Logarithmic Error Approach to D-type Iterative Learning Control for SISO-Bilinear Systems

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Abstract: Bilinear sysetms is a nonlinear systems. This paper studies the iterative learning control (ILC) problem for a class of single input and single output (SISO) bilinear system. A D-type iterative learning algorithm is designed based on the expression of system solution. And then the convergence of ILC algorithm is proved by introducing iterative error with logarithmic form. The simulation results show the effectiveness of the proposed algorithm.

Key Words: Bilinear Systems, Iterative Learning Control, Convergence, Logarithmic Error

1 Introduction

A great deal of achievements concerning iterative learning control (ILC) has been obtained, since the idea of ILC first proposed by Arimoto in 1984 (see Ref.[1]) and attracted high attention of numerous scholars. As we all know, ILC has been applied to a wide range of systems, which perform repetitive motion in a finite time (see Ref.[2, 3]), such as repeated track the desired trajectory, iterative learning control method can be used to implement effective control for system. Learning controller of ILC has a simple structure and is applicable to the controlled object of nonlinear or uncertain system. With increasing iteration number, its performance would be better, need not exactly known the information of systmes, and the learning process could proceed both online and off line. The control method could adjust the current track by using previous learning information through learning controller's successive operation. Due to its structural simplicity and effective learning ability in the process of controller design, ILC has become an important branch of intelligent control, which has been widely used in industries for control of repetitive motions. Nowadays, iterative learning control has made great new progress, such as in Ref.[4] high-order nonlinear multi-agent systems based a new distributed adaptive iterative learning control scheme is studied. In Ref.[5], it proposed a non-model based approach to ILC via extremum seeking. In Ref.[6], the optimality and flexibility for varying tasks of ILC is considered. In Ref.[7],it explored P-type iterative learning control law with initial state learning for impulsive differential equations and so on.

Study of Bilinear systems (BLS) (see Refs.[8, 9]) could date back to the $1960s$. With object of study getting more complicated in practical problems that linear control model can not meet the requirements, BLS came into being, which could be able to demonstrate nonlinear characteristic. BLS have a simple structure and is most similar to linear systems, which makes it have advantages in terms of controllability and optimization. It is linear for state and control respectively, while is nonlinear on the whole (see Refs.[10, 11]), which is the greatest hallmark of BLS. For these characteristics and background of practical application, BLS became a research hot spot in the control area, and provided theoretical supports and practical application for further development (see Ref.[12]). Such as in Ref.[13], a class of bilinear systems with a single control where the input matrix is rank on is considered. In Ref.[14], it dealt with the problem of trajectory tracking of a class of bilinear systems with time-varying measurable disturbance. In Ref.[15], it considered a control system whose control cannot be implemented exactly but is shifted by a time independent constant in a discrete list of possibilities. In recent years, many researches also have focused on robustness of generalized control systems, controllability of discrete systems, and optimal control algorithm of quantum systems and so on. For instance, in Refs[16, 17] controllability of continuous bilinear systems and discrete systems has been discussed. In Ref.[18] has described the research of state feedback stabilization controller of generalized bilinear systems. In Refs.[19, 20], optimal bilinear control of nonlinear equation is studied. Robust stabilization of generalized bilinear systems with uncertain parameters has been studied in Refs.[21, 22].

The advantage of ILC is that solving problems which are hard to be solved by conventional control such as nonlinear control, ILC has simple algorithm and can achieve the purpose of optimization by simple iteration.It is well known that BLS is a kind of simple nonlinear system. And technically, systems of practical engineering are all nonlinear, so it is very necessary to study iterative learning control. For example, in biological reproductive process, state variable x indicates the number of organisms population, and control variable u represents net reproductive rate controlled artificially, then breeding process of biomass can be described by a bilinear system in form of $\dot{x}(t) = ux$. Looking forward

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to the future, a large number of topics remain to do further research in the field of ILC for BLS.

However, bilinear systems control theory remains in research and developmental stage, much work is yet to be further done by researchers, especially in terms of using iterative learning control to study convergence problem of bilinear systems. In this paper, iterative learning method has been used in this paper to study SISO BLS. A D-type iterative learning algorithm has been given, which could make the controlled object of SISO BLS track the desired trajectory precisely in a finite time.

2 Preliminaries and Problem Formulation

Consider the following time invariant bilinear systems (see Ref.[6])

$$\dot{x}(t) = Ax(t) + u(t)Bx(t), x(0) = x_0 \in R^n \quad (1)$$

where $A \in R^{n\times n}$, $B \in R^{n\times n}$ are constant matrices. $x(\cdot) \in R^n$, $u(\cdot)\in R^n$ are the state and input variables, respectively. t denotes the time variable in [0,T]. x_0 is the initial condition.

Lemma 1[5]**.** If $AB = BA$, $x(0) = x_0 \in R^n$, then $e^{A+B} = e^A e^B$ and the solution of (1) holds as follows

$$x(t) = e^{\int_0^t (A+u(s)B)ds} x_0. \quad (2)$$

When the system (1) is operated during a finite time interval repetitively, the bilinear systems can be described as

$$\dot{x}_k(t) = Ax_k(t) + u_k(t)Bx_k(t), \quad (3)$$

where k denotes the iteration number.

In this paper, we considered simply case, that is, A, B, u take scalar repetitively, the SISO-bilinear systems can be described as

$$\dot{x}_k(t) = ax_k(t) + u_k(t)bx_k(t) \quad (4)$$

Similar as others systems, we find this desire control $u_d(t)$, used on ILC, such that output $y_k(t)$ to tracking the desire output $y_d(t)$, while iterative number $k \to \infty$. In order to this, we give the following assumptions throughout of the paper.

Assumption 1. The desired trajectory $x_d(t)$ is iteration invariant, and every operation begins at an identical initial condition i.e., $x_d(0)$=$x_k(0)$ for all k.

Assumption 2. For a given desired trajectory $x_d(t)$ there exists a desired control input $u_d(t)$ such that

$$\dot{x}_d(t) = ax_d(t) + u_d(t)bx_d(t). \quad (5)$$

We consider the D-type ILC algorithm as follows:

$$u_{k+1}(t) = u_k(t) + \gamma\dot{e}_k(t). \quad (6)$$

where $e_k(t)$ is tracking error, and γ is a constant learning gain. In the following, we consider how to choose the learning gain γ, such that the convergence of the output tracking error can be guaranteed. When (4) runs repeatedly to design iterative learning controllers in a finite time interval [0,T], tracking error is defined as follows

$$e_k(t) = \ln\left[\frac{x_d(t)}{x_k(t)}\right]. \quad (7)$$

Remark 1. For linear system, the solution of $\dot{x}(t)$=$ax(t)$ is $x(t)$=$e^{at}x_0$=$\left[\sum_{n=1}^{\infty}\frac{(at)^n}{n!}\right]x_0$. For Lemma 1, if when $x_k(0)$= 0 and $x_d(0) = 0$, it can be obtained respectively $x_k(t) = 0$ and $x_d(t) = 0$. In this case system (4) can not be iterative learning process, so we don't consider this situation.

Remark 2. Note the iterative error (7), which is different from the aforementioned studies such as in Refs.[2, 23] that they usually assume $e_k(t) = x_d(t) - x_k(t)$. By the Lemma 1 $x(t) = e^{\int_0^t (a+u(s)b)ds}x_0$, $x_d(t) > 0$ and $x_k(t) > 0$ are deduced. From remark 1, $x_d(t) \neq$ and $x_k(t) \neq 0$ are obtained. According to the above two conditions, $\left[\frac{x_d(t)}{x_k(t)}\right] > 0$ is obtained. So the logarithmic error algorithm $e_k(t) = \ln\left[\frac{x_d(t)}{x_k(t)}\right]$ is well defined.

3 Convergence Analysis of Iterative Learning Control Algorithm

Now, we can give the following theorem.

Theorem 1. For the system (4), when the ILC algorithm (6) is used. Consider the Assumption 1 and 2 are satisfied, then the state tracking error is convergent that is $\lim_{k\to\infty} e_k(t) = 0$, $t \in [0, T]$. if following conditions hold,

1) $|1 - \gamma b| < 1$,

2) $x_k(0) = x_d(0)$.

Proof. From (7), we have

$$e_{k+1}(t) = \ln(x_d(t)/x_{k+1}(t)), \quad (8)$$

Based on (7) and (8), we can obtain

$$\begin{aligned} & e_{k+1}(t) - e_k(t) \\ = & \ln(x_d(t)/x_{k+1}(t)) - \ln(x_d(t)/x_k(t)) \\ = & \ln(x_k(t)/x_{k+1}(t)) \end{aligned} \quad (9)$$

hence

$$e_{k+1}(t) = e_k(t) + \ln\left[\frac{x_k(t)}{x_{k+1}(t)}\right]. \quad (10)$$

Based on (2) and Assumption 1, we have

$$\begin{aligned} \ln\left[\frac{x_k(t)}{x_{k+1}(t)}\right] &= \ln\left[\frac{e^{\int_0^t (a+u_k(s)b)ds}x_k(0)}{e^{\int_0^t (a+u_{k+1}(s)b)ds}x_{k+1}(0)}\right] \\ &= \ln\left[\frac{e^{\int_0^t (a+u_k(s)b)ds}}{e^{\int_0^t (a+u_{k+1}(s)b)ds}}\right] \\ &= \ln e^{\int_0^t (u_k(s)-u_{k+1}(s))bds} \\ &= -\int_0^t [u_{k+1}(s) - u_k(s)]bds \\ &= -\int_0^t \gamma\dot{e}_k(s)bds \\ &= -\gamma b\left[e_k(t) - e_k(0)\right]. \end{aligned}$$

From (7), we have $e_k(0)$=$\ln\left[\frac{x_d(0)}{x_k(0)}\right]$. According to $x_d(0)$ =$x_k(0)$ of Theorem 1 condition, we can have $e_k(0)$=0. Therefore

$$\ln\left[\frac{x_k(t)}{x_{k+1}(t)}\right] = -\gamma be_k(t) \quad (11)$$

substituting (9) into (11), we can obtain

$$\begin{aligned} e_{k+1}(t) &= e_k(t) + \ln\left[\frac{x_k(t)}{x_{k+1}(t)}\right] \\ &= e_k(t) - \gamma b e_k(t) \\ &= (1-\gamma b) e_k(t). \end{aligned} \quad (12)$$

Now, we prove the convergence of $| e_k(t) |$. From (12), the following inequality can be obtained:

$$|e_{k+1}(t)| = |(1-\gamma b)e_k(t)| \le |1-\gamma b||e_k(t)|. \quad (13)$$

Let $|(1-\gamma b)|=\alpha$, and two side of (13) take $\sup_{0\leqslant t\leqslant T}$ norm, it can be obtained

$$\|e_{k+1}\| \le \alpha \|e_k\|. \quad (14)$$

where $\|e_k\| = \sup_{0\leqslant t\leqslant T}\{e_k(t)\}$.
Furthermore,

$$\|e_{k+1}\| \le \alpha^k \|e_1\|. \quad (15)$$

Based on $0 < \alpha < 1$, we can obtain

$$\lim_{k\to\infty} e_k(t) = 0, t \in [0, T].$$

. The proof of Theorem 1 is complete.

4 Simulation

In this section, an example is used to verify our conclusion. Let us consider the SISO-bilinear systems as follows

$$\dot{x}(t) = ax(t) + ubx(t) \quad (16)$$

where $t \in [0, 1]$, $a = 1$, $b = 1$.
Example 1. $x_k(0) > 0$. The desired repetitive reference trajectory is $x_{d1}(t) = 12t^2(1-t) + 1.8$ and $t \in [0, 1]$. The initial condition are given as $x_k(0) = 0.1$ for all k. The ILC scheme is constructed as $u_{k+1}(t) = u_k(t) + 0.22\dot{e}_k(t)$. Using Theorem 1, we have $| 1-\gamma b |= 0.78 < 1$. The convergence condition in Theorem 1 is satisfied. The simulation results showed in Fig.1-Fig.2.
Example 2.$x_k(0) < 0$.The desired repetitive reference trajectory is $x_{d2}(t) = -1.5t - 1.8$ and $t \in [0, 1]$. The initial condition are given as $x_k(0) = -1.8$ for all k. The simulation results showed in Fig.3-Fig.4.

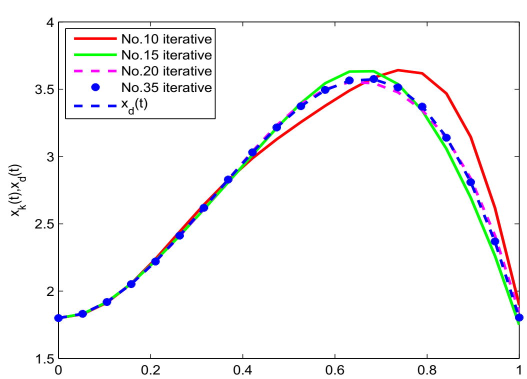

Fig. 1: Actual output and desire output curve $x_1(t)$

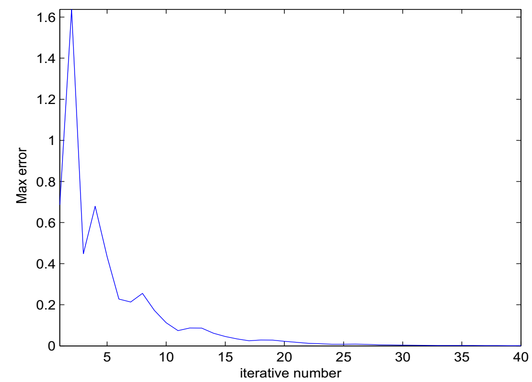

Fig. 2: Maximum error-iterative number curve $x_1(t)$

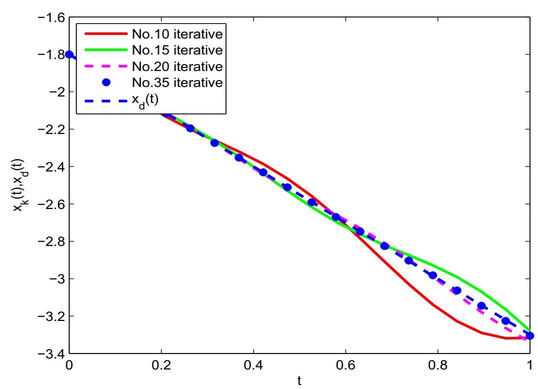

Fig. 3: Actual output and desire output curve $x_2(t)$

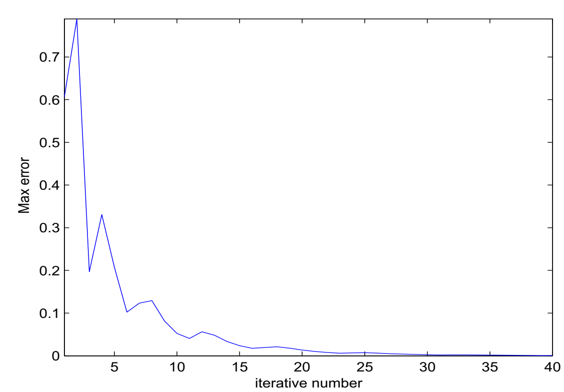

Fig. 4: Maximum error-iterative number curve $x_2(t)$

Fig.1 and Fig.3 are different iterative numbers output and desired curve, respectively. Fig.2 and Fig 4 show the convergence of iteratively error on the iteration domain. In numerical,maximum error are 3.5088×10^{-3},2.9433×10^{-3} at iterative thirty times, for $x_1(t), x_2(t)$,respectively.

5 Conclusions

The problem of iterative learning control for SISO-bilinear systems has been discussed. It has been shown that under some conditions, the D-type ILC algorithm can guarantee the asymptotic convergence of the output tracking error between the given desired track and the actual curve through the iterative learning process.The theoretical analysis is supported by simulations. It must be noted that this proposed method may be only suitable that desire curve is definite positive or definite negative. In the future work, we will consider multi-input multi-output bilinear systems.

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