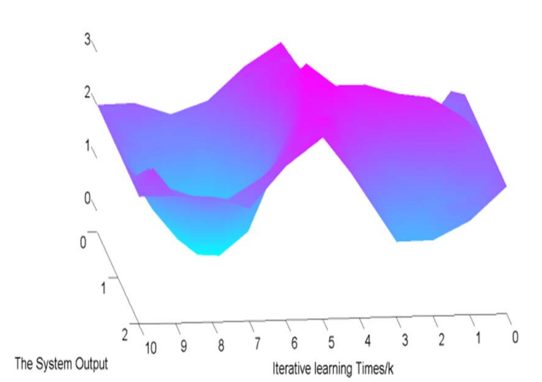

Fig. 1 The output after 20 times iterative output

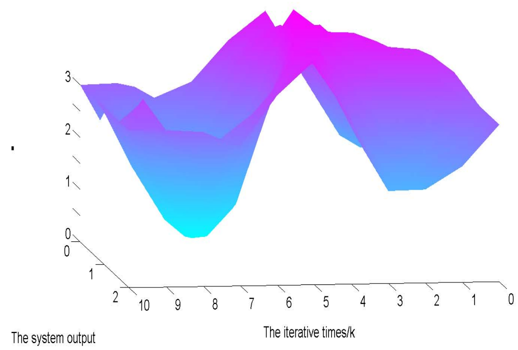

Fig. 2 The output after 10 times iterative output

Compare with iterative learning control algorithm for second order strong hyperbolic distributed parameter systems, with the increase of iterations, the digital simulation displays that this system has good control performance.

4 CONCLUSIONS

The issues of strong hyperbolic distributed parameter systems for iterative learning control have been studied and results have been presented. Current we focused on the parabolic system, but the paper studied the strong hyperbolic(second order) control problem. The control approach of the paper not only expands the applied range in distributed parameter systems but also has important theory and practical significance for iterative learning control

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