High Order Internal Model Iterative Learning Control for Impulsive Differential Systems

Zijian Luo[1], Jinrong Wang[1], Wei Wei[1,2]
1. College of Science, GuiZhou University, Guiyang, 550025
E-mail: sci.jrwang@gzu.edu.cn
2. Guizhou Minzu University, Guiyang, 550025, China
E-mail: wwei@gzu.edu.cn

Abstract: In this paper, we design high-order internal models learning law for impulsive differential systems to track the varying reference accurately by adopting a few iterations in a finite time interval. We present convergence analysis for the P-type and D-type updating law. Finally, two examples are given to illustrate our theoretical results.

Key Words: high-order internal mode, iterative learning control, varying reference trajectory, impulsive differential systems

1 Introduction

Since Uchiyama [1] and Arimoto [2, 3] put forward the iterative learning control (ILC for short), ILC has been extended to tracking tasks with iteratively varying reference trajectories [4, 5, 6, 7] and fractional ILC issues [8, 9, 10, 11]. It should be noted that the contribution of [12] pointed out that how to deal with iteration-varying factors became a focus of ILC research. To deal with iteratively varying reference trajectories, high-order internal models (HOIM for short) is an effective method. A method (HOIM) can be formulated as a polynomial, which will help ILC with HOIM able to complete various tasks, such as XY-table, robotic manipulator and so on. It is remarkable that Liu et al. [13] applied the learning law with HOIM, and gave the convergence analysis. His work has important implications for our research.

Impulsive phenomenon can be understood as a kind of instantaneous mutation. In modern science and technology, the practical problems in various fields are ubiquitous. The mathematical model can be attributed to impulsive differential systems [14, 15, 16, 17], which can reflect the changes of things more deeply and more accurately. Hence, it is necessary for us to study the control systems with impulsive terms.

In this work, we explore the extension of HOIM for impulsive differential systems based on the paper [13], and provide the convergence analysis of ILC with HOIM for impulsive differential systems. We try to design some ILC law to generate the control input $u_k(\cdot)$ such that the impulsive system output $y_k(\cdot)$ tracks the iteratively varying reference trajectories $r_k(\cdot)$ as accurately as possible as $k \to \infty$ for a.e. $t \in [0,T]$ in the sense of λ-norm.

This work is partially supported by Natural Science Foundation of China (11261011,11201091), Unite Foundation of Guizhou Province ([2015]7640) and Outstanding Scientific and Technological Innovation Talent Award of Education Department of Guizhou Province ([2014]240).

The rest of this paper is organized as follows. In Section 2, we present problem formulation and lemmas. In Section 3, we give result of P-type ILC for impulsive differential systems. In Section 4, we give result of D-type ILC. Examples are given in Section 5 to demonstrate the application of our main results.

2 Problem formulation

Throughout this paper, let $\mathcal{PC}([0,T],R^n)$ be the space of all piecewise continuous continuous functions from $[0,T]$ into R^n endowed with λ norm: $\|x\|_\lambda = \sup_{t\in[0,T]} e^{-\lambda t}\|x(t)\|$, where $\lambda > 0$. Let $\|x\|$ denotes a vector norm such as the Euclidean norm for a n-dimensional vector x, and $\|E(t)\| = \max_{t\in[0,T]} \sum_{1\le i,j\le n} |e_{i,j}(t)|$ denotes a norm for a matrix function $E(t) = (e_{i,j}(t))$, where $e_{i,j} \in \mathcal{C}([0,T],R)$.

Motivated by [13, Definition 2], we consider the iteratively varying reference trajectories with HOIM:

$$r_{k+1} = H(w^{-1})r_k, \tag{1}$$

where r_{k+1} the $k+1$-times iteratively varying reference trajectories, $H(w^{-1}) = H_1 + H_2 w^{-1} + \cdots + H_m w^{-m+1}$, $H_j = h_j I_{n\times n}$, $j = 1, \cdots m$ are diagonal matrices, h_j are coefficients of a stable polynomial (including marginal stability)

$$1 - h_1 w^{-1} - h_2 w^{-2} - \cdots - h_m w^{-m}, \tag{2}$$

where function of factor w^{-1} is $w^{-1} r_{k+1} = r_k$.

From (1) and (2), the reference trajectories with the HOIM can be expressed in a matrix form

$$r_{k+1}(t) = H_1 r_k(t) + \cdots + H_m r_{k-m+1}(t). \tag{3}$$

Obviously, m initial trajectories $r_0(t), \cdots, r_{1-m}(t)$ are required to determine the regressor. The condition for initial states required by HOIM-based ILC is $x_{k+1}(0) =$

978-1-4673-9714-8/16/$31.00

$H(w^{-1})x_k(0)$. When $H(w^{-1}) = 1$, it means that $x_{k+1}(0) = x_k(0),\ k = 1, 2, \cdots$.
In this paper, we mainly consider impulsive systems

$$\begin{cases} \dot{x_k}(t) = A(t)x_k(t) + B(t)u_k(t), \\ \quad t \in [0,T]\backslash\{t_1, \cdots, t_N\}, \\ x_k(t_j^+) - x_k(t_j^-) = I_j(x_k(t_j^-)),\ j = 1, 2, \cdots, N, \\ y_k(t) = C(t)x_k(t) + D(t)u_k(t), \end{cases} \quad (4)$$

As for (4), the subscript k denotes the kth learning iteration, $t \in [0,T]$, $A(\cdot), B(\cdot), C(\cdot), D(\cdot) \in \mathcal{C}([0,T], R^{n\times n})$, and both of them are uniform bounded. $I_j : R^{n\times 1} \to R^{n\times 1}$ is Lipschitz continuous with the constant $L_j > 0$ and I_j has linear property. Fixed time t_j satisfying $0 = t_0 < t_1 < \cdots < t_N < t_{N+1} = T$, the symbols $x(t_j^+) := \lim_{\epsilon\to 0^+} x(t_j + \epsilon)$ and $x(t_j^-) := \lim_{\epsilon\to 0^-} x(t_j + \epsilon)$ represent the right and left limits of $x(t)$ at $t = t_j$ respectively. The vectors $x_k(\cdot), y_k(\cdot)$ and $u_k(\cdot) \in R^{n\times 1}$, $x_k(\cdot)$, $u_k(\cdot)$ and $y_k(\cdot)$ are state, input and output vectors, respectively. Let $r_k(t)$ be the iteratively varying reference trajectories and $e_k(t) = r_k(t) - y_k(t)$ be the tracking error. For the systems (4), we consider P-type ILC law with the mth-order internal model(see [13, Formula (7)])

$$\begin{aligned} u_{k+1}(t) &= H(w^{-1})u_k + \Gamma(w^{-1})e_k \\ &= H_1u_k(t) + + \cdots + H_mu_{k-m+1}(t) \\ &\quad + \Gamma_1e_k(t) + + \cdots + \Gamma_me_{k-m+1}(t) \quad (5) \end{aligned}$$

$\Gamma(w^{-1}) = \Gamma_1 + \Gamma_2w^{-1} + \cdots + \Gamma_mw^{-m+1}$, Γ_j be the diagonal learning gain matrices.
Next, we still consider the impulsive systems (4) with $D(t) = 0$, we consider D-type ILC law with the mth-order internal model(see [13, Formula (17)])

$$\begin{aligned} u_{k+1}(t) &= H_1u_k(t) + \cdots + H_mu_{k-m+1}(t) \\ &\quad + \Gamma_1\dot{e}_k(t) + \cdots + \Gamma_m\dot{e}_{k-m+1}(t) \quad (6) \end{aligned}$$

or $u_{k+1} = H(w^{-1})u_k + \Gamma(w^{-1})\dot{e}_k$.

Lemma 2.1. *(see [18]) Let for $t \geq 0$, the following inequality hold*

$$x(t) \leq a(t) + b\int_0^t x(s)ds + \sum_{0<t<t_k} \zeta_k x(t_k),$$

where $x, a \in PC\big([0,\infty], R^+\big)$, and a is nondecreasing and $b, \zeta_k > 0$. Then, for $t \geq 0$, the following inequality is valid:

$$x(t) \leq a(t) \prod_{0<t<t_k} (1 + \zeta_k)e^{bt}.$$

Lemma 2.2. *(see [19, Lemma 3]) Let $\{a_k\}$ be a real sequence defined as $a_k \leq p_1a_{k-1} + p_2a_{k-2} + \cdots + p_ma_{k-m} + d_k$, $k > m$, where d_k is a specified real sequence, $\overline{d}$ is constant. If $p_1, p_2, \cdots, p_m$ are nonnegative numbers satisfying $\sum_{j=1}^m p_j < 1$. Then $\limsup_{k\to\infty} d_k \leq \overline{d}$ implies that*

$$\limsup_{k\to\infty} a_k \leq \frac{\overline{d}}{1 - \sum_{j=1}^m p_j}.$$

3 Convergence analysis of P-type

At beginning, we impose the following assumptions:
(A_1) $x_k(0) = x_d(0)$.
(A_2) $I_j : R^{n\times 1} \to R^{n\times 1},\ j = 1, \cdots N$ is Lipschitz continuous and I_j has linear property. It means that there is the constant $L_j > 0$ such that

$$\|I_j(x) - I_j(\hat{x})\| \leq L_j\|x_k - \hat{x}\|$$
$$and$$
$$I_j(H(w^{-1})x) = H(w^{-1})I_j(x).$$

for any $x,\ \hat{x} \in R^n$ and all $t \in [0,T]$.
(A_3) Learning gains Γ_j are chosen such that for $\sum_{j=1}^m \eta_j < 1$ where $\eta_j := \|H_j - D(t)\Gamma_j\|,\ j = 1, 2, \cdots, m$. Now we are ready to present the main result.

Theorem 3.1. *For the system (4) and reference trajectories (1), assumptions $(A_1) - (A_3)$ hold, The P-type ILC law (5) guarantees that $\lim_{k\to\infty} \sup \|r_k - y_k\| \to 0$ in the sense of λ-norm.*

Proof. By [13, (9)], we have

$$\begin{aligned} \|e_{k+1}(t)\| &\leq \|H_1 - D(t)\Gamma_1\|\|e_k(t)\| + \cdots \\ &\quad + \|H_m - D(t)\Gamma_m\|\|e_{k-m+1}(t)\| \\ &\quad + \|C(t)\|\|x_{k+1}(t) - H(w^{-1})x_k(t)\|. \quad (7) \end{aligned}$$

Now we need to evaluate the term $x_{k+1} - H(w^{-1})x_k$ of (7). Let $\Delta x_k(t) := x_{k+1}(t) - H(w^{-1})x_k(t)$. For $t \in [0,T]$, we have

$$\begin{aligned} \|\Delta x_k(t)\| &\leq \textstyle\int_0^t [\|A(t)\|\|x_{k+1}(s) - H(w^{-1})x_k(s)\| \\ &\quad + \|B(t)\|\|u_{k+1}(s) - H(w^{-1})u_k(s)\|]ds \\ &\quad + \textstyle\sum_{0<t_j<t} \|I_j(x_{k+1}(t_j^-)) - H(w^{-1})I_j(x_k(t_j^-))\|. \quad (8) \end{aligned}$$

According to (A_2), we could obtain

$$\begin{aligned} &\|I_j(x_{k+1}(t_j^-)) - H(w^{-1})I_j(x_k(t_j^-))\| \\ \leq\ & L_j\|x_{k+1}(t_j^-) - H(w^{-1})x_k(t_j^-)\|. \quad (9) \end{aligned}$$

Submitting (9) to (8), we obtain

$$\begin{aligned} \|\Delta x_k(t)\| &\leq \int_0^t \|B(t)\|\|u_{k+1}(s) - H(w^{-1})u_k(s)\|ds \\ &\quad + \int_0^t \|A(t)\|\|x_{k+1}(s) - H(w^{-1})x_k(s)\|ds \\ &\quad + \sum_{0<t_j<t} L_j\|x_{k+1}(t_j^-) - H(w^{-1})x_k(t_j^-)\|. \end{aligned}$$

Using Lemma 2.1, the above inequality becomes:

$$\begin{aligned} \|\Delta x_k(t)\| &\leq \|B(t)\| \prod_{0<t_k<t} (1 + L_j)e^{\|A(t)\|t} \\ &\quad \times \int_0^t \|u_{k+1}(s) - H(w^{-1})u_k(s)\|ds. \end{aligned}$$

Linking with (5), we get

$$\|\Delta x_k(t)\| \leq \|B(t)\| \prod_{0<t_k<t} (1 + L_j)e^{\|A(t)\|t}$$

$$\times \int_0^t \|\Gamma(w^{-1})e_k(s)\|ds$$
$$\leq \|B(t)\| \prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t} \times \int_0^t [\|\Gamma_1\|\|e_k(s)\| + \cdots + \|\Gamma_m\|\|e_{k-m+1}(s)\|]ds. \quad (10)$$

Liking (7) and (10) we get

$$\begin{aligned}\|e_{k+1}(t)\| \leq\ & \|H_1 - D(t)\Gamma_1\|\|e_k(t)\| + \cdots \\ +\ & \|H_m - D(t)\Gamma_m\|\|e_{k-m+1}(t)\| \\ +\ & \rho_1 \prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t}\int_0^t \|e_k(s)\|ds \\ +\ & \cdots + \rho_m \prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t} \\ \times\ & \int_0^t \|e_{k-m+1}(s)\|ds,\end{aligned}$$

where $\rho_j = \|C(t)\|\|B(t)\|\|\Gamma_j\|, j = 1, 2, \cdots, m$. Thus,

$$\begin{aligned}\|e_{k+1}(t)\| \leq\ & \|H_1 - D(t)\Gamma_1\|\|e_k(t)\| + \cdots \\ +\ & \|H_m - D(t)\Gamma_m\|\|e_{k-m+1}(t)\| \\ +\ & \rho_1\|e_k\|_\lambda \frac{e^{\lambda t}}{\lambda} \prod_{0<t_k<t}(1+L_j)e^{\|A(s)\|t} \\ +\ & \cdots + \rho_m\|e_{k-m+1}\|_\lambda \frac{e^{\lambda t}}{\lambda} \\ \times\ & \prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t}, \quad (11)\end{aligned}$$

where we use the fact

$$\int_0^t \|e_k(s)\|ds \leq \|e_k\|_\lambda \frac{e^{\lambda t}}{\lambda}.$$

So, we take λ-norm for inequality (11), we obtain

$$\begin{aligned}\|e_{k+1}\|_\lambda \leq\ & (\eta_1+\delta_1)\|e_k\|_\lambda + (\eta_2+\delta_2)\|e_{k-1}\|_\lambda \\ & + \cdots + (\eta_m+\delta_m)\|e_{k-m+1}\|_\lambda, \quad (12)\end{aligned}$$

where

$$\delta_i = \frac{\rho_i}{\lambda}\prod_{0<t_k<T}(1+L_j)e^{\|A(t)\|T}, i = 1, 2, \cdots, m.$$

Since δ_i are arbitrarily small with sufficiently large λ and $\sum_{j=1}^m \eta_j < 1$, by Lemma 2.2 one can complete the proof. □

In the following sequels, we consider one possible extension of HOIM-based ILC.

We consider impulsive systems with integrator

$$\begin{cases} \dot{x_k}(t) = A(t)x_k(t) + B(t)u_k(t) + p(t), \\ \quad t \in [0,T]\backslash\{t_1, \cdots, t_N\}, \\ x_k(t_j^+) - x_k(t_j^-) = I_j(x_k(t_j^-)),\ j = 1, 2, \cdots, N, \\ y_k(t) = C(t)x_k(t) + D(t)u_k(t) + d(t), \end{cases} \quad (13)$$

where $p(t)$ denotes iteration-invariant state disturbances and $d(t)$ denotes output disturbances.

Like [13], we set

$$\begin{aligned} H_a(w^{-1}) &= (I+H_1) + (H_2 - H_1)w^{-1} + \cdots \\ &+ (H_m - H_{m-1})w^{m-1} - H_m w^{-m}, \\ \Gamma_a(w^{-1}) &= \Gamma_1 + \Gamma_2 w^{-1} + \cdots + \Gamma_m w^{-m+1}. \end{aligned}$$

We take P-type ILC law that employs the mth-order internal model like

$$u_{k+1} = H_a(w^{-1})u_k + \Gamma_a(w^{-1})e_k. \quad (14)$$

From [13, Formula (24)], we know that $w^{-1}d(t) = d(t)$ and $H_a(w^{-1})d(t) = d(t)$.

Theorem 3.2. *For the system (13) and reference trajectories (3), apply the P-type ILC law (14). Assume that assumptions$(A_1) - (A3)$ hold, The P-type ILC law (14) guarantees that* $\lim_{k\to\infty}\sup\|r_k - y_k\| \to 0$ *in the sense of λ-norm.*

Proof. According to [13] and the property $H_a(w^{-1})d(t) = d(t)$, the output tracking errors can be expressed as

$$\begin{aligned} e_{k+1}(t) &= H_a(w^{-1})e_k(t) \\ &- D(t)[u_{k+1} - (t)H_a(w^{-1})u_k(t)] \\ &- C(t)[x_{k+1}(t) - H_a(w^{-1})x_k(t)]. \end{aligned}$$

Then,

$$\begin{aligned} & \|e_{k+1}(t)\| \\ =\ & \|H_a(w^{-1})e_k(t) - D(t)[u_{k+1}(t) - H_a(w^{-1})u_k(t)] \\ & -C(t)[x_{k+1}(t) - H_a(w^{-1})x_k(t)]\| \\ \leq\ & \|H_a(w^{-1})e_k(t)\| + \\ & \|D(t)\|\|u_{k+1}(t) - H_a(w^{-1})u_k(t)\| \\ & +\|C(t)\|\|x_{k+1}(t) - H_a(w^{-1})x_k(t)\|. \end{aligned}$$

Set $\Delta x_k(t) := x_{k+1}(t) - H_a(w^{-1})x_k(t)$, $\Delta u_k(t) := u_{k+1}(t) - H_a(w^{-1})u_k(t)$. Analogous to Theorem 3.1 in deriving $\|\Delta x_k(t)\|$, we have

$$\begin{aligned} \|\Delta x_k(t)\| \leq\ & \textstyle\int_0^t [\|A(t)\|\|x_{k+1}(s) - H_a(w^{-1})x_k(s)\| \\ & +\|B(t)\|\|u_{k+1}(s) - H_a(w^{-1})u_k(s)\|]ds \\ & +\textstyle\sum_{0<t_j<t}\|I_j(x_{k+1}(t_j^-)) - H_a(w^{-1})I_j(x_k(t_j^-))\|, \end{aligned}$$

because $p - H_a(w^{-1})p = 0$.

One can make fundamental computation to derive that

$$\begin{aligned} & \|I_j(x_{k+1}(t_j^-)) - H_a(w^{-1})I_j(x_k(t_j^-))\| \\ \leq\ & L_j\|x_{k+1}(t_j^-) - H_a(w^{-1})x_k(t_j^-)\| \\ & +\|I_j(H_a(w^{-1})x_k(t_j^-)) - H_a(w^{-1})I_j(x_k(t_j^-))\|, \end{aligned}$$

and

$$\|I_j(H(w^{-1})x_k(t_j^-)) - H_a(w^{-1})I_j(x_k(t_j^-))\| = 0.$$

Therefore, we obtain

$$\|\Delta x_k(t)\| \leq \textstyle\int_0^t \|B(t)\|\|u_{k+1}(s) - H_a(w^{-1})u_k(s)\|ds$$

$$+\int_0^t \|A(t)\|\|x_{k+1}(s)-H_a(w^{-1})x_k(s)\|ds$$
$$+\sum_{0<t_j<t} L_j\|x_{k+1}(t_j^-)-H_a(w^{-1})x_k(t_j^-)\|.$$

As a consequence, the asymptotic learning convergence can be derived by following the same procedure in the proof of Theorem 3.1 with $H(w^{-1})$ replaced by $H_a(w^{-1})$ and $\Gamma(w^{-1})$ replaced by $\Gamma_a(w^{-1})$. □

Remark 3.3. *As for nonlinear impulsive differential system*

$$\begin{cases} \dot{x_k}(t)=f(t,x_k(t),u_k(t)),\ t\in[0,T]\backslash\{t_1,\cdots,t_N\},\\ x_k(t_j^+)-x_k(t_j^-)=I_j(x_k(t_j^-)),\ j=1,2,\cdots,N,\\ y_k(t)=C(t)x_k(t)+D(t)u_k(t), \end{cases}$$

If nonlinear term f satisfies Lipschitz condition, there is a positive constant $L_f>0$ such that

$$\begin{aligned} &\|f(t,x_{k+1}(t),u_{k+1}(t))\\ &-f(t,H(w^{-1})x_k(t),H(w^{-1})u_k(t))\|\\ \le\ & L_f\Big(\|x_{k+1}(t)-H(w^{-1})x_k(t)\|\\ &+\|u_{k+1}(t)-H(w^{-1})u_k(t)\|\Big), \end{aligned}$$

for each $t\in(t_j,\ t_{j+1}],\ j=1,2,\cdots N-1$, then we can use P-type ILC to study the convergence of the output of the system and obtain similar results.

4 Convergence analysis of D-type

In this section, we study systems (4) with $D(t)=0$, the matrix $C(t)B(t)\in\mathcal{C}([0,T],R^{q\times q})$ is of full rank, D-type ILC law (6) would be used. The learning convergence can be derived analogous to the P-type.

(A_4) Learning gains Γ_j are chosen such that for $\sum_{j=1}^m\eta_j<1$ where $\eta_j:=\|H_j-C(t)B(t)\Gamma_j\|,\ j=1,2,\cdots,m$ and $e_k(0)=0$.

Theorem 4.1. *For the system (4) with $D(t)=0$ and reference trajectories (3). Assumptions$(A_1)-(A2)$ and (A_4) hold, The D-type ILC law (6) guarantees that $\lim_{k\to\infty}\sup\|r_k-y_k\|\to 0$ in the sense of λ-norm.*

Proof. By [13, (18)] and the matrix $C(t)B(t)$ is of full rank implies that $H_i-C(t)B(t)\Gamma_i$ is well defined, we have

$$\begin{aligned} &\|e_{k+1}(t)\|\le\|H_1-C(t)B(t)\Gamma_1\|\|\dot{e}_k(t)\|\\ &+\cdots+\|H_m-C(t)B(t)\Gamma_m\|\|\dot{e}_{k-m+1}(t)\|\\ &+\|C(t)\|\|A(t)\|\|x_{k+1}(t)-H(w^{-1})x_k(t)\|. \end{aligned}\quad(15)$$

Let $\Delta x_k(t)\triangleq x_{k+1}(t)-H(w^{-1})x_k(t)$, and repeat the similar procedure of Theorem 3.1 via Lemma 2.1, one can obtain

$$\|\Delta x_k(t)\|\le\|B(t)\|\prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t}$$
$$\times\int_0^t[\|\Gamma_1\|\|\dot{e}_k(s)\|+\cdots+\|\Gamma_m\|\|\dot{e}_{k-m+1}(s)\|]ds.$$

Hence, we get

$$\|\dot{e}_{k+1}(t)\|\le\|H_1-C(t)B(t)\Gamma_1\|\|\dot{e}_k(t)\|+\cdots$$
$$+\|H_m-C(t)B(t)\Gamma_m\|\|\dot{e}_{k-m+1}(t)\|$$
$$+\rho_1\prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t}\int_0^t\|\dot{e}_k(s)\|ds+\cdots$$
$$+\rho_m\prod_{0<t_k<t}(1+L_j)e^{\|A(t)\|t}\int_0^t\|\dot{e}_{k-m+1}(s)\|ds,$$

where $\rho_j=\|C(t)\|\|A(t)\|\|B(t)\|\|\Gamma_j\|,\ j=1,2,\cdots,m$. Similar to the proof of Theorem 3.1 , we obtain

$$\begin{aligned}\|\dot{e}_{k+1}\|_\lambda\ &\le\ (\eta_1+\delta_1)\|\dot{e}_k\|_\lambda+(\eta_2+\delta_2)\|\dot{e}_{k-1}\|_\lambda\\ &+\ \cdots+(\eta_m+\delta_m)\|\dot{e}_{k-m+1}\|_\lambda,\end{aligned}\quad(16)$$

where

$$\delta_i=\frac{\rho_i}{\lambda}\prod_{0<t_k<T}(1+L_j)e^{\|A(t)\|T},\quad i=1,2,\cdots,m.$$

Comparing (16) with (12), we can see the analogy except for the substitution of quantity e_k with $\dot{e}_k$, and $D(t)$ by $C(t)B(t)$. The convergence of the sequence $\dot{e}_k$ can be concluded straightforward. As a result, the convergence of e_k is easy to obtained by using integration by parts. The proof is completed. □

5 Simulation examples

In this section, a numerical example is presented to demonstrate the validity of the designed method. For simplicity, the following example, we consider first order HOIM as $H(w^{-1})=-1$, it is used in the robot arm.

Example 4.1 Consider the following impulsive differential equations:

$$\begin{cases} \dot{x_k}(t)=\sin t\cdot x_k(t)+\sin(t+1)u_k(t),\\ \quad t\in[0,1]\backslash\{0.3,0.6\},\\ x_k(t_1^+)-x_k(t_1^-)=0.2x_k(t_1^-),\ t_1=0.3,\\ y_k(t)=x_k(t)+\cos tu_k(t), \end{cases}\quad(17)$$

and choose the P-type iterative learning control law as follows:

$$u_{k+1}(t)=-u_{k+1}(t)-0.8e_k(t).$$

The original reference trajectory is given as:

$$r(t)=\begin{cases}2\sin(2\pi t)t,\ t\in[0,0.3],\\ 2\sin(2\pi t)t+1,\ t\in(0.3,0.6],\\ 2\sin(2\pi t)t+2,\ t\in(0.6,1].\end{cases}$$

As for $t\in[0,1]$, we set $A(t)=sin(t)$, $B(t)=sin(t+1)$, $C(t)=1$, $D(t)=cos(t)$, $I_1(x_k(t_1^-))=0.2x_k(t_1^-),\ t_1=0.3$. Obviously, $L_1=0.2$. Set $H_1=-1$ and $\Gamma_1=-0.8$. It is not difficult to verify that $|H_1-D(t)\Gamma_1|<1$, $(A_1)-(A_3)$ are satisfied. Obviously, All the conditions of Theorem 3.1 are satisfied, then $\lim_{k\to\infty}\sup\|r_k-y_k\|\to 0$.

The upper figure of Figure 1 shows the system (17) output y_k of the 10th iterations and the reference trajectory y_d. The lower figure of Figure 1 shows the L^2-norm of the tracking error in each iteration.

Example 5.2 Consider the following differential equations:

$$\begin{cases} \dot{x_k}(t)=e^tx_k(t)+\sin(t)u_k(t),\ t\in[0,1]\backslash\{0.5\},\\ x_k(t_1^+)-x_k(t_1^-)=0.02x_k(t_1^-),\ \ t_1=0.5,\\ y_k(t)=1.2x_k(t), \end{cases}\quad(18)$$

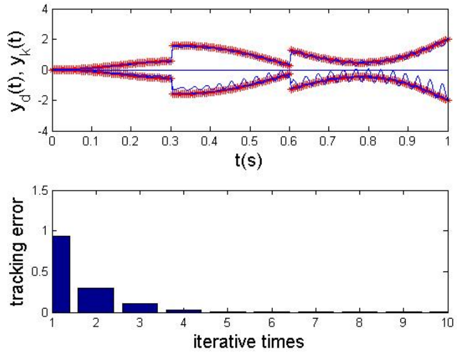

Figure 1: The system output and the tracking error.

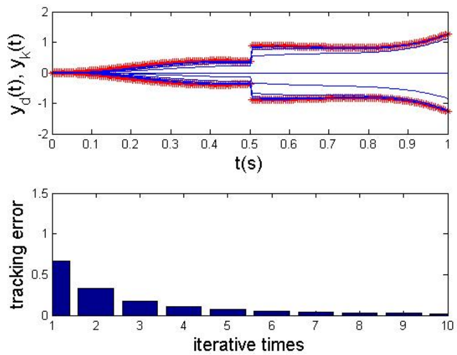

Figure 2: The system output and the tracking error.

and choose the D-type iterative learning control law as follows:

$$u_{k+1}(t) = -u_{k+1}(t) - 6.5\dot{e}_k(t).$$

The original reference trajectory is given as:

$$r(t) = \begin{cases} 12t^2(0.85-t)^2,\ t \in [0, 0.5], \\ 12t^2(0.85-t)^2 + t,\ t \in (0.5, 1]. \end{cases}$$

As for $t \in [0, 1]$, we set $A(t) = e^t$, $B(t) = sin(t)$, $C(t) = 1.2$, $D(t) = 0$, $I_1(x_k(t_1^-)) = (-1)^k(0.02x_k(t_1^-) + 0.05)$, $t_1 = 0.5$. Obviously, $L_1 = 0.02$. Set $H_1 = -1$ and $\Gamma_1 = -0.65$. It is not difficult to verify that $|H_1 - C(t)B(t)\Gamma_1| < 1$, $(A_1) - (A_2)$ and (A_5) are satisfied. Obviously, All the conditions of Theorem 4.1 are satisfied, then $\lim_{k\to\infty} \sup \|r_k - y_k\| \to 0$.
The following figure of Figure 2 shows the system (18) output y_k of the 10th iterations and the reference trajectory y_d. The lower figure of Figure 2 shows the L^2-norm of the tracking error in each iteration.

REFERENCES

[1] M. Uchiyama, Formulation of high-speed motion pattern of a mechanical arm by trial, Transactions of the Society of Instrument and Control Engineers, Vol.14, No.6, 706-712, 1978.

[2] S. Arimoto, S. Kawamura, Bettering operation of robots by learning, Journal of Robotic system, Vol.1, No.2, 123-140, 1984.

[3] S. Arimoto, Mathematical theory of learning with applications to robot control, Adaptive and Learning Systems, Springer, US, 379-388, 1986.

[4] S. S. Saab, W. G. Vogt, M. H. Mickle, Learning control algorithm for tracking slowly varying trajectories, IEEE Transactions on Systems, Man and Cybernetics, Part B, Vol.27, No.4, 657-670, 1997.

[5] Y. Q. Chen, K. L. Moore, Harnessing the nonrepetitiveness in iterative learning control, Proceedings of 41st IEEE conference on Decision and Control, Las Vegas, NV, US, Vol.3, 3350-3355, 2002.

[6] J. X. Xu, J. Xu, On iterative learning for different tracking tasks in the presence of time-varying uncertainties, IEEE Transactions on Systems, Man and Cybernetics Part B, Vol.34, No.1, 589-597, 2004.

[7] H. S. Ahn, K. L. Moore, Y. Q. Chen, Iterative learning control: robustness and monotonic convergence in the iteration domain, Springer Science and Business Media, Springer, Berlin, 2007.

[8] Y. Li, Y. Q. Chen, H. S. Ahn, Fractional-order iterative learning control for fractional-order linear systems, Asian Journal of Control, Vol.13, No.1, 54-63, 2011.

[9] Y. Li, Y. Q. Chen, H. S. Ahn, G. Tian, A survey on fractional-order iterative learning control, Journal of Optimization Theory and Applications, Vol.156, No.1, 127-140, 2013.

[10] Y. H. Lan, Y. Zhou, Iterative learning control with initial state learning for fractional order nonlinear systems, Computers and Mathematics with Applications, Vol.64, No.10, 3210-3216, 2012.

[11] Y. H. Lan, Y. Zhou, D-type iterative learning control for fractional order linear time-delay systems, Asian Journal of Control, Vol.15, No.3, 669-677, 2013.

[12] K. L. Moore, Y. Q. Chen, H. S. Ahn, Iterative learning control: Atutorial and big picture view, Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, US, 2352-2357, 2006.

[13] C. Liu, J. X. Xu, J. Wu, On iterative learning control with high-order internal models, 2009 IEEE International Conference on Control and Automation, Vol.24, 731-742, 2009.

[14] L. Hetel, J. Daafouz, S. Tarbouriech, C. Prieur, Stabilization of linear impulsive systems through a nearly-periodic reset, Nonlinear Analysis: Hybrid Systems, Vol.7, No.1, 4-15, 2013.

[15] P. Chen, Y. Li, H. Yang, Perturbation method for nonlocal impulsive evolution equations, Nonlinear Analysis: Hybrid Systems, Vol.8, 22-30, 2013.

[16] T. Fang, J. Sun, Stability of complex-valued impulsive and switching system and application to the Lü system, Nonlinear Analysis: Hybrid Systems, Vol.14, 38-46, 2014.

[17] S. Liu, J. Wang, W. Wei, A Study on iterative learning control for impulsive differential systems, Communications in Nonlinear Science and Numerical Simulation, Vol.24, No.1, 4-10, 2015.

[18] A. M. Samoilenko, N. A. Perestyuk, Stability of solutions of differential equations with impulse effect, Differential'nye Uravneniya, Vol.13, 1981-1992, 1997.

[19] X. Ruan, Z. Bien, Pulse compensation for PD-type iterative learning control against initial state shift, International Journal of Systems Science, Vol.43, No.11, 2172-2184, 2012.