On Adaptive Iterative Learning From Tracking Tasks with Different Magnitude and Time Scales

Xuefang Li[1], Jian-Xin Xu[1]

1. Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576
E-mail: xuefangli@u.nus.edu; elexujx@nus.edu.sg

Abstract: In this paper, we introduce a new adaptive iterative learning control (AILC) scheme based on a time scaling factor, which enables learning from control tasks with different magnitude and time scales. The proposed AILC scheme overcomes the limitation of traditional ILC that the target trajectory must be identical in all iterations. In addition, the requirement on classic ILC that every trial must repeat in a fixed time duration is removed. For nonlinear systems with time-invariant and time-varying parametric uncertainties, the new learning algorithm works effectively to nullify the tracking error. It is shown that the new AILC is capable of fully utilizing all the learned knowledge despite the iteratively varying tracking tasks. In the end, an illustrative example is presented to demonstrate the performance and the effectiveness of the proposed AILC scheme.

Key Words: Adaptive Iterative learning control; Time scaling factor; Time scale transformation

1 INTRODUCTION

Iterative learning control (ILC) is one kind of control methodology effectively dealing with repeated tracking control problem or periodic disturbance rejected problems. ILC was initially proposed in 1984 [1], and now has been well established in terms of both the underlying theory and experimental applications [2]-[7]. However, in most of previous works, a fundamental requirement is that the target trajectory must be invariant in all iterations. If however there is a change in the target trajectory due to the variation of control objectives or task specifications, no matter how small it might be, the control system will have to restart the learning process from the very beginning and the previous tracking information including control input signals and tracking errors can no longer be used.

Nevertheless, in practice a control system may implement different but highly correlated motion tasks. For instance, consider that a XY-table draws a number of circles in specified time periods [8]. There are three different kinds of operation specifications: 1) draw all the circles with the same radius but different periods; 2) draw all the circles with the same periods but different radii; 3) draw all the circles which differ one from another in both radii and periods. Obviously, those control signals are inherently correlated because: 1) they are generated by the same robotic dynamics and 2) each motion pattern is related (or proportional) to another either in spatial distribution or in time scale. Now, the control problem is whether a control system can learn consecutively from different but highly correlated tracking tasks.

In the existing literature, there are some works that have investigated learning control problems for iteration-varying control tasks. In [9], D-type, PD-type, and PID-type learning algorithms were presented for tracking trajectories "slowly" varying in the iteration domain. In that work, the difference between two consecutive iterations is assumed to be bounded by a small constant. Due to the presence of non-parametric system uncertainties, only a bounded tracking error is guaranteed if the target trajectory keeps changing in the iteration axis. In [8], [10]-[12], direct learning control and recursive direct learning control schemes were developed to make the use of previously obtained control information to generate control input for a new trajectory. However, a difficulty encountered in further expansion of these direct learning control schemes is the requirement for the perfect preceding control information and the open-loop control nature. In [13], the authors developed a new iterative learning control method based on composite energy function, where the target trajectories of any two consecutive iterations can be different, but the dynamics must repeat in a fixed time interval. In a sense, it is a special case of control tasks in the same time scale but different magnitude scales. To the best of our knowledge, there are no works dealing with learning control problems for target trajectories that are different in both magnitude and time scales. For these kinds of non-repeatable learning control problems, in spite of the variations of the trajectory patterns, it should be noted that the underlying dynamic properties of the controlled system remain the same. We need to explore the inherent relations of different trajectory patterns and the learning control scheme could potentially be both plant-dependent and trajectory-dependent.

In this paper, a new adaptive iterative learning control (AILC) scheme is designed by introducing a time scale factor to deal with control tasks with different magnitude and time scales. By adopting the time scale transformations, all the target trajectories are scaled into the same time scale,

Invited Session: Advances in Iterative Learning Control Theory and its Applications.

and the convergence of tracking errors for nonlinear systems with time-invariant and time-varying parametric uncertainties are obtained based on Lyapunov theory. In this way, the learning control system is capable of fully utilizing all the learned knowledge to solve different but somehow correlated control problems. The main contribution of this work is to show that the limitations of traditional ILC, that the target trajectory must be identical in all iterations and every trial must repeat in a fixed time duration, can be removed.

The paper is organized as follows. In Section II, we formulate learning control problems for control tasks with different magnitude and time scales. In Section III, the controller design and convergence analysis are presented. Further, the proposed AILC law is extended to nonlinear systems with time-varying parametric uncertainties in Section IV. Lastly, Section V gives an illustrative example. Throughout this paper, $\dot{f}$ is the derivative of function f with respect to t.

2 PROBLEM FORMULATION

Consider the following nonlinear dynamic system

$$\frac{dx}{dt} = \theta^T \mathbf{f}(x) + v(t), \tag{1}$$

where $x \in \mathbf{R}$ is the system state; $v \in \mathbf{R}$ is the control input; $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$ is the unknown vector-valued parameter; and $\mathbf{f}(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T$ is a known vector-valued function, which is a local Lipschitz function with respect to x.

The target trajectory in the ith iteration is denoted as $x_i^r(t_i)$, $t_i \in [0, T_i]$, that could be different from iteration to iteration, where t_i is the time scale and T_i is the trial length of ith iteration. Throughout this paper, we assume that the given trajectories $x_i^r(t_i)$ are at least continuously differentiable with respect to the time t_i.

Before addressing the non-repeatable learning control problem, let us provide some notations and definitions that would be useful in the derivation of our main result.

Definition 1 *[8] Trajectory $x_i(t_i)$, $t_i \in [0, T_i]$ is said to be proportional to another trajectory $x(t)$, $t \in [0, T]$ both in magnitude and time scales if and only if*

$$\kappa_i^{-1}(t_i) x_i(t_i) = x(t)$$

where $\kappa_i(t_i) \neq 0$ is the time-varying magnitude scaling factor and $\rho_i(t) = t_i$ is the time scaling factor, which is continuously differentiable and satisfies $\rho_i(0) = 0$ and $\rho_i(T) = T_i$.

Assumption 1 *$d\rho_i(t)/dt > 0$ and the inverse function $\rho_i^{\{-1\}}(t_i)$ of $\rho_i(t)$ exists and is known, and the magnitude scaling factor $\kappa_i(t_i)$ is continuously differentiable.*

Remark 1 *$d\rho_i(t)/dt > 0$ implies that $\rho_i(t)$ is a monotonically increasing function with respect to the time t. The conditions $\rho_i(0) = 0$ and $\rho_i(T) = T_i$ are satisfied for this kind of monotonically increasing function.*

Assume that $x_i^r(t_i)$, $i = 1, 2, 3, \ldots$ are proportional to a trajectory $x^r(t)$, $t \in [0, T]$. According to the Definition 1, there has $\kappa_i^{-1}(t_i) x_i^r(t_i) = x^r(t)$. Multiply $\kappa_i(t_i)$ on both sides, we can obtain $x_i^r(t_i) = \kappa_i(t_i) x^r(t) = \kappa_i(\rho_i(t)) x^r(t)$. Denote $\gamma_i(t) \triangleq \kappa_i(\rho_i(t)) x^r(t)$, $i = 1, 2, 3, \ldots$, it shows that $x_i^r(t_i) = \gamma_i(t)$, $i = 1, 2, 3, \ldots$.

Since the target trajectory in the ith iteration is $x_i^r(t_i)$ and the time scale is $t_i \in [0, T_i]$, the dynamics in the ith iteration is

$$\frac{dx_i}{dt_i} = \theta^T \mathbf{f}(x_i) + v_i(t_i), \quad t_i \in [0, T_i]. \tag{2}$$

Considering that $\gamma_i(t) = \kappa_i(\rho_i(t)) x^r(t) = x_i^r(t_i)$ and applying a time scale transformation $dt_i = \dot{\rho}_i(t) dt$, we can rewrite system (2) into the following form with the time scale $t \in [0, T]$

$$\begin{aligned} \frac{dx_i}{dt} &= \frac{dt_i}{dt}(\theta^T \mathbf{f}(x_i) + v_i(\rho_i(t))) \\ &= \dot{\rho}_i(t) \theta^T \mathbf{f}(x_i) + \dot{\rho}_i(t) u_i(t), \end{aligned} \tag{3}$$

where $u_i(t) \triangleq v_i(\rho_i(t))$.

Denote $x_i(t)$, $t \in [0, T]$ the solution of system (3) and $e_i(t) \triangleq x_i(t) - \gamma_i(t)$, $t \in [0, T]$ the tracking error. The error dynamics in the ith iteration is

$$\begin{aligned} \frac{de_i(t)}{dt} &= \frac{x_i(t)}{dt} - \frac{\gamma_i(t)}{dt} \\ &= \dot{\rho}_i(t) \theta^T \mathbf{f}(x_i) + \dot{\rho}_i(t) u_i(t) - \dot{\gamma}_i(t). \end{aligned} \tag{4}$$

The control objective is to tune $u_i(t)$ such that the tracking error $e_i(t)$ converges to zero as the iteration number $i \to \infty$. As is common in ILC field, the following assumption is made.

Assumption 2 *Initial error satisfies the identical initialization condition $e_i(0) = 0$, $\forall i \in \mathbf{Z}^+$.*

3 ILC DESIGN AND CONVERGENCE ANALYSIS

In this section, based on the assumptions and notations that were given in Section II, ILC design and convergence analysis are addressed, respectively.

The proposed controller is

$$u_i(t) = -k e_i(t) + \dot{\gamma}_i(t)/\dot{\rho}_i(t) - \hat{\theta}_i^T \mathbf{f}(x_i), \quad t \in [0, T] \tag{5}$$

with the updating law

$$\dot{\hat{\theta}}_i(t) = \dot{\rho}_i(t) \mathbf{f}(x_i) e_i(t) \tag{6}$$

with $\hat{\theta}_0(0) = 0, \hat{\theta}_{i+1}(0) = \hat{\theta}_i(T)$, where $k > 0$ is a feedback gain and $\hat{\theta}_i(t)$ is the estimation of θ in the ith iteration. Then, our main result is presented in the following theorem.

Theorem 1 *For the nonlinear system (1), under the assumption 2, the AILC scheme (5) and (6) guarantees that the tracking error converges to zero in $L^2[0, T]$ norm, i.e.,*

$$\lim_{i \to \infty} ||e_i(t)||_{L^2[0,T]} = 0,$$

which leads to $x_i(t_i) \to x_i^r(t_i)$, as $i \to \infty$ in $L^2[0, T]$ norm.

Proof. Define the Lyapunov function for the ith iteration

$$V(e_i,\phi_i)=\frac{1}{2}e_i^2(t)+\frac{1}{2}\phi_i^T(t)\phi_i(t),\ \ i\in\mathbf{Z}^+, \tag{7}$$

where $\phi_i(t)\triangleq\hat{\theta}_i(t)-\theta$ is the estimation error. Note that the error dynamics at the ith iteration, with the control law (5), is

$$\begin{aligned}\dot{e}_i(t) &= \dot{x}_i(t)-\dot{\gamma}_i(t)\\ &= -k\dot{\rho}_i(t)e_i(t)-\dot{\rho}_i(t)\phi_i^T(t)\mathbf{f}(x_i). \end{aligned}\tag{8}$$

Differentiating V with respect to t and substituting the error dynamics (8) and the adaptation law (6) yield

$$\begin{aligned}\dot{V}(e_i,\phi_i) &= e_i(t)\dot{e}_i(t)+\phi_i^T(t)\dot{\phi}_i(t)\\ &= -k\dot{\rho}e_i^2(t)\le 0. \end{aligned}\tag{9}$$

Integrate the derivative of V over $[0,T]$, and use the fact $|e_i(0)|=0\le|e_i(T)|$ and $\hat{\theta}_{i+1}(0)=\hat{\theta}_i(T)$,

$$\begin{aligned}V(0,\phi_i(T)) &\le V(e_i(T),\phi_i(T)) \qquad (10)\\ &= V(e_i(0),\phi_i(0))+\int_0^T\dot{V}d\tau\\ &= V(0,\phi_{i-1}(T))-k\int_0^T\dot{\rho}_i(\tau)e_i^2(\tau)d\tau.\end{aligned}$$

By using the mathematical induction, we have

$$V(0,\phi_i(T))\le V(0,\phi_0(T))-k\sum_{j=1}^{i}\int_0^T\dot{\rho}_j(\tau)e_j^2(\tau)d\tau. \tag{11}$$

From the finiteness of $V(0,\phi_0(T))$ and the positive definiteness of $V(0,\phi_i(T))$, we can obtain

$$\lim_{i\to\infty}\int_0^T\dot{\rho}_i(\tau)e_i^2(\tau)d\tau=0. \tag{12}$$

Since $\dot{\rho}_i(t)$ is continuous with respect to t, $t\in[0,T]$, there exists a constant $\alpha>0$ such that $\dot{\rho}_i(t)\ge\alpha$. Then, it gives

$$\alpha\int_0^T e_i^2(\tau)d\tau\le\int_0^T\dot{\rho}_i(\tau)e_i^2(\tau)d\tau. \tag{13}$$

Thus, according to (12), it implies that $\lim_{i\to\infty}\int_0^T e_i^2(\tau)d\tau=0$, i.e., the tracking error $e_i(t)$ is convergent in $L^2[0,T]$ norm. ■

4 EXTENSION TO SYSTEMS WITH TIME-VARYING PARAMETER

In this section, the proposed ILC scheme is extended to nonlinear systems with time-varying parameter

$$\frac{dx}{dt}=\theta^T(t)\mathbf{f}(x)+v(t),\ t\in[0,T], \tag{14}$$

where $x\in\mathbf{R}$ is the system state; $v\in\mathbf{R}$ is the control input; $\theta(t)=[\theta_1(t),\theta_2(t),\dots,\theta_n(t)]^T$ is the unknown time-varying parameter; and $\mathbf{f}(x)=[f_1(x),f_2(x),\dots,f_n(x)]^T$ is a known vector-valued function, which is a local Lipschitz function with respect to x. In addition, there exists a known continuous function $g(x)>0$ such that $\|\mathbf{f}(x)\|\le g(x)$.

The dynamics in the $i$the iteration is

$$\frac{dx_i}{dt_i}=\theta^T(t_i)\mathbf{f}(x_i)+v(t_i),\ t_i\in[0,T_i], \tag{15}$$

It is worth noting that if we apply the time scale transformation $dt_i=\dot{\rho}_i(t)dt$ directly to the system (15), the system can be rewritten as

$$\frac{dx_i}{dt}=\zeta_i^T(t)\mathbf{f}(x_i)+u_i(t),\ t\in[0,T], \tag{16}$$

where $u_i(t)\triangleq v_i(\rho_i(t))$ and $\zeta_i(t)\triangleq\theta(\rho_i(t))$ is iteration-time-varying. To solve the learning control problem for system (16) with iteration-time-varying uncertainties $\zeta_i(t)$, we give the following assumption.

Assumption 3 *Assume that the time-varying parameters $\theta_j(t)$, $j=1,2,\dots,n$, are smooth and can be approximated as:*

$$\theta_j(t)=\Phi_j^T(t)\eta_j+\varepsilon_j(t),\ j=1,2,\dots,n, \tag{17}$$

in a sufficiently large interval $[0,\overline{T}]$, where $\overline{T}\ge\max_{i\ge1}\{T,T_i\}$, $\Phi_j(t)=[\varphi_{j,1}(t),\varphi_{j,2}(t),\dots,\varphi_{j,l_j}(t)]^T$, $l_j\in\mathbf{Z}^+$ is a known vector-valued function, $\{\varphi_{j,k}(t),\ \ k=1,2,\dots,l_j\}$ is an orthonormal basis, $\eta_j=[\eta_{j,1},\eta_{j,2},\dots,\eta_{j,l_j}]^T$ are unknown constant coefficients, and $\varepsilon_j(t)$, $j=1,2,\dots,n$, are the approximate errors.

Due to the boundedness of $\theta_j(t)$ for $t\in[0,\overline{T}]$, we have that there exists a constant $\overline{\lambda}_j>0$ such that $|\varepsilon_j(t)|\le\overline{\lambda}_j$, $t\in[0,\overline{T}]$. Let

$$\Phi(t)=\begin{bmatrix}\Phi_1(t)&0&\cdots&0\\0&\Phi_2(t)&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\Phi_n(t)\end{bmatrix}\in\mathbf{R}^{n\times\sum_{j=1}^n l_j}, \tag{18}$$

$\eta=[\eta_1^T,\eta_2^T,\dots,\eta_n^T]^T$, and $\varepsilon(t)=[\varepsilon_1(t),\varepsilon_2(t),\dots,\varepsilon_n(t)]^T$, then $\theta(t)$ can be rewritten in the matrix form

$$\theta(t)=\Phi^T(t)\eta+\varepsilon(t). \tag{19}$$

Further, according to $|\varepsilon_j(t)|\le\overline{\lambda}_j$, $t\in[0,\overline{T}]$, there exists a constant $\lambda>0$ such that $\|\varepsilon(t)\|\le\lambda$ for $t\in[0,\overline{T}]$.
From (19), we have

$$\theta(t_i)=\Phi^T(t_i)\eta+\varepsilon(t_i),\ t_i\in[0,T_i]. \tag{20}$$

Substituting (20) into the system (15) yields

$$\frac{dx_i}{dt_i}=\eta^T\Phi(t_i)\mathbf{f}(x_i)+\varepsilon^T(t_i)\mathbf{f}(x_i)+v_i(t_i). \tag{21}$$

Similarly as the previous case with time-invariant parameters, apply the time scale transformation $dt_i=\dot{\rho}_i(t)dt$, $t\in[0,T]$ to system (21), we can obtain

$$\frac{dx_i}{dt}=\dot{\rho}_i(t)\eta^T\Xi_i(t)\mathbf{f}(x_i)+\dot{\rho}_i(t)\xi_i^T(t)\mathbf{f}(x_i)+\dot{\rho}_i(t)u_i(t), \tag{22}$$

where $\Xi_i(t) \triangleq \Phi(\rho_i(t))$, $\xi_i(t) \triangleq \varepsilon(\rho_i(t))$ and $u_i(t) \triangleq v_i(\rho_i(t))$. Since $\|\varepsilon(t)\| \leq \lambda$ for $t \in [0,\overline{T}]$ and $\|\mathbf{f}(x)\| \leq g(x)$, we have

$$|\varepsilon^T(t)\mathbf{f}(x)| \leq \lambda g(x), t \in [0,\overline{T}]. \tag{23}$$

Further, according to $[0,T_i] \subseteq [0,\overline{T}]$, it is obvious that

$$|\xi_i^T(t)\mathbf{f}(x_i)| = |\varepsilon(\rho_i(t))\mathbf{f}(x_i)| = |\varepsilon(t_i)\mathbf{f}(x_i)| \leq \lambda g(x_i). \tag{24}$$

Denote $\hat{\eta}_i(t)$ and $\hat{\lambda}_i(t)$ the estimation of η and λ at the ith iteration, respectively. The revised learning control law in the ith iteration is

$$\begin{aligned} u_i(t) \quad = \quad & -ke_i(t) + \dot{\gamma}_i(t)/\dot{\rho}_i(t) - \hat{\eta}_i^T \Xi_i(t)\mathbf{f}(x_i) \\ & -\mathrm{sgn}(e_i(t))\hat{\lambda}_i g(x_i),\ t \in [0,T], \end{aligned} \tag{25}$$

where $k > 0$ is a feedback gain. The updating laws for $\hat{\eta}_i(t)$ and $\hat{\lambda}_i(t)$ are

$$\dot{\hat{\eta}}_i(t) = \dot{\rho}_i(t)\Xi_i(t)\mathbf{f}(x_i)e_i(t), \tag{26}$$
$$\dot{\hat{\lambda}}_i(t) = \dot{\rho}_i(t)g(x_i)|e_i(t)| \tag{27}$$

with $\hat{\eta}_0(0) = 0,\ \hat{\eta}_{i+1}(0) = \hat{\eta}_i(T)$ and $\hat{\lambda}_0(0) = 0,\ \hat{\lambda}_{i+1}(0) = \hat{\lambda}_i(T)$, respectively.

The convergence property of the proposed learning controller is derived in the following theorem.

Theorem 2 *For the nonlinear system (14), under the assumptions 2, 3, the AILC scheme (25) with (26) and (27) guarantees that the tracking error $e_i(t)$ is convergent in $L^2[0,T]$ norm.*

Proof. The proof can be performed similarly as in the proof of Theorem 1. Denote $\phi_i(t) \triangleq \hat{\eta}_i(t) - \eta$ and $\psi_i(t) \triangleq \hat{\lambda}_i(t) - \lambda$ the estimation errors and consider the Lyapunov function in the ith iteration

$$V(e_i,\phi_i,\psi_i) = \frac{1}{2}e_i^2(t) + \frac{1}{2}\phi_i^T(t)\phi_i(t) + \frac{1}{2}\psi_i^2(t). \tag{28}$$

The error dynamics is

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{\gamma}_i(t) \qquad (29) \\ &= -k\dot{\rho}_i(t)e_i(t) - \dot{\rho}_i(t)\phi_i^T \Xi_i(t)\mathbf{f}(x_i) \\ &\quad +\dot{\rho}_i(t)\xi_i^T(t)\mathbf{f}(x_i) - \dot{\rho}_i(t)\hat{\lambda}_i g(x_i)\mathrm{sgn}(e_i(t)), \end{aligned}$$

where the revised controller (25) is applied. Differentiating V with respect to t and substituting the error dynamics (30), the adaptation laws (26) and (27) yield

$$\begin{aligned} \dot{V}(e_i,\phi_i,\psi_i) &= e_i(t)\dot{e}_i(t) + \phi_i^T(t)\dot{\phi}_i(t) + \psi_i(t)\dot{\psi}_i(t) \\ &= -k\dot{\rho}_i(t)e_i^2(t) + \dot{\rho}_i(t)\xi_i^T(t)\mathbf{f}(x_i)e_i(t) \\ &\quad -\dot{\rho}_i(t)\lambda g(x_i)|e_i(t)| \\ &\leq -k\dot{\rho}_i e_i^2(t). \end{aligned} \tag{30}$$

Finally, similar as the time-invariant case, integrate both sides of (30) over $[0,T]$ and use the mathematical induction, we have

$$\begin{aligned} V(0,\phi_i(T),\psi_i(T)) \leq\ & V(0,\phi_0(T),\psi_0(T)) \\ & -k\sum_{j=1}^{i}\int_0^T \dot{\rho}_j(\tau)e_j^2(\tau)d\tau. \end{aligned} \tag{31}$$

Thus, it follows that the tracking error is convergent in $L^2[0,T]$ norm due to the finiteness of $V(0,\phi_0(T),\psi_0(T))$ and the positive definiteness of $V(0,\phi_i(T),\psi_i(T))$. ■

In general, the discontinuous control scheme (25) is avoided (if possible), since it causes not only the problem of existence and uniqueness of solutions ([14]-[15]), but also chattering ([16]) that may excite high-frequency unmodeled dynamics ([17]). It motivates us to seek an appropriate smooth approximation of (25) that can guarantee the boundedness of the parameter estimations $\hat{\eta}_i(t)$ and $\hat{\lambda}_i(t)$, as well as the convergence of $e_i(t)$ to a reasonably small neighborhood of the origin.

Let $\overline{\varepsilon} > 0$ be a constant and consider the following smooth learning scheme

$$\begin{aligned} u_i(t) \quad = \quad & -ke_i(t) + \dot{\gamma}_i(t)/\dot{\rho}_i(t) - \hat{\eta}_i^T \Xi_i(t)\mathbf{f}(x_i) \\ & -\hat{\lambda}_i\omega(x_i,e_i),\ t \in [0,T], \end{aligned} \tag{32}$$

with the updating laws

$$\dot{\hat{\eta}}_i(t) = \dot{\rho}_i(t)\Xi_i(t)\mathbf{f}(x_i)e_i(t) - \sigma_1(\hat{\eta}_i(t) - \eta^0), \tag{33}$$
$$\dot{\hat{\lambda}}_i(t) = \dot{\rho}_i(t)e_i(t)\omega(x_i,e_i) - \sigma_2(\hat{\lambda}_i(t) - \lambda^0) \tag{34}$$

with $\hat{\eta}_0(0) = 0,\ \hat{\eta}_{i+1}(0) = \hat{\eta}_i(T)$ and $\hat{\lambda}_0(0) = 0,\ \hat{\lambda}_{i+1}(0) = \hat{\lambda}_i(T)$, where $\omega(x_i,e_i) \triangleq g(x_i)\tanh(\dot{\rho}_i g(x_i)e_i(t)/\overline{\varepsilon})$, $\sigma_1,\sigma_2 > 0$, and $\eta^0,\ \lambda^0$ are design constants. The updating laws (33) and (34) incorporate a leakage term based on a variant of σ modification [19][20]. By applying the controller (32) with the updating laws (33) and (34), the convergence property is summarize in the following theorem.

Theorem 3 *For the nonlinear system (14), under the assumptions 2, 3, the AILC scheme (32) with (33) and (34) guarantees that the tracking error $e_i(t)$ is ultimately bounded in $L^2[0,T]$ norm.*

Proof. Let $\phi_i(t) \triangleq \hat{\eta}_i(t) - \eta$ and $\chi_i(t) \triangleq \hat{\lambda}_i(t) - \lambda^*$, where $\lambda^* \triangleq \max\{\lambda,\lambda^0\}$. Similarly as the previous case, the error dynamics is

$$\begin{aligned} \dot{e}_i(t) \quad = \quad & -k\dot{\rho}_i(t)e_i(t) - \dot{\rho}_i(t)\phi_i^T \Xi_i(t)\mathbf{f}(x_i) \\ & +\dot{\rho}_i(t)\xi_i^T(t)\mathbf{f}(x_i) - \dot{\rho}_i(t)\hat{\lambda}_i\omega(x_i,e_i), \end{aligned} \tag{35}$$

where the controller (32) is applied. Differentiate the Lyapunov function

$$V(e_i,\phi_i,\chi_i) = \frac{1}{2}e_i^2(t) + \frac{1}{2}\phi_i^T(t)\phi_i(t) + \frac{1}{2}\chi_i^2(t), \tag{36}$$

in the ith iteration, we have

$$\begin{aligned} \dot{V}(e_i,\phi_i,\chi_i) \quad = \quad & -k\dot{\rho}_i e_i^2(t) + \dot{\rho}_i(t)\xi_i^T(t)\mathbf{f}(x_i)e_i(t) \\ & -\dot{\rho}_i(t)\hat{\lambda}_i(t)e_i(t)\omega(x_i,e_i) \\ & +\dot{\rho}_i(t)\chi_i(t)e_i(t)\omega(x_i,e_i) \\ & -\sigma_1\phi_i^T(t)(\hat{\eta}_i(t) - \eta^0) \\ & -\sigma_2\chi_i(t)(\hat{\lambda}_i(t) - \lambda^0), \end{aligned} \tag{37}$$

where the error dynamics (35) and two updating laws (33) and (34) are used. By completing the square, and

$|\xi_i^T(t)\mathbf{f}(x_i)e_i(t)| \le \lambda^* g(x_i)|e_i(t)|$, $t \in [0,T]$, we can obtain

$$\begin{aligned}\dot{V}(e_i,\phi_i,\psi_i) \le\ & -k\dot{\rho}_i e_i^2(t) - \frac{1}{2}\sigma_1\phi_i^T(t)\phi_i(t) - \frac{1}{2}\sigma_2\chi_i^2(t) \\ & +\lambda^*[\dot{\rho}_i(t)g(x_i)|e_i(t)| - \dot{\rho}_i(t)e_i(t)\omega(x_i,e_i)] \\ & +\frac{1}{2}\sigma_1(\eta-\eta^0)^T(\eta-\eta^0) \\ & +\frac{1}{2}\sigma_2(\lambda^*-\lambda^0)^2, \qquad (38)\end{aligned}$$

Claim[18]: The inequality $0 \le |u| - u\tanh(\frac{u}{\varepsilon}) \le \delta\varepsilon$ holds for any $\varepsilon > 0$ and for any $u \in \mathbb{R}$, where δ is a constant that satisfies $\delta = e^{-(\delta+1)}$, i.e., $\delta = 0.2785$.
According to Assumption 1, we have $\dot{\rho}_i(t) > 0$ for $t \in [0,T]$. Now, using the claim, it gives

$$\begin{aligned}& \lambda^*[\dot{\rho}_i(t)g(x_i)|e_i(t)| - \dot{\rho}_i(t)e_i(t)\omega(x_i,e_i)] \\ =\ & \lambda^*[\dot{\rho}_i(t)g(x_i)|e_i(t)| - \dot{\rho}_i e_i(t)g(x_i)\tanh(\frac{\dot{\rho}_i g(x_i)e_i(t)}{\overline{\varepsilon}})] \\ \le\ & \lambda^*\delta\overline{\varepsilon} \le \frac{1}{2}\lambda^*\overline{\varepsilon}. \qquad (39)\end{aligned}$$

Therefore, we obtain from (38)

$$\dot{V}(e_i,\phi_i,\chi_i) \le -c[V(e_i,\phi_i,\chi_i) - \frac{\gamma}{c}], \qquad (40)$$

where $c \triangleq \min\{2k\alpha,\sigma_1,\sigma_2\}$ and $\gamma \triangleq \frac{1}{2}\lambda^*\overline{\varepsilon} + \frac{1}{2}\sigma_1(\eta-\eta^0)^T(\eta-\eta^0) + \frac{1}{2}\sigma_2(\lambda^*-\lambda^0)^2$. Integrate (40) over $[0,T]$ and apply the mathematical induction, we have

$$\begin{aligned}V(0,\phi_i(T),\psi_i(T)) \le\ & V(0,\phi_0(T),\psi_0(T)) \qquad (41) \\ & -c\sum_{j=1}^{i}\int_0^T [V(e_i(\tau),\phi_i(\tau),\chi_i(\tau)) \\ & -\frac{\gamma}{c}]d\tau.\end{aligned}$$

Thus, according to the finiteness of $V(0,\phi_0(T),\psi_0(T))$ and the positive definiteness of $V(0,\phi_i(T),\psi_i(T))$, it gives that

$$\lim_{i\to\infty}\int_0^T V(e_i(\tau),\phi_i(\tau),\chi_i(\tau))d\tau = \frac{\gamma T}{c}. \qquad (42)$$

Finally, we have that the tracking error $e_i(t)$, $t \in [0,T]$, and the estimate errors $\phi_i(t)$, $\chi_i(t)$, $t \in [0,T]$ are ultimately bounded in $L^2[0,T]$ norm.
In addition, from (38) and (39) , it shows

$$\dot{V}(e_i,\phi_i,\psi_i) \le -k\dot{\rho}_i e_i^2(t) + \gamma \qquad (43)$$

Similar as (42), we can get

$$\begin{aligned}V(0,\phi_i(T),\psi_i(T)) \le\ & V(0,\phi_0(T),\psi_0(T)) \qquad (44) \\ & -k\sum_{j=1}^{i}\int_0^T [\dot{\rho}_i(\tau)e_i^2(\tau) - \frac{\gamma}{k}]d\tau.\end{aligned}$$

Since $V(0,\phi_0(T),\psi_0(T))$ is finite and $V(0,\phi_i(T),\psi_i(T))$ is positive definite, it follows

$$\int_0^T [\dot{\rho}_i(\tau)e_i^2(\tau) - \frac{\gamma}{k}]d\tau = 0. \qquad (45)$$

Therefore, we have $\lim_{i\to\infty}\int_0^T e_i^2 d\tau \le \frac{\gamma T}{k\alpha}$, i.e., the boundedness of tracking error $e_i(t)$, $t \in [0,T]$ in $L^2[0,T]$ norm is $\gamma T/(k\alpha)$. ■

Remark 2 *Theorem 3 reveals that the ultimate bound of tracking error is related to the feedback gain k and the parameter γ. By increasing k and decreasing γ, the ultimate bound of tracking error can be sufficient small, where γ is determined by $\overline{\varepsilon}$, σ_1, σ_2, η^0 and λ^0.*

5 ILLUSTRATIVE EXAMPLE

In order to show effectiveness of our proposed AILC algorithm, we consider the following nonlinear system with time-varying parameters

$$\frac{dx_i}{dt_i} = \theta(t_i)x_i^3 + v_i(t_i), \qquad (46)$$

where $\theta(t_i) = e^{\sin(t_i)} + 2\sin(\cos(t_i))$, $t_i \in [0,T_i]$, and T_i is the trial length of ith iteration. Similarly as example 1, let the reference trajectory in the ith iteration be

$$x_i^r(t_i) = \kappa_i(t_i)\sin(\lambda_i t_i),\ \ t_i \in [0, 2\pi/\lambda_i], \qquad (47)$$

where $\lambda_i = |\sin(i)| + 1/2$, $\kappa_i(t_i) = \cos(t_i) + 3/2$, and $t_i = \rho_i(t) = t/\lambda_i$. Assume

$$\theta(t_i) = \Phi^T(t_i)\eta + \varepsilon(t_i), \qquad (48)$$

where $\Phi(t_i) = [1, \sin(t_i), \cos(t_i), \sin(2t_i), \cos(2t_i)]$, $\eta = [\eta_1,\eta_2,\eta_3,\eta_4,\eta_5]^T$ and $|\varepsilon(t_i)x_i^3| < \lambda g(x_i)$.
Set the feedback gain $k = 1$ and $g(x_i) = 0.5|x_i|^3$ in the controller (25) with (26) and (27). The performance of the tracking error $\|e_i\|_{L_2}$ and the tracking performance for 1st and 10th iterations are presented in Fig. 1 and Fig. 2, respectively. In addition, Fig. 3 gives the control input signal in the 100th iteration. It can be seen that since the sign function is used in the controller (25), the input signal oscillates at high frequencies.

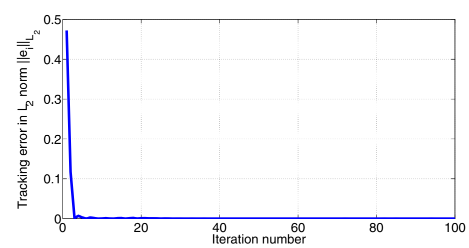

Figure 1: Tracking error in L_2-norm by using controller (25) with (26) and (27).

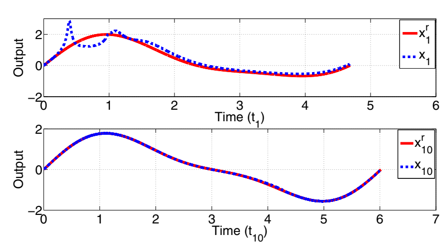

Figure 2: Output tracking profile by using controller (25) with (26) and (27).

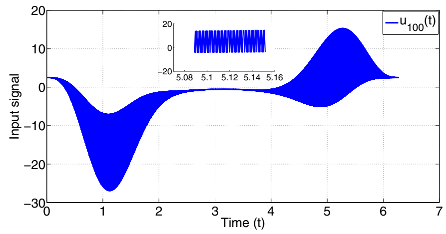

Figure 3: Input signal in the 50th iteration by using controller (25) with (26) and (27).

To avoid the chattering phenomena and demonstrate the effect of the function $\tanh(\bullet)$, we fix the feedback gain $k = 1$ and $g(x_i) = 0.5|x_i|^3$ in (32) and set $\bar{\varepsilon} = 0.1$. In addition, choose $\sigma_1 = 10^{-3}$, $\sigma_2 = 10^{-2}$, $\eta^0 = \lambda^0 = 1$ in the updating laws (33) and (34). The simulation results are shown in Figs. 4 and 5. Although convergence speed by applying the controller (32) is slower than that of (25), the control signal with the controller (32), as shown in Fig. 5, is smooth. It implies that the controller (32) with (33) and (34) is more applicable to practical systems.

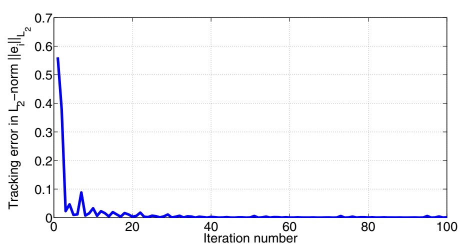

Figure 4: Tracking error in L_2-norm by using controller (32) with (33) and (34).

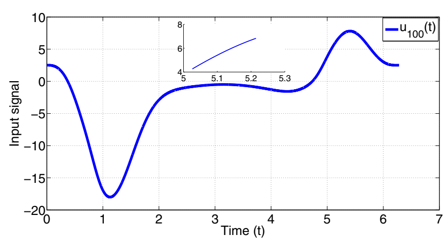

Figure 5: Input signal in the 50th iteration by using controller (32) with (33) and (34).

6 Conclusion

This paper presents the AILC design and analysis results for trajectories with different magnitude and time scales. Due to the variation of the magnitude and time scales, a new AILC scheme is developed by introducing time scale transformations and the convergence of tracking error is derived based on Lyapunov theory. The proposed AILC scheme overcomes the limitation of traditional ILC that the target trajectory must be identical in all iterations. In addition, the requirement on classic ILC that every trial must repeat in a fixed time duration is absolutely removed. The design method is novel and it is shown that the learning control system is capable of fully utilizing all the learned knowledge despite the iteratively varying tracking tasks.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, 1(2), pp. 123-140, 1984.
[2] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Syst. Mag.*, 26(3), pp. 96-114, 2006.
[3] Y. Q. Chen, K. L. Moore, J. Yu, and T. Zhang, "Iterative learning control and repetitive control in hard disk drive industry - a tutorial," *Int. J. Adaptive Control and Signal Processing*, 22, pp. 325-343, 2008.
[4] C. -J. Chien, "A discrete iterative learning control for a class of nonlinear time-varying systems," *IEEE Trans. Automat. Contr.*, 43(5), pp. 748-752, 1998.
[5] M. X. Sun and D. Wang, "Iterative learning control with initial rectifying action," *Automatica*, 38(8), pp. 1177-1182, 2002.
[6] K. -H. Park, "An average operator-based PD-type iterative learning control for variable initial state error," *IEEE Trans. Automat. Contr.*, 50(6), pp. 865-869, 2005.
[7] X. Li, J.-X. Xu and D. Huang, "An iterative learning control approach for linear systems with randomly varying trial lengths," *IEEE Trans. Automat. Contr.*, DOI 10.1109/TAC.2013.2294827.
[8] J.-X. Xu and T. Zhu, "Dual-scale direct learning control of trajectory tracking for a class of nonlinear uncertain systems," *IEEE Trans. Automat. Contr.* 44(10), pp. 1884-1888, 1999.
[9] W. G. V. Samer, S. Saab, and M. H. Mickle, "Learning control algorithms for tracking slowly varying trajectories," *IEEE Trans. Syst. Man Cybern. B, Cybern.* 27(4), pp. 657- 670, 1997.
[10] J.-X. Xu, "Direct learning of control efforts for trajectories with different magnitude scales," *Automatica* 33(12), pp. 2191-2195, 1997.
[11] J.-X. Xu, "Direct learning of control efforts for trajectories with different time scales," *IEEE Trans. Automat. Contr.* 43(7), pp. 1027-1030, 1998.
[12] J.-X. Xu , J. Xu and B. Viswanathan, "Recursive direct learning of control efforts for trajectories with different magnitude scales," *Asian J. Control* 4(1), pp. 49-59, 2002.
[13] J.-X. Xu and J. Xu, "On iterative learning from different tracking tasks in the presence of time-varying uncertainties," *IEEE Trans. Syst. Man Cybern. B, Cybern.* 34(1), pp. 589 - 597, 2004.
[14] Hàjek, "Discontinuous differential equations," Part I, *J. Diff. Eqns* 32, pp. 149 -170, 1979.
[15] M. M. Polycarpou and P. A. Ioannou, "On the existence and uniqueness of solutions in adaptive control systems," *IEEE Trans. Automat. Contr.* 38(3), pp. 474-479, 1993.
[16] V. I. Utkin, *Sliding Modes and Their Applications to Variable Structure Systems*, MIR, Moscow, 1978.
[17] J. -J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, NJ, 1991.
[18] M. M. Polycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica* 32(3), pp. 423-427, 1996.
[19] P. A. Ioannou and A. Datta, "Robust adaptive aontrol: a unified approach," *Proc. IEEE* 79, pp. 1736-1768, 1991.
[20] P. A. Ioannou and P. V. Kokotovic, *Adaptive Systems with Reduced Models*, Springer-Verlag, New York, 1983.