Iterative Learning Control for Switched Linear Parabolic Systems in Space $W^{1,2}$

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Abstract: This paper deals with the problem of iterative learning control for a class of switched linear parabolic systems in space $W^{1,2}$. Here, the considered switched systems with arbitrary switching rules are operated during a finite time interval repetitively. According to the characteristics of the systems, iterative learning control laws are proposed for such switched parabolic systems based on the P-type learning algorithm. Using the contraction mapping method, it is shown that the algorithm can guarantee the output tracking errors on $W^{1,2}$ space converge along the iteration axis. A simulation example illustrates the effectiveness of the proposed algorithm.

Key Words: Switched linear parabolic systems, Iterative learning control, P-type learning algorithm, $W^{1,2}$ space

1 Introduction

Since the complete algorithm of iterative learning control (ILC) was first proposed by Arimoto et al [1], it has become the hot issues of cybernetics and has attracted considerable attention in recent years. In the process of ILC design, D-type [1-4] or P-type [5-7] learning algorithms are often adopted so that the output trajectory of the iterative system can converge to the desired reference trajectory. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications, e.g., see [8] for detailed results. Nowadays, ILC is playing an increasingly important role in controlling repeatable processes.

In recent years, there have been many works reported on ILC for distributed parameter systems (DPSs) [9-14]. Papers [9-11] designed the ILC algorithms for parabolic DPSs by using the P-type learning algorithm. Paper [12] discussed a D-type ILC algorithm for a class of irregular parabolic DPSs. In [13], ILC was applied to temporal-spatial discretized first order hyperbolic partial differential equations (PDEs), guaranteeing stability of the closed loop systems and satisfying the requirements of performance. Paper [14] proposed an ILC algorithm for a class of second order nonlinear hyperbolic DPSs. It is well known that the Sobolev space is a more appropriate space to discuss the problem of weak solution for PDE. And the ILC problem for DPSs in space $W^{1,2}$ was considered in [15], the systems in which are governed by parabolic PDEs or second order hyperbolic PDEs, and the convergence condition was given based on the P-type learning algorithm.

Switched systems, each of which consists of a certain number of subsystems and a switching law, have attracted much research attention in control theory field during recent years. Papers [16-20] designed the ILC algorithms for switched systems, and the corresponding convergence conclusions of the ILC algorithms were given. Recently, paper [21] discussed the ILC problem for switched parabolic DPSs in space L^2, the convergence conclusion in L^2 norm was proposed and the convergence analysis was given based on the P-type learning algorithm. Motivated by the works in [15,21], this paper further studies the problem of ILC algorithm for switched parabolic systems, and the convergence conclusion in $W^{1,2}$ norm is proposed. According to the characteristics of the systems, ILC laws are proposed for such switched parabolic systems based on the P-type learning algorithm, and the convergence condition in $W^{1,2}$ norm is established. Using the contraction mapping method, it is shown that the algorithm can guarantee the output tracking errors converge to zero on $W^{1,2}$ space.

In this paper, the notational conventions are adopted as follows: for function $Q(x,t) \in R^n \cap L^2[0,1]$, $(x,t) \in [0,1]\times[0,T]$, $\|Q(x,t)\|$ denotes Euclidean norm of $Q(x,t)$; take the norm: $\|Q(\cdot,t)\|_{L^2[0,1]} = \sqrt{\int_0^1 \|Q(x,t)\|^2 \mathrm{d}x}$ and define $\|Q\|_{L^2[0,1],s} = \sup\limits_{t\in[0,T]} \|Q(\cdot,t)\|^2_{L^2[0,1]}$. Denote $W^{1,2}[0,1] = \left\{ Q(x,t) \,|\, Q(x,t) \in L^2[0,1], \frac{\partial Q(x,t)}{\partial x} \in L^2[0,1] \right\}$.

2 Problem description

Consider the following switched parabolic system [21]:

$$\begin{cases} \frac{\partial Q(x,t)}{\partial t} = D_{\sigma(t)} \frac{\partial^2 Q(x,t)}{\partial x^2} + A_{\sigma(t)} Q(x,t) + B_{\sigma(t)} u(x,t) \\ y(x,t) = C_{\sigma(t)} Q(x,t) + G_{\sigma(t)} u(x,t) \end{cases} \tag{1}$$

where $(x,t) \in (0,1)\times[0,T]$, $\sigma(t)$ is a random switching law defined by $\sigma(t)$: $\{[0,t_1],[t_1,t_2],\cdots,[t_{l-1},T]\} \to M = \{1,2,\cdots,m\}$, this means that the matrices $\left(D_{\sigma(t)}, A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, G_{\sigma(t)}\right)$ are allowed to take values, at an arbitrary time, in the finite set $\{(D_1,A_1,B_1,C_1,G_1),\cdots,(D_m,A_m,B_m,C_m,G_m)\}$.

Without loss of generality, the arbitrary switching law $\sigma(t)$ can be assumed as follows [16,17]:

$$\sigma(t) = i = \begin{cases} 1, & t\in[0,t_1] \\ 2, & t\in[t_1,t_2] \\ \vdots & \\ m, & t\in[t_{m-1},T] \end{cases}$$

then the system (1) can be represented as:

$$\begin{cases} \frac{\partial Q(x,t)}{\partial t} = D_i \frac{\partial^2 Q(x,t)}{\partial x^2} + A_i Q(x,t) + B_i u(x,t) \\ y(x,t) = C_i Q(x,t) + G_i u(x,t) \end{cases} \tag{2}$$

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with initial-boundary conditions: $Q(x,0)$, $Q(0,t)$ (or $\left.\frac{\partial Q(x,t)}{\partial x}\right|_{x=0}$), $Q(1,t)$ (or $\left.\frac{\partial Q(x,t)}{\partial x}\right|_{x=1}$). Where $i \in M$, $Q(x,t) \in R^n$, $u(x,t), y(x,t) \in R$ represent the state, control input and output of the system, respectively.$D_i = \mathrm{diag}(d_{i1}, d_{i2}, \cdots, d_{in})$ represent positive diagonal matrices, and A_i, B_i, C_i, G_i represent constant matrices of appropriate dimensions.

The system (2) is assumed to satisfy the following assumptions:

Assumption 1 For a given trajectory $y_r(x,t)$, there exists a $u_r(x,t)$ such that

$$\begin{cases} \frac{\partial Q_r(x,t)}{\partial t} = D_i \frac{\partial^2 Q_r(x,t)}{\partial x^2} + A_i Q_r(x,t) + B_i u_r(x,t) \\ y_r(x,t) = C_i Q_r(x,t) + G_i u_r(x,t) \end{cases}$$

It is assumed that the system (2) is repeatable over $t \in [0,T]$. Rewrite the system (2) at each iteration as:

$$\begin{cases} \frac{\partial Q_k(x,t)}{\partial t} = D_i \frac{\partial^2 Q_k(x,t)}{\partial x^2} + A_i Q_k(x,t) + B_i u_k(x,t) \\ y_k(x,t) = C_i Q_k(x,t) + G_i u_k(x,t) \end{cases} \tag{3}$$

where the subscript k is employed to mark the iteration index.

Assumption 2 The initial-boundary resetting conditions hold for all iterations, i.e., $Q_k(x,0) = Q_r(x,0)$; $Q_k(0,t) = Q_r(0,t)$ (or $\left.\frac{\partial Q_k(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial Q_r(x,t)}{\partial x}\right|_{x=0}$), $Q_k(1,t) = Q_r(1,t)$ (or $\left.\frac{\partial Q_k(x,t)}{\partial x}\right|_{x=1} = \left.\frac{\partial Q_r(x,t)}{\partial x}\right|_{x=1}$). $k = 0, 1, 2, \cdots$.

The learning control target is to find an appropriate learning algorithm, so that the output tracking error $e_k(x,t)$ uniformly converges to zero on $W^{1,2}[0,1]$ space, that is

$$\lim_{k\to\infty} \|e_k\|_{L^2[0,1],s} = 0, \ \lim_{k\to\infty} \left\| \frac{\partial e_k}{\partial x} \right\|_{L^2[0,1],s} = 0$$

where $e_k(x,t) = y_r(x,t) - y_k(x,t)$.

Lemma 1[22] Suppose $\{a_k\}$, $\{b_k\}$ are two non-negative real sequences satisfying

$$a_{k+1} \le \rho a_k + b_k, \ 0 \le \rho < 1$$

if $\lim_{k\to\infty} b_k = 0$, then $\lim_{k\to\infty} a_k = 0$.

3 Main result

Constructing the learning algorithm for the system (3) as follows:

$$u_{k+1}(x,t) = u_k(x,t) + q e_k(x,t) \tag{4}$$

where $q \in R$ is the learning gain, then we have the following theorem:

Theorem 1 Let Assumptions 1-2 are satisfied . If there exist a learning gain q and a positive number ε such that

$$\rho = \max_{1\le i\le m} |1 - qG_i| < \frac{1}{\sqrt{1+\varepsilon}} \tag{5}$$

then the iterative process of the system (3) is convergent on $W^{1,2}[0,1]$ space, under the effect of control law (4), i.e.,

$$\lim_{k\to\infty} \|e_k\|_{L^2[0,1],s} = 0, \ \lim_{k\to\infty} \left\| \frac{\partial e_k}{\partial x} \right\|_{L^2[0,1],s} = 0.$$

Proof Due to space limitations and the similar conclusion has been given in [21], the proof of

$$\lim_{k\to\infty} \|e_k\|_{L^2[0,1],s} = 0 \tag{6}$$

is omitted.

Next, we prove $\lim_{k\to\infty} \left\| \frac{\partial e_k}{\partial x} \right\|_{L^2[0,1],s} = 0$. Denote $\delta Q_k(x,t) = Q_{k+1}(x,t) - Q_k(x,t)$, $\delta u_k(x,t) = u_{k+1}(x,t) - u_k(x,t)$. It follows from (3) and (4) that

$$\begin{aligned} e_{k+1}(x,t) &= e_k(x,t) + y_k(x,t) - y_{k+1}(x,t) \\ &= e_k(x,t) - C_i \delta Q_k(x,t) - G_i \delta u_k(x,t) \\ &= (1 - qG_i) e_k(x,t) - C_i \delta Q_k(x,t) \end{aligned}$$

Differentiate both sides of the above expression to x obtains

$$\frac{\partial e_{k+1}(x,t)}{\partial x} = (1 - qG_i) \frac{\partial e_k(x,t)}{\partial x} - C_i \frac{\partial(\delta Q_k(x,t))}{\partial x}$$

From (5), we have

$$\left| \frac{\partial e_{k+1}(x,t)}{\partial x} \right| \le \rho \left| \frac{\partial e_k(x,t)}{\partial x} \right| + \|C_i\| \left\| \frac{\partial(\delta Q_k(x,t))}{\partial x} \right\|$$

Using the basic inequality, we have

$$\begin{aligned} \left| \frac{\partial e_{k+1}(x,t)}{\partial x} \right|^2 &\le (1+\varepsilon)\rho^2 \left| \frac{\partial e_k(x,t)}{\partial x} \right|^2 \\ &+ \left(1 + \frac{1}{\varepsilon}\right) \|C_i\|^2 \left\| \frac{\partial(\delta Q_k(x,t))}{\partial x} \right\|^2 \end{aligned} \tag{7}$$

Integrating both sides with respect to x from 0 to 1, it yields

$$\begin{aligned} \left\| \frac{\partial e_{k+1}(\cdot,t)}{\partial x} \right\|^2_{L^2[0,1]} &\le (1+\varepsilon)\rho^2 \left\| \frac{\partial e_k(\cdot,t)}{\partial x} \right\|^2_{L^2[0,1]} \\ &+ \left(1 + \frac{1}{\varepsilon}\right) \|C_i\|^2 \left\| \frac{\partial(\delta Q_k(\cdot,t))}{\partial x} \right\|^2_{L^2[0,1]} \end{aligned} \tag{8}$$

By (3) and combining with (4), we have

$$\begin{aligned} \frac{\partial(\delta Q_k(x,t))}{\partial t} &= D_i \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} + A_i \delta Q_k(x,t) \\ &+ qB_i e_k(x,t) \end{aligned}$$

Using $\left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}}$ to left-multiply both sides of the above expression, it yields

$$\begin{aligned} &\left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}} \frac{\partial(\delta Q_k(x,t))}{\partial t} \\ &= \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}} D_i \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \\ &+ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}} A_i \delta Q_k(x,t) \\ &+ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}} qB_i e_k(x,t) \end{aligned} \tag{9}$$

Integrating by parts and combining with the boundary resetting conditions given in Assumption 2, we have

$$\begin{aligned} &\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\right)^{\mathrm{T}} \frac{\partial\left(\delta Q_k(x,t)\right)}{\partial t} \right\} \mathrm{d}x \\ &= -\frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \left\| \frac{\partial\left(\delta Q_k(\cdot,t)\right)}{\partial x} \right\|^2_{L^2[0,1]} \end{aligned} \tag{10}$$

and

$$\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right)^{\mathrm{T}} A_i \delta Q_k(x,t) \right\} \mathrm{d}x$$
$$= -\int_0^1 \left\{ \left(\frac{\partial(\delta Q_k(x,t))}{\partial x} \right)^{\mathrm{T}} A_i \frac{\partial(\delta Q_k(x,t))}{\partial x} \right\} \mathrm{d}x \quad (11)$$

So, integrating both sides of (9) respect to x from 0 to 1 and combining with (10), (11), we can get

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \left\| \frac{\partial\left(\delta Q_k(\cdot,t)\right)}{\partial x} \right\|^2_{L^2[0,1]} \\ &+2\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right)^{\mathrm{T}} D_i \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right\} \mathrm{d}x \\ =\ & 2\int_0^1 \left\{ \left(\frac{\partial(\delta Q_k(x,t))}{\partial x} \right)^{\mathrm{T}} A_i \frac{\partial(\delta Q_k(x,t))}{\partial x} \right\} \mathrm{d}x \\ &-2\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right)^{\mathrm{T}} qB_i e_k(x,t) \right\} \mathrm{d}x \end{aligned} \quad (12)$$

Note $D_i = \mathrm{diag}(d_{i1}, d_{i2}, \cdots, d_{in})$, so

$$\begin{aligned} &\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right)^{\mathrm{T}} D_i \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right\} \mathrm{d}x \\ \geq\ & d \left\| \frac{\partial^2(\delta Q_k(\cdot,t))}{\partial x^2} \right\|^2_{L^2[0,1]} \end{aligned} \quad (13)$$

where $d = \min\limits_{\substack{1\leq i\leq m \\ 1\leq j\leq n}} \{d_{ij}\}$. Using the basic inequality, we have

$$\begin{aligned} &-2\int_0^1 \left\{ \left(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right)^{\mathrm{T}} qB_i e_k(x,t) \right\} \mathrm{d}x \\ \leq\ & 2\int_0^1 \left\{ \left\| \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right\| \|qB_i e_k(x,t)\| \right\} \mathrm{d}x \\ \leq\ & \int_0^1 \left\{ d \left\| \frac{\partial^2(\delta Q_k(x,t))}{\partial x^2} \right\|^2 + \frac{\|qB_i\|^2}{d} \|e_k(x,t)\|^2 \right\} \mathrm{d}x \\ =\ & d \left\| \frac{\partial^2(\delta Q_k(\cdot,t))}{\partial x^2} \right\|^2_{L^2[0,1]} + \frac{\|qB_i\|^2}{d} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned} \quad (14)$$

Substituting (13) and (14) into (12), we can obtain

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \left\| \frac{\partial\left(\delta Q_k(\cdot,t)\right)}{\partial x} \right\|^2_{L^2[0,1]} \\ \leq\ & \frac{\mathrm{d}}{\mathrm{d}t} \left\| \frac{\partial\left(\delta Q_k(\cdot,t)\right)}{\partial x} \right\|^2_{L^2[0,1]} + d \left\| \frac{\partial^2(\delta Q_k(\cdot,t))}{\partial x^2} \right\|^2_{L^2[0,1]} \\ \leq\ & 2\int_0^1 \left\{ \left(\frac{\partial(\delta Q_k(x,t))}{\partial x} \right)^{\mathrm{T}} A_i \frac{\partial(\delta Q_k(x,t))}{\partial x} \right\} \mathrm{d}x \\ &+ \frac{\|qB_i\|^2}{d} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \\ \leq\ & 2\|A_i\| \left\| \frac{\partial(\delta Q_k(\cdot,t))}{\partial x} \right\|^2_{L^2[0,1]} \\ &+ \frac{\|qB_i\|^2}{d} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned} \quad (15)$$

Applying Gronwall lemma to (15) over $[0,t]$, $0 \leq t \leq t_1$, and combining with the initial resetting condition given in Assumption 2, it yields

$$\begin{aligned} &\left\| \frac{\partial(\delta Q_k(\cdot,t))}{\partial x} \right\|^2_{L^2[0,1]} \\ \leq\ & \int_0^t \mathrm{e}^{2\|A_1\|(t-\tau)} \frac{\|qB_1\|^2}{d} \|e_k(\cdot,\tau)\|^2_{L^2[0,1]} \mathrm{d}\tau \\ \leq\ & \frac{\mathrm{e}^{2\|A_1\|t_1} \|qB_1\|^2}{d} \sup_{t\in[0,t_1]} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned}$$

Therefore

$$\begin{aligned} &\sup_{t\in[0,t_1]} \left\| \frac{\partial(\delta Q_k(\cdot,t))}{\partial x} \right\|^2_{L^2[0,1]} \\ \leq\ & \frac{\mathrm{e}^{2\|A_1\|t_1} \|qB_1\|^2}{d} \sup_{t\in[0,t_1]} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned} \quad (16)$$

From (8), (16), we have

$$\begin{aligned} &\sup_{t\in[0,t_1]} \left\| \frac{\partial e_{k+1}(\cdot,t)}{\partial x} \right\|^2_{L^2[0,1]} \\ \leq\ & (1+\varepsilon)\rho^2 \sup_{t\in[0,t_1]} \left\| \frac{\partial e_k(\cdot,t)}{\partial x} \right\|^2_{L^2[0,1]} \\ &+ \left(1+\frac{1}{\varepsilon}\right) \frac{\mathrm{e}^{2\|A_1\|t_1} \|C_1\|^2 \|qB_1\|^2}{d} \\ &\times \sup_{t\in[0,t_1]} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned}$$

Since $(1+\varepsilon)\rho^2 < 1$ by (5), combining with (6) and Lemma 1, we obtain

$$\lim_{k\to\infty} \sup_{t\in[0,t_1]} \left\| \frac{\partial e_k(\cdot,t)}{\partial x} \right\|^2_{L^2[0,1]} = 0 \quad (17)$$

Taking $t = t_1$ in (16) and by (6), we get

$$\lim_{k\to\infty} \left\| \frac{\partial(\delta Q_k(\cdot,t_1))}{\partial x} \right\|^2_{L^2[0,1]} = 0 \quad (18)$$

Applying Gronwall lemma to (15) over $[t_1, t]$, $t_1 \leq t \leq t_2$, we have

$$\begin{aligned} &\left\| \frac{\partial(\delta Q_k(\cdot,t))}{\partial x} \right\|^2_{L^2[0,1]} \\ \leq\ & \mathrm{e}^{2\|A_2\|(t-t_1)} \left\| \frac{\partial(\delta Q_k(\cdot,t_1))}{\partial x} \right\|^2_{L^2[0,1]} \\ &+ \int_{t_1}^t \frac{\mathrm{e}^{2\|A_2\|(t-\tau)} \|qB_2\|^2}{d} \|e_k(\cdot,\tau)\|^2_{L^2[0,1]} \mathrm{d}\tau \\ \leq\ & \mathrm{e}^{2\|A_2\|(t_2-t_1)} \left\| \frac{\partial(\delta Q_k(\cdot,t_1))}{\partial x} \right\|^2_{L^2[0,1]} \\ &+ \frac{\mathrm{e}^{2\|A_2\|(t_2-t_1)} \|qB_2\|^2}{d} \sup_{t\in[t_1,t_2]} \|e_k(\cdot,t)\|^2_{L^2[0,1]} \end{aligned}$$

Therefore

$$\begin{aligned}&\sup_{t\in[t_1,t_2]}\left\|\frac{\partial(\delta Q_k(\cdot,t))}{\partial x}\right\|^2_{L^2[0,1]}\\&\le \mathrm{e}^{2\|A_2\|(t_2-t_1)}\left\|\frac{\partial(\delta Q_k(\cdot,t_1))}{\partial x}\right\|^2_{L^2[0,1]}\\&\quad+\frac{\mathrm{e}^{2\|A_2\|(t_2-t_1)}\|qB_2\|^2}{d}\sup_{t\in[t_1,t_2]}\|e_k(\cdot,t)\|^2_{L^2[0,1]}\end{aligned}$$

Combining with (6) and (18), we know

$$\lim_{k\to\infty}\sup_{t\in[t_1,t_2]}\left\|\frac{\partial(\delta Q_k(\cdot,t))}{\partial x}\right\|^2_{L^2[0,1]}=0 \tag{19}$$

Taking $t=t_2$ in (19), we get

$$\lim_{k\to\infty}\left\|\frac{\partial(\delta Q_k(\cdot,t_2))}{\partial x}\right\|^2_{L^2[0,1]}=0$$

Taking $t\in[t_1,t_2]$, and combining with (8), it yields

$$\begin{aligned}&\sup_{t\in[t_1,t_2]}\left\|\frac{\partial e_{k+1}(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}\\&\le(1+\varepsilon)\rho^2\sup_{t\in[t_1,t_2]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}\\&\quad+\left(1+\frac{1}{\varepsilon}\right)\|C_2\|^2\sup_{t\in[t_1,t_2]}\left\|\frac{\partial(\delta Q_k(\cdot,t))}{\partial x}\right\|^2_{L^2[0,1]}\end{aligned}$$

Since $(1+\varepsilon)\rho^2<1$ by (5), combining with (19) and Lemma 1, we have

$$\lim_{k\to\infty}\sup_{t\in[t_1,t_2]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}=0 \tag{20}$$

For $t\in[t_2,t_3]$, by repeating the same procedure as that after (18), we can obtain

$$\lim_{k\to\infty}\sup_{t\in[t_2,t_3]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}=0 \tag{21}$$

Further we have

$$\begin{aligned}&\lim_{k\to\infty}\sup_{t\in[t_3,t_4]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}=0\\&\quad\vdots\\&\lim_{k\to\infty}\sup_{t\in[t_{m-1},T]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}=0\end{aligned} \tag{22}$$

From (17), (20)-(22), we know

$$\lim_{k\to\infty}\left\|\frac{\partial e_k}{\partial x}\right\|^2_{L^2[0,1],s}=\lim_{k\to\infty}\sup_{t\in[0,T]}\left\|\frac{\partial e_k(\cdot,t)}{\partial x}\right\|^2_{L^2[0,1]}=0$$

This completes the proof.

Remark 1 Especially, when $\sigma(t)=1$ for all $\{[0,t_1],[t_1,t_2],\cdots,[t_{m-1},T]\}$, the conclusion of Theorem 1 is the same as the Theorem 1 in [15].

4 Simulation example

Consider the following switched linear parabolic system, which contains two subsystems:

$$\begin{cases}\frac{\partial Q(x,t)}{\partial t}=D_{\sigma(t)}\frac{\partial^2 Q(x,t)}{\partial x^2}+A_{\sigma(t)}Q(x,t)+B_{\sigma(t)}u(x,t)\\ y(x,t)=C_{\sigma(t)}Q(x,t)+G_{\sigma(t)}u(x,t)\end{cases} \tag{23}$$

where $\sigma(t)$ is an arbitrary switching sequence with $\sigma(t)=\{1,2\}$, and D_1=1,$A_1=0$, $B_1=C_1=G_1=1$, $D_2=3$, $A_2=2$, $B_2=C_2=G_2=1$, the subscript k is employed to mark the iteration index. Take $T=1$, $q=\frac{1}{2}$, $\varepsilon=1$, then

$$|1-qG_1|=|1-qG_2|=\frac{1}{2}<\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{1+\varepsilon}}$$

Take the given desired trajectory as: $y_r(x,t)=2\sin(x+t)+\cos(x+t)$, then $Q_r(x,t)=\sin(x+t)$, $u_r(x,t)=\sin(x+t)+\cos(x+t)$. We produce a random sequence $\sigma(t)$, as shown in Fig.1. If $\sigma(t)=1$ the system (23) is (D_1,A_1,B_1,C_1,G_1), otherwise, if $\sigma(t)=2$, the system (23) is (D_2,A_2,B_2,C_2,G_2). Take the initial control $u_0(x,t)=1$, it is easy to see that the output tracking errors on $W^{1,2}$ space tend to zero as $k\to\infty$ (shown in Fig. 2,3).

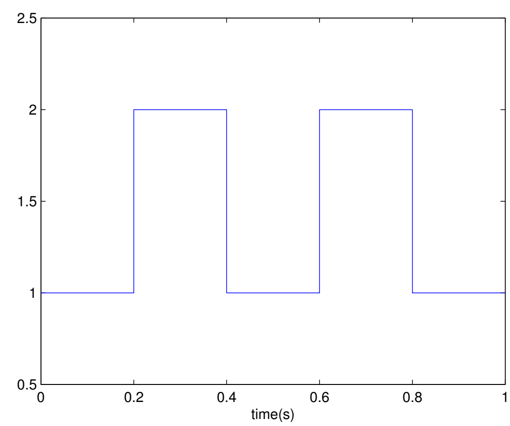

Fig. 1: The random switching rule $\sigma(t)$

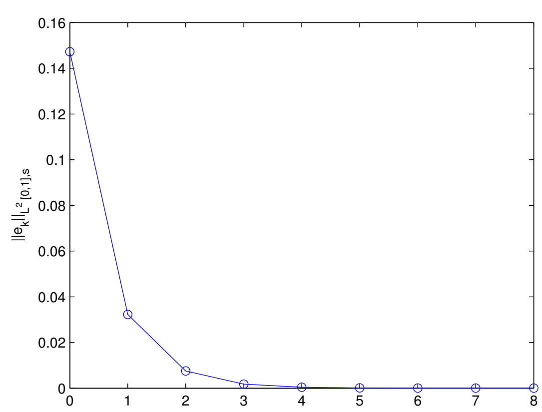

Fig. 2: Iterations for the output tracking errors

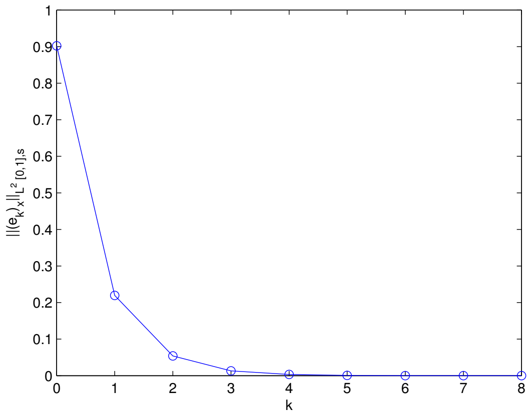

Fig. 3: Iterations for the output tracking errors

5 Conclusion

This paper considers the iterative learning control problem for a class of switched linear parabolic systems in space $W^{1,2}$. By using the P-type learning algorithm, the convergence theorem of the output tracking errors on $W^{1,2}$ space is established based on the contraction mapping method. The simulation results are consistent with theoretical analysis.

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