2-D H_∞ Based Iterative Learning Control Design for Linear Discrete-Time Uncertain Systems with Multiple High Order Internal Models

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Abstract: In this work we focus on the iterative learning control (ILC) design problem for linear discrete-time uncertain systems with iteration-varying factors, including reference trajectories, initial states, and exogenous disturbances. First, multiple high-order internal models (HOIMs) are given for various iteration-varying factors. Second, a new ILC scheme is constructed according to an augmented HOIM that is the aggregation of all HOIMs. Third, the HOIM-based ILC is transformed into a controller design problem of 2-D Roesser model. Fourth, the H_∞ performance of 2-D Roesser model is studied under a non-zero boundary condition. Then, a HOIM-based ILC design criterion is presented to achieve perfect tracking and 2-D H_∞ tracking performance which yields a high-order ILC (HO-ILC). Utilizing information provided by multiple HOIMs, it is shown that HO-ILC laws outperform low-order ILC (LO-ILC) laws in presence of iteration-varying factors. In addition, a composite HOIM-based law is proposed to improve the initial phase tracking performance. Finally, a numerical example is given to illustrate the efficiency of the proposed HOIM-based ILC design method.

Key Words: Iterative learning control, 2-D H_∞ theory, Linear discrete-time uncertain systems, Iteration-varying factors, Multiple high order internal models

1 Introduction

ILC is an effective control scheme for dynamical systems with repetitive operation over a fixed time interval. To date, numerous ILC techniques have been developed in the control field ([1]). In general, the system invariance property (i.e., the invariance of initial states, parameters, disturbances, and control tasks in the iteration domain) in classic ILC is indispensable to achieve perfect tracking. However, for many practical control systems, the system invariance property is hardly achievable. Therefore, the ILC design with iteration-varying factors is a problem of considerable importance in both theory and practical applications. For example, the iteration-varying initial states ([2]), reference trajectories ([3]-[4]), and parameters ([5]) have been frequently encountered. In practice, many disturbances along the iterative axis can be described with HOIMs ([6]-[7]), for example unknown leaving traffic flow of a freeway and power demand of a region are varying from long term perspective at a temporal scale of daily, weekly, monthly or quarterly span ([8]). Such long term disturbances are in general predictable based on historical data. Therefore, among various kinds of iteration-varying factors, we deal with factors that are specified by known multiple HOIMs .

In recent years, the 2-D system theory was successfully introduced to the ILC field ([9]-[12]). However, the system invariance property is still a fundamental assumption for perfect tracking. For instance, in [13], for a piecewise linearizable nonlinear batch process with iteration-invariant initial state and reference trajectory, state-feedback and output-feedback ILC design criteria were presented based on a 2-D H_∞ control method. Furthermore, to improve the tracking performance, an effective approach is to design HO-ILC laws by using more of the past control information ([14]-[15]). However, it is also reported that the HO-ILC may not be helpful to improve the tracking performance ([16]-[17]). For example, it was shown in [18] that an optimal HO-ILC does not add to the optimality of LO-ILC if the iteration-varying factors are modeled as zero-mean white noise.

It is worth noting that HO-ILC has demonstrated to outperform LO-ILC in many experimental applications (see e.g., [19]). It motivates the authors to consider the following two key issues and provide solu-

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tions. First, a HOIM-based ILC is constructed, as a kind of HO-ILC law, to improve the tracking performance when the information of iteration-varying factors is available. Second, the H_∞ theory of 2-D systems is applied to design the law such that the effect of iteration-varying factors can be further reduced by incorporating the current-cycle feedback with the state information.

The paper is organized as follows. In Section II, for linear systems with iteration-varying factors specified by multiple HOIMs, a special HOIM-based ILC is introduced. Then, the ILC design problem is described as a controller design problem of a 2-D Roesser model. In Section III, the H_∞ performance of 2-D Roesser model is analyzed. In Section IV, a HOIM-based ILC design criterion is established based on the 2-D H_∞ theory, and a composite HOIM-based ILC is presented to improve the initial phase tracking performance. In the end, two illustrative examples are provided in Section V.

Notations. Denote $\|\cdot\|$ the usual Euclidean norm and $\|\cdot\|_{[\alpha,\beta]}$ the ℓ_2 norm over $\ell_2[\alpha,\beta]$, i.e., $\|\boldsymbol{w}\|_{[\alpha,\beta]} = \sqrt{\sum_{t=\alpha}^{\beta}\|\boldsymbol{w}(t)\|^2}$. In addition, the notation $*$ represents the elements below the main diagonal of a symmetric matrix, $\oplus$ stands for the direct sum, $\mathcal{N}_a$ denotes the integers greater than or equal to a, and $\mathcal{N}_{a,b}$ denotes the integer sequence $a, a+1, \ldots, b$ with $b > a$.

2 Problem Formulation

Consider the following discrete-time linear system

$$\left\{\begin{array}{rcl} \boldsymbol{x}_k(t+1) & = & (A+\Delta A(t))\boldsymbol{x}_k(t) + (B+ \\ & & \Delta B(t))\boldsymbol{u}_k(t) + \boldsymbol{w}_k(t), \\ \boldsymbol{y}_k(t) & = & C\boldsymbol{x}_k(t) + \boldsymbol{v}_k(t), \end{array}\right. \quad (1)$$

where $t \in \mathcal{N}_{0,T}$ is the discrete-time index, $k \in \mathcal{N}_0$ is the iteration index, $\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{u} \in \mathbb{R}^p, \boldsymbol{y} \in \mathbb{R}^q, \boldsymbol{w} \in \mathbb{R}^n, \boldsymbol{v} \in \mathbb{R}^q$ denote the state, input, output, state disturbance, and output disturbance, respectively, and $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}$, and $C \in \mathbb{R}^{q\times n}$ are known real constant matrices. Furthermore, $\Delta A(t), \Delta B(t)$ are unknown matrices representing parameter uncertainties, and assumed to be the form $[\Delta A(t)\ \Delta B(t)] = DF(t)[E_1\ E_2]$, where $D \in \mathbb{R}^{n\times s}, E_1 \in \mathbb{R}^{d\times n}, E_2 \in \mathbb{R}^{d\times p}$ are known real constant matrices and $F : \mathcal{N}_{0,T} \to \mathbb{R}^{s\times d}$ are unknown matrix function satisfying $F^T(t)F(t) \le I_d, \forall t \in \mathcal{N}_{0,T}$. The reference trajectory, initial state, and exogenous disturbances are iteration-varying and satisfy the following restrictions:

A1) The iteration-varying reference trajectories are generated by the following stable HOIM

$$\left\{\begin{array}{rcll} \boldsymbol{r}_{k+1}(t) & = & H_1(z^{-1})\boldsymbol{r}_k(t), & k \in \mathcal{N}_{m_1-1}, \\ \boldsymbol{r}_k(t) & = & \boldsymbol{r}_k^0(t), & k \in \mathcal{N}_{0,m_1-1}, \end{array}\right. \quad (2)$$

where m_1 denotes the order, $H_1(z^{-1}) = h_1 + h_2 z^{-1} + \ldots + h_{m_1} z^{-m_1+1} (h_j \in \mathbb{R})$, z denotes the shift operator in the iteration domain with the property $z^{-1}\boldsymbol{r}_k(t) = \boldsymbol{r}_{k-1}(t)$, and $\boldsymbol{r}_k^0(t), k \in \mathcal{N}_{0,m_1-1}$ denote the initial reference trajectories.

A2) The initial state and exogenous disturbances satisfy

$$\left\{\begin{array}{l} \sum_{k=m_2-1}^{\infty}\|\boldsymbol{x}_{k+1}(0) - H_2(q^{-1})\boldsymbol{x}_k(0)\|^2 < \infty, \\ \sum_{k=m_3-1}^{\infty}\|\boldsymbol{w}_{k+1} - H_3(q^{-1})\boldsymbol{w}_k\|_{[0,T]}^2 < \infty, \\ \sum_{k=m_4-1}^{\infty}\|\boldsymbol{v}_{k+1} - H_4(q^{-1})\boldsymbol{v}_k\|_{[0,T]}^2 < \infty, \end{array}\right. \quad (3)$$

where $H_i(z^{-1})$ with order $m_i (i = 2, 3, 4)$ are stable HOIMs.

To handle iteration-varying factors satisfying assumptions A1)-A2), the following HOIM-based ILC is proposed:

$$\left\{\begin{array}{rcl} \boldsymbol{u}_{k+1}(t) & = & G_\varrho(z^{-1})\boldsymbol{u}_k(t) + \delta\boldsymbol{u}_k(t), \\ & & k \in \mathcal{N}_{\varrho-1}, \\ \boldsymbol{u}_k(t) & = & \boldsymbol{u}_k^0(t), k \in \mathcal{N}_{0,\varrho-1}, \end{array}\right. \quad (4)$$

where ϱ is the order of ILC, $\delta\boldsymbol{u}$ is the modification of control input that will be given later, $\boldsymbol{u}_k^0, k \in \mathcal{N}_{0,\varrho-1}$ denote the initial inputs that are generally set as zero for implementation, and $G_\varrho(z^{-1}) = \beta_1 + \beta_2 z^{-1} + \ldots + \beta_\varrho z^{-\varrho+1}$ is the HOIM of ILC and satisfies the following relationship:

$$\begin{array}{l} 1 - G_\varrho(z^{-1})z^{-1} \text{ can be divided exactly by} \\ \quad 1 - H_1(z^{-1})z^{-1}, \ldots, 1 - H_4(z^{-1})z^{-1}. \end{array} \quad (5)$$

To ensure the HOIM-based ILC (4) can be used to achieve perfect tracking, we first give the following property.

Property 2.1 *For the iteration-varying factors satisfying assumptions A1)-A2) and the HOIM $G_\varrho(z^{-1})$ satisfying the relationship (5), the following equality and inequalities hold:*

$$\left\{\begin{array}{l} \boldsymbol{r}_{k+1}(t) = G_\varrho(q^{-1})\boldsymbol{r}_k(t), \quad \forall k \in \mathcal{N}_{\varrho-1}, \\ \sum_{k=\varrho-1}^{\infty}\|\boldsymbol{x}_{k+1}(0) - G_\varrho(q^{-1})\boldsymbol{x}_k(0)\|^2 < \infty, \\ \sum_{k=\varrho-1}^{\infty}\|\boldsymbol{w}_{k+1} - G_\varrho(q^{-1})\boldsymbol{w}_k\|_{[0,T]}^2 < \infty, \\ \sum_{k=\varrho-1}^{\infty}\|\boldsymbol{v}_{k+1} - G_\varrho(q^{-1})\boldsymbol{v}_k\|_{[0,T]}^2 < \infty. \end{array}\right. \quad (6)$$

Next, let us transform the closed-loop ILC system (1) and (4) into a 2-D system. To facilitate ILC design that allows for iteration-varying factors satisfying (6), we define

$$\begin{array}{rcl} \boldsymbol{e}_k(t) & = & \boldsymbol{r}_k(t) - \boldsymbol{y}_k(t), \\ \delta\boldsymbol{f}_k(t) & = & \boldsymbol{f}_{k+1}(t-1) - G_\varrho(z^{-1})\boldsymbol{f}_k(t-1), \end{array}$$

where $\boldsymbol{f}$ might be the state $\boldsymbol{x}$ and exogenous disturbances $\boldsymbol{w}, \boldsymbol{v}$. Note that $\boldsymbol{r}_{k+1}(t) = G_\varrho(z^{-1})\boldsymbol{r}_k(t)$. Then, from (1) and (4), we can easily obtain $\boldsymbol{e}_{k+1}(t) = G_\varrho(z^{-1})\boldsymbol{e}_k(t) - C\delta\boldsymbol{x}_k(t+1) - \delta\boldsymbol{v}_k(t+1)$ and $\delta\boldsymbol{x}_k(t+1) = (A+\Delta A(t))\delta\boldsymbol{x}_k(t) + (B+\Delta B(t))\delta\boldsymbol{u}_k(t-1) +$

$\delta \boldsymbol{w}_k(t)$. It follows that

$$\begin{cases} \begin{bmatrix} \delta \boldsymbol{x}_k(t+1) \\ \hat{\boldsymbol{e}}_{k+1}(t) \end{bmatrix} = \mathcal{A} \begin{bmatrix} \delta \boldsymbol{x}_k(t) \\ \hat{\boldsymbol{e}}_k(t) \end{bmatrix} + \mathcal{B}_1 \\ \quad \delta \boldsymbol{u}_k(t-1) + \mathcal{B}_2 \boldsymbol{\varpi}_k(t) + \mathcal{D}\boldsymbol{\psi}_k(t), \\ \boldsymbol{z}_k(t) = \boldsymbol{e}_k(t) = \mathcal{L} \begin{bmatrix} \delta \boldsymbol{x}_k(t) \\ \hat{\boldsymbol{e}}_k(t) \end{bmatrix}, \\ t \in \mathcal{N}_{1,T}, k \in \mathcal{N}_{\varrho-1}, \end{cases} \tag{7}$$

where $\hat{\boldsymbol{e}}_k(t) = [\boldsymbol{e}_k^T(t)\ \boldsymbol{e}_{k-1}^T(t)\ \ldots\ \boldsymbol{e}_{k-\varrho+1}^T(t)]^T, \boldsymbol{\varpi}_k(t) = [\delta \boldsymbol{w}_k^T(t)\ \ \delta \boldsymbol{v}_k^T(t+1)]^T, \mathcal{B}_1 = [B, -CB, 0, \ldots, 0]^T$, $\mathcal{D} = [D, -CD, 0, \ldots, 0]^T, \boldsymbol{\psi}_k(t) = F(t)[E_1 \delta \boldsymbol{x}_k(t) + E_2 \delta \boldsymbol{u}_k(t-1)]$, $\Theta = \begin{bmatrix} E_1^T E_1 & E_1^T E_2 \\ * & E_2^T E_2 \end{bmatrix}, \mathcal{L} = [0, I_q, 0, \ldots, 0]$, and

$$\boldsymbol{\psi}_k^T(t)\boldsymbol{\psi}_k(t) \le \begin{bmatrix} \delta \boldsymbol{x}_k(t) \\ \delta \boldsymbol{u}_k(t-1) \end{bmatrix}^T \Theta \begin{bmatrix} \delta \boldsymbol{x}_k(t) \\ \delta \boldsymbol{u}_k(t-1) \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} A & 0 & \ldots & 0 & 0 \\ -CA & \beta_1 I_q & \ldots & \beta_{\varrho-1} I_q & \beta_\varrho I_q \\ 0 & I_q & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I_q & 0 \end{bmatrix},$$

$$\mathcal{B}_2 = \begin{bmatrix} I_n & 0 \\ -C & -I_q \\ 0 & 0 \\ \vdots & . \\ 0 & 0 \end{bmatrix}.$$

3 H_∞ performance of 2-D Roesser model

Inspired by the structure of 2-D ILC system (7), the H_∞ performance of the following 2-D Roesser model is analyzed in this section:

$$\begin{cases} \begin{bmatrix} \bar{\boldsymbol{x}}^h(i+1,j) \\ \bar{\boldsymbol{x}}^v(i,j+1) \end{bmatrix} = \bar{A} \begin{bmatrix} \bar{\boldsymbol{x}}^h(i,j) \\ \bar{\boldsymbol{x}}^v(i,j) \end{bmatrix} + \bar{B}\bar{\boldsymbol{\varpi}}(i,j) \\ \qquad + \bar{D}\bar{\boldsymbol{\psi}}(i,j), \\ \bar{\boldsymbol{\psi}}^T(i,j)\bar{\boldsymbol{\psi}}(i,j) \le \bar{\boldsymbol{x}}^T(i,j)\bar{\Theta}\bar{\boldsymbol{x}}(i,j), \\ \bar{\boldsymbol{z}}(i,j) = \bar{L}\bar{\boldsymbol{x}}(i,j),\ i \in \mathcal{N}_{1,T}, j \in \mathcal{N}_0, \end{cases} \tag{8}$$

where $\bar{\boldsymbol{x}} = [\bar{\boldsymbol{x}}^{hT}\ \bar{\boldsymbol{x}}^{vT}]^T \in \mathbb{R}^{\bar{n}}$ is the state and $\bar{\boldsymbol{x}}^h \in \mathbb{R}^{\bar{n}_1}, \bar{\boldsymbol{x}}^v \in \mathbb{R}^{\bar{n}_2}, \bar{n}_1 + \bar{n}_2 = \bar{n}$ represent the horizontal and vertical states, respectively. Further, $\bar{\boldsymbol{z}} \in \mathbb{R}^{\bar{q}}, \bar{\boldsymbol{\varpi}} \in \mathbb{R}^{\bar{m}}, \bar{\boldsymbol{\psi}} \in \mathbb{R}^{\bar{s}}$ denote the controlled output, disturbance, and uncertainty, respectively, and $\bar{A}, \bar{B}, \bar{D}, \bar{L}$, and $\bar{\Theta}$ are appropriate dimensional real matrices.

Note the fact that the 2-D ILC system (7) satisfies $\sum_{k=\varrho-1}^{\infty} \|\delta \boldsymbol{x}_k(1)\|^2 < \infty$ and $\sum_{k=\varrho-1}^{\infty} \|\boldsymbol{\varpi}_k\|_{[1,T]}^2 < \infty$ from (6). Therefore, the boundary state and disturbance of 2-D system (8) satisfy

$$\sum_{j=0}^{\infty} \|\bar{\boldsymbol{x}}^h(1,j)\|^2 < \infty, \sum_{j=0}^{\infty} \|\bar{\boldsymbol{\varpi}}(j)\|_{[1,T]}^2 < \infty. \tag{9}$$

Similar to Definition 2.2 in [20] for the zero boundary condition, the following definition of 2-D H_∞ performance is given for the non-zero boundary condition given in (9).

Definition 3.1 *Suppose that the 2-D Roesser model (8) satisfies (9). The 2-D system is said to have 2-D H_∞ performance if*

1) *The 2-D system is asymptotically stable, i.e, $\lim_{j\to\infty} \bar{\boldsymbol{x}}(i,j) = 0, \forall i \in \mathcal{N}_{1,T}$, when $\bar{\boldsymbol{\varpi}}(i,j) = 0$.*
2) *The 2-D system satisfies*

$$\begin{aligned} &\textstyle\sum_{j=0}^{\infty} \|\bar{\boldsymbol{z}}(j)\|_{[1,T]}^2 < \gamma_1 \|\bar{\boldsymbol{x}}^v(0)\|_{[1,T]}^2 \\ &\quad + \gamma_2 \textstyle\sum_{j=0}^{\infty} \|\bar{\boldsymbol{x}}^h(1,j)\|^2 \\ &\quad + \gamma_3 \textstyle\sum_{j=0}^{\infty} \|\bar{\boldsymbol{\varpi}}(j)\|_{[1,T]}^2 \end{aligned} \tag{10}$$

for any non-zero bounded disturbance $\bar{\boldsymbol{\varpi}}$ and all admissible uncertainty $\bar{\boldsymbol{\psi}}$, where $\gamma_1 > 0$ and $\gamma_2 > 0$ are prescribed scalars.

In order to obtain the 2-D H_∞ performance of (8), we first analyze the asymptotic stability of the system (8) with $\bar{\boldsymbol{\varpi}} = 0$.

Theorem 3.1 *Suppose that the 2-D Roesser model (8) satisfies (9). The 2-D system with $\bar{\boldsymbol{\varpi}} = 0$ is asymptotically stable if there exist a positive scalar μ, symmetric positive definite matrices $P_1 \in \mathbb{R}^{\bar{n}_1 \times \bar{n}_1}, P_2 \in \mathbb{R}^{\bar{n}_2 \times \bar{n}_2}, P \triangleq P_1 \oplus P_2$ such that*

$$\begin{bmatrix} \bar{A}^T P \bar{A} - P + \mu\bar{\Theta} & \bar{A}^T P \bar{D} \\ * & -\mu I_{\bar{s}} \end{bmatrix} < 0. \tag{11}$$

Next, we need to get the inequality (10) to complete the analysis of 2-D H_∞ performance of (8). Based on the asymptotic stability criterion in Theorem 3.1, the following theorem is established.

Theorem 3.2 *Suppose that the 2-D Roesser model (8) satisfies (9). If there exist a positive scalar μ, symmetric positive definite matrices $P_1 \in \mathbb{R}^{\bar{n}_1 \times \bar{n}_1}, P_2 \in \mathbb{R}^{\bar{n}_2 \times \bar{n}_2}, P \triangleq P_1 \oplus P_2$, and $Q \in \mathbb{R}^{\bar{p} \times \bar{p}}$ such that*

$$\begin{bmatrix} \begin{matrix} \bar{A}^T P \bar{A} - P \\ + \bar{L}^T \bar{L} + \mu\bar{\Theta} \end{matrix} & \bar{A}^T P \bar{B} & \bar{A}^T P \bar{D} \\ * & \bar{B}^T P \bar{B} - Q & \bar{B}^T P \bar{D} \\ * & * & \bar{D}^T P \bar{D} - \mu I_{\bar{s}} \end{bmatrix} < 0, \tag{12}$$

then the 2-D Roesser model (8) has the 2-D H_∞ performance (10) with $\gamma_1 = \lambda_{\max}(P), \gamma_2 = \lambda_{\max}(Q)$.

4 2-D H_∞ based ILC design with HOIMs

In this section, a HOIM-based ILC design criterion is presented by using the result of Theorem 3.2 for the 2-D ILC system (7). Choose the HOIM-based ILC (4) as

$$\begin{aligned} \boldsymbol{u}_{k+1}(t) &= G_\varrho(z^{-1})\boldsymbol{u}_k(t) + K_0 \left[\boldsymbol{x}_{k+1}(t) \right. \\ &\quad \left. - G_\varrho(z^{-1})\boldsymbol{x}_k(t)\right] + \textstyle\sum_{i=1}^{\varrho} K_i \boldsymbol{e}_{k-i+1}(t+1), \end{aligned} \tag{13}$$

where $K_0, K_1, \ldots, K_\varrho$ are design parameters. It is not difficult to see that the sum of latter two items of above equation is the modification $\delta \boldsymbol{u}(t)$ in (4). Then, we have $\delta \boldsymbol{u}_k(t-1) = \mathcal{K}[\delta^T \boldsymbol{x}_k(t)\ \hat{\boldsymbol{e}}_k^T(t)]^T$ that is a state-feedback controller of the 2-D ILC system (7), where $\mathcal{K} = [K_0\ K_1\ \ldots\ K_\varrho]$.

By using Theorem 3.2, we can establish the following HOIM-based ILC design criterion, which is the main contribution of this paper.

Theorem 4.1 *Suppose that the linear system (1) satisfies assumptions A1)-A2) and the HOIM $G_\varrho(z^{-1})$ of ILC (13) satisfies the relationship (5). If, for given positive scalars μ, α_1, α_2 satisfying $\alpha_1 + \alpha_2 = 1$, there exist symmetric positive definite matrices $W_1 \in \mathbb{R}^{n\times n}, W_2 \in \mathbb{R}^{\varrho q\times \varrho q}, W \triangleq W_1 \oplus W_2, Q \in \mathbb{R}^{(n+q)\times(n+q)}$, and $R \in \mathbb{R}^{p\times(n+\varrho q)}$ such that the following* generalised eigenvalue problem

$$\begin{cases} \min(\gamma) \\ s.t\ the\ constraints: \\ \Sigma < 0, I_{n+\varrho q} < \gamma\alpha_1 W, Q < \gamma\alpha_2 I_{n+q} \end{cases} \tag{14}$$

is feasible, where

$$\Sigma = \begin{bmatrix} -W & \Sigma_{12} & \mathcal{B}_2 & \mathcal{D} & 0 & 0 \\ * & -W & 0 & 0 & W\mathcal{L}^T & \Sigma_{26}^T \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & -\mu I_s & 0 & 0 \\ * & * & * & * & -I_p & 0 \\ * & * & * & * & * & -\mu^{-1}\Theta^{-1} \end{bmatrix},$$

with $\Sigma_{12} = \mathcal{A}W + \mathcal{B}_1 R, \Sigma_{26} = \begin{bmatrix} [I_n\ 0]W \\ R \end{bmatrix}$, *then the closed-loop ILC system (1) and (13) with $[K_0\ K_1\ \ldots\ K_\varrho] = RW^{-1}$ satisfies the 2-D H_∞ performance*

$$\begin{aligned} &\textstyle\sum_{k=\varrho-1}^{\infty} \|\boldsymbol{e}_k\|^2_{[1,T]} < \gamma_1 \sum_{k=0}^{\varrho-1} \|\boldsymbol{e}_k\|^2_{[1,T]} \\ &\quad +\gamma_2 \textstyle\sum_{k=\varrho-1}^{\infty} \|\delta \boldsymbol{x}_k(1)\|^2 \\ &\quad +\gamma_3 \textstyle\sum_{k=\varrho-1}^{\infty} \|\boldsymbol{\varpi}_k\|^2_{[1,T]} \end{aligned} \tag{15}$$

with $\gamma_1 = \lambda_{\max}(W_2^{-1}), \gamma_2 = \lambda_{\max}(W_1^{-1}), \gamma_3 = \lambda_{\max}(Q)$,

It is worth noting that there are ϱ initial inputs in the HOIM-based ILC (4). There is no learning in the initial interval $k \in \mathcal{N}_{0,\varrho-1}$, which may seriously affects the initial phase tracking performance. As such, we will introduce a *composite HOIM-based ILC law* for implementation. Without loss of generality, we assume that there exist $\kappa(\kappa \le 4)$ iteration-varying factors with HOIMs $H_i(z^{-1})$ whose orders satisfy $1 \le m_1 \le \ldots \le m_\kappa$. Then, the composite HOIM-based ILC law is given as:

$$\begin{cases} \boldsymbol{u}_{k+1}(t) = G_{\varrho_\kappa}(z^{-1})\boldsymbol{u}_k(t) + K^0_{\varrho_\kappa}[\boldsymbol{x}_{k+1}(t) \\ \quad -G_{\varrho_\kappa}(z^{-1})\boldsymbol{x}_k(t)] + \sum_{i=1}^{\varrho_\kappa} K^i_{\varrho_\kappa}\boldsymbol{e}_{k-i+1}(t+1), \\ \quad k \in \mathcal{N}_{\varrho_\kappa - 1}, \\ \boldsymbol{u}_{k+1}(t) = G_{\varrho_j}(z^{-1})\boldsymbol{u}_k(t) + K^0_{\varrho_j}[\boldsymbol{x}_{k+1}(t) \\ \quad -G_{\varrho_j}(z^{-1})\boldsymbol{x}_k(t)] + \sum_{i=1}^{\varrho_j} K^i_{\varrho_j}\boldsymbol{e}_{k-i+1}(t+1), \\ \quad k \in \mathcal{N}_{\varrho_j-1,\varrho_{j+1}-2}, \\ \quad j = 1, \ldots, \kappa-1, \\ \boldsymbol{u}_k(t) = \boldsymbol{u}^0_k(t), k \in \mathcal{N}_{0,\varrho_1-1}, \end{cases} \tag{16}$$

where $[K^0_{\varrho_i}\ K^1_{\varrho_i}\ \ldots\ K^{\varrho_i}_{\varrho_i}](i \in \mathcal{N}_{1,\kappa})$ are the learning gains obtained from LMI (14) with $G_\varrho(z^{-1}) = G_{\varrho_i}(z^{-1})$, here $G_{\varrho_i}(z^{-1})$ is the HOIM with minimum order ϱ_i that allows $1 - G_{\varrho_i}(z^{-1})z^{-1}$ to be divided exactly by $1 - H_1(z^{-1})z^{-1}, \ldots, 1 - H_i(z^{-1})z^{-1}$.

Remark 4.1 *The perfect tracking and 2-D H_∞ performance is guaranteed by the first ILC law in (16) that rejecting all the iteration-varying factors, and the initial phase tracking performance is improved by the other ILC laws in (16) that rejecting the parts of those factors. In other words, to improve the tracking performance, the information of iteration-varying factors is fully utilized in the composite HOIM-based ILC law (16).*

5 Numerical examples

Consider the linear system (1) with

$$A = \begin{bmatrix} -0.5 & 0.3 \\ 0.2 & -0.7 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0.3 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.45 & -0.1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, E_2 = 0.2.$$

Assume that $\boldsymbol{r}_k(t) \equiv \sin(0.1t)$, $F(t) = 0.5\cos(0.1t) + 0.5\mathfrak{R}_{[-1,1]}(t)$, $\boldsymbol{x}_k(0) = (-1)^k[1\ 2]^T$, $\boldsymbol{w}_k(t) \equiv 0$, and $v_k(t) \equiv 0$ for any $t \in \mathcal{N}_{0,20}$, where $\mathfrak{R}_{[-1,1]}$ denotes a random number in the interval $[-1, 1]$. It implies that $\kappa = 2, H_1(z^{-1}) = 1$ and $H_2(z^{-1}) = -1$. Then, from $1 - G_1(z^{-1})z^{-1} = 1 - H_1(z^{-1})z^{-1}$ and $1 - G_2(z^{-1})z^{-1} = (1 - H_1(z^{-1})z^{-1})(1 - H_2(z^{-1})z^{-1})$, we can get $G_1(z^{-1}) = 1$ and $G_2(z^{-1}) = z^{-1}$, respectively. By solving the LMI (14) with $G_\varrho = G_2(z^{-1})$, we obtain a second-order ILC law

$$\begin{aligned} \boldsymbol{u}_{k+1}(t) = \boldsymbol{u}_{k-1}(t) + K_0[\boldsymbol{x}_{k+1}(t) - \boldsymbol{x}_{k-1}(t)] \\ + \textstyle\sum_{i=1}^{2} K_i \boldsymbol{e}_{k-i+1}(t+1) \end{aligned} \tag{17}$$

with $K_0 = [0.54\ \ -0.39]^T, K_1 = 0, K_2 = 1.78$. Furthermore, by solving the LMI (14) with $G_\varrho = G_1(z^{-1})$, a first-order ILC law is given:

$$\begin{aligned} \boldsymbol{u}_{k+1}(t) = \boldsymbol{u}_k(t) + K_0[\boldsymbol{x}_{k+1}(t) + \boldsymbol{x}_k(t)] \\ +K_1\boldsymbol{e}_k(t+1) \end{aligned} \tag{18}$$

with $K_0 = [0.54\ \ -0.38]^T, K_1 = 1.78$. Under the first-order ILC law (18), second-order ILC law (17), and the composite HOIM-based ILC law (17)-(18), the convergence profiles of the total square error $\mathcal{E}(k) = \sum_{t=1}^{T} |\boldsymbol{r}_k(t) - \boldsymbol{y}_k(t)|$ along the iteration axis k are given in Fig. 1.

From Fig.1, we see that the first-order ILC only achieves bounded tracking because of $\sum_{k=0}^{\infty} |\boldsymbol{x}_{k+1}(0) - \boldsymbol{x}_k(0)|^2 \to \infty$, but the second-order ILC can achieves perfect tracking because of $\sum_{k=1}^{\infty} |\boldsymbol{x}_{k+1}(0) - G_{\varrho_2}(z^{-1})\boldsymbol{x}_k(0)|^2 = \sum_{k=1}^{\infty} |\boldsymbol{x}_{k+1}(0) - \boldsymbol{x}_{k-1}(0)|^2 = 0$. Furthermore, by compositing the first-order ILC and second-order ILC, the tracking performance is greatly improved.

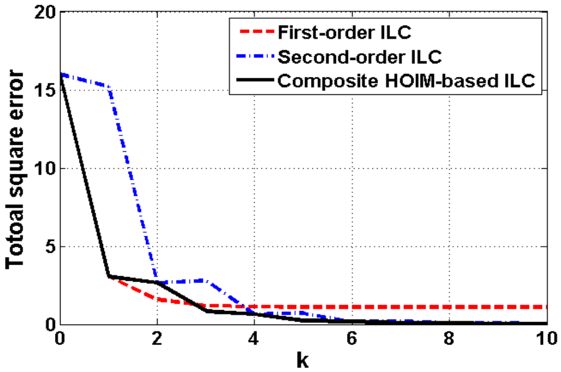

Fig. 1: The convergence profiles of total square error of Example ?? along iteration axis k.

6 Conclusions

A 2-D H_∞ based ILC design method is presented for linear discrete-time systems with iteration-varying factors, i.e., reference trajectories, initial states, and exogenous disturbances. For these iteration-varying factors that are generated by or ultimately satisfy multiple HOIMs, a new ILC scheme is presented with an augmented HOIM that is the aggregation of all HOIMs. Then, a HOIM-based ILC design criterion is presented by establishing the H_∞ theory of 2-D Roesser model under the non-zero boundary condition. It is shown that the HO-ILC law could contributes to improve the tracking performance, because the more iteration-varying information utilized the better tracking performance achieved. Finally, a composite HOIM-based ILC is proposed to achieve the perfect tracking and improve the initial phase tracking performance.

Appendix A: *Proof of Property 2.1*

Only the proof of inequality $\sum_{k=\varrho-1}^{\infty}|\boldsymbol{x}_{k+1}(0)-G_\varrho(z^{-1})\boldsymbol{x}_k(0)|^2<\infty$ is given in detail. The remaining one equality and two inequalities can be proved in similar ways.

Note that $1-G_\varrho(z^{-1})z^{-1}$ can be divided exactly by $1-H_2(z^{-1})z^{-1}$. Then, we define $1-G_\varrho(z^{-1})z^{-1}=(1-H_2(z^{-1})z^{-1})\bar{G}_o(z^{-1})$ and $\bar{G}_o(z^{-1})=1+\chi_1 z^{-1}+\ldots+\chi_{\varrho-m_2}z^{-\varrho+m_2}(\chi_i\in\mathbb{R})$. Thus, we have $\sum_{k=\varrho-1}^{\infty}|\boldsymbol{x}_{k+1}(0)-G_\varrho(z^{-1})\boldsymbol{x}_k(0)|^2=\sum_{k=\varrho-1}^{\infty}|\bar{G}_o(z^{-1})(1-H_2(z^{-1})z^{-1})\boldsymbol{x}_{k+1}(0)|^2\le(1+\sum_{i=1}^{\rho-m_2}|\chi_i|)\sum_{k=m_2-1}^{\infty}|(1-H_2(z^{-1})z^{-1})\boldsymbol{x}_{k+1}(0)|^2<\infty$, since $\sum_{k=m_2-1}^{\infty}|\boldsymbol{x}_{k+1}(0)-H_2(z^{-1})\boldsymbol{x}_k(0)|^2<\infty$. □

Appendix B: *Proof of Theorem 3.1*

Let $V_1(\boldsymbol{x})=\boldsymbol{x}^TP_1\boldsymbol{x}, V_2(\boldsymbol{x})=\boldsymbol{x}^TP_2\boldsymbol{x}$, and $\Delta V(\bar{\boldsymbol{x}})=V_1(\bar{\boldsymbol{x}}^h(i+1,j))+V_2(\bar{\boldsymbol{x}}^v(i,j+1))-V_1(\bar{\boldsymbol{x}}^h(i,j))-V_2(\bar{\boldsymbol{x}}^v(i,j))$. Then, computing the difference $\Delta V(\bar{\boldsymbol{x}})$ along the solution of (8) with $\bar{\boldsymbol{\varpi}}=0$ yields $\Delta V(\bar{\boldsymbol{x}})\le\Delta V(\bar{\boldsymbol{x}})+\mu[\bar{\boldsymbol{x}}^T(i,j)\bar{\Theta}\bar{\boldsymbol{x}}(i,j)-\bar{\boldsymbol{\psi}}^T(i,j)\bar{\boldsymbol{\psi}}(i,j)]=-\boldsymbol{\zeta}^T(i,j)\Upsilon\boldsymbol{\zeta}(i,j)$, where $\mu>0$, $\boldsymbol{\zeta}=[\bar{\boldsymbol{x}}\ \bar{\boldsymbol{\psi}}]^T$, and $(-\Upsilon)$ is the left-hand side of inequality (11). Thus, we have $\Delta V(\bar{\boldsymbol{x}})<0$ and $\Delta V(\bar{\boldsymbol{x}})\le-\frac{\lambda_{\min}(\Upsilon)}{\lambda_{\max}(\Upsilon)}V_2(\bar{\boldsymbol{x}}^v(i,j))$, if the inequality (11) holds.

From $\Delta V(\bar{\boldsymbol{x}})<0$, we can see $\sum_{i=1}^{q}\sum_{j=0}^{p}\Delta V(\bar{\boldsymbol{x}})=\sum_{j=0}^{p}[\sum_{i=1}^{q}V_1(\bar{\boldsymbol{x}}^h(i+1,j))-\sum_{i=1}^{q}V_1(\bar{\boldsymbol{x}}^h(i,j))]+\sum_{i=1}^{q}[\sum_{j=0}^{p}V_2(\bar{\boldsymbol{x}}^v(i,j+1))-\sum_{j=0}^{p}V_2(\bar{\boldsymbol{x}}^v(i,j))]=\sum_{j=0}^{p}[V_1(\bar{\boldsymbol{x}}^h(q+1,j))-V_1(\bar{\boldsymbol{x}}^h(1,j))]+\sum_{i=1}^{q}[V_2(\bar{\boldsymbol{x}}^v(i,p+1))-V_2(\bar{\boldsymbol{x}}^v(i,0))]<0$ for any $p\in\mathcal{N}_1, q\in\mathcal{N}_{1,T-1}$. It follows that $\sum_{j=0}^{p}V_1(\bar{\boldsymbol{x}}^h(q+1,j))<\sum_{j=0}^{p}V_1(\bar{\boldsymbol{x}}^h(1,j))+\sum_{i=1}^{q}V_2(\bar{\boldsymbol{x}}^v(i,0))$. Thus, we have $\sum_{j=0}^{\infty}V_1(\bar{\boldsymbol{x}}^h(q+1,j))<\infty, q\in\mathcal{N}_{1,T-1}$ and $\lim_{j\to\infty}\bar{\boldsymbol{x}}^h(i,j)=0, i\in\mathcal{N}_{2,T}$, if $\sum_{j=0}^{\infty}|\bar{\boldsymbol{x}}^h(1,j)|^2<\infty$.

Let $\gamma=1-\frac{\lambda_{\min}(\Upsilon)}{\lambda_{\max}(\Upsilon)}\in(0,1)$. Then, we have $V_2(\bar{\boldsymbol{x}}^v(i,j+1))\le\gamma V_2(\bar{\boldsymbol{x}}^v(i,j))-V_1(\bar{\boldsymbol{x}}^h(i+1,j))+V_1(\bar{\boldsymbol{x}}^h(i,j))$ that implies $\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,j+1))\le\gamma\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,j))+V_1(\bar{\boldsymbol{x}}^h(1,j))$. It follows that $\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,p))\le\gamma^p\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,0))+\sum_{j=1}^{p}\gamma^{j-1}V_1(\bar{\boldsymbol{x}}^h(1,p-j))$ for any integer $p\ge1$. Hence, we have $\sum_{p=1}^{\infty}\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,p))\le\frac{\gamma}{1-\gamma}\sum_{i=1}^{T}V_2(\bar{\boldsymbol{x}}^v(i,0))+\frac{1}{1-\gamma}\sum_{j=0}^{\infty}V_1(\bar{\boldsymbol{x}}^h(1,j))$, and $\lim_{j\to\infty}\bar{\boldsymbol{x}}^v(i,j)=0,\forall i\in\mathcal{N}_{1,T}$ if $\sum_{j=0}^{\infty}|\bar{\boldsymbol{x}}^h(1,j)|^2<\infty$.

Therefore, the asymptotic stability is proved. □

Appendix C: *Proof of Theorem 3.2*

Introduce $J(i,j)=\Delta V(\bar{\boldsymbol{x}})+\bar{\boldsymbol{z}}^T(i,j)\bar{\boldsymbol{z}}(i,j)-\bar{\boldsymbol{\varpi}}^T(i,j)Q\bar{\boldsymbol{\varpi}}(i,j)$, where $\Delta V(\bar{\boldsymbol{x}})$ is defined in proceeding Appendix, Proof of Theorem 3.1. Then, for any scalar $\mu>0$, we have $J(i,j)\le\Delta V(\bar{\boldsymbol{x}})+\bar{\boldsymbol{z}}^T(i,j)\bar{\boldsymbol{z}}(i,j)-\bar{\boldsymbol{\varpi}}^T(i,j)Q\bar{\boldsymbol{\varpi}}(i,j)+\mu[\bar{\boldsymbol{x}}^T(i,j)\bar{\Theta}\bar{\boldsymbol{x}}(i,j)-\bar{\boldsymbol{\psi}}^T(i,j)\bar{\boldsymbol{\psi}}(i,j)]=-\boldsymbol{\xi}^T(i,j)\Pi\boldsymbol{\xi}(i,j)$, where $\mu>0$, $\boldsymbol{\xi}(=[\bar{\boldsymbol{x}}^T\ \bar{\boldsymbol{\varpi}}^T\ \bar{\boldsymbol{\psi}}^T]^T$, and $(-\Pi)$ is the left-hand side of inequality (12). Note that $\Pi>0$ implies the inequality (11). Thus, the 2-D system (8) with $\bar{\boldsymbol{\varpi}}=0$ is asymptotic stability. Furthermore, we also can get $\sum_{i=1}^{T}\sum_{j=0}^{\infty}(\Delta V(\bar{\boldsymbol{x}})+\bar{\boldsymbol{z}}^T(i,j)\bar{\boldsymbol{z}}(i,j)-\bar{\boldsymbol{\varpi}}^T(i,j)Q\bar{\boldsymbol{\varpi}}(i,j))\le-\lambda_{\min}(\Pi)\sum_{i=1}^{T}\sum_{j=0}^{\infty}|\bar{\boldsymbol{\varpi}}(i,j)|^2$.

Next, we show that, subject to (9), the 2-D system (8) satisfies (10) for all non-zero $\bar{\boldsymbol{\varpi}}$. Note that, for any integers $p\ge1, q\ge0$, $\sum_{i=1}^{p}\sum_{j=0}^{q}\Delta V(\bar{\boldsymbol{x}})=\sum_{j=0}^{q}[\bar{\boldsymbol{x}}^{hT}(p+1,j)P_1\bar{\boldsymbol{x}}^h(p+1,j)-\bar{x}^{hT}(1,j)P_1\bar{\boldsymbol{x}}^h(1,j)]+\sum_{i=1}^{p}[\bar{\boldsymbol{x}}^{vT}(i,q+1)P_2\bar{\boldsymbol{x}}^v(i,q+1)-\bar{x}^{vT}(i,0)P_2\bar{\boldsymbol{x}}^v(i,0)]$. Then, we have $\sum_{i=1}^{T}\sum_{j=0}^{\infty}\Delta V(\bar{\boldsymbol{x}})\ge-\lambda_{\max}(P)(\sum_{i=1}^{T}|\bar{\boldsymbol{x}}^v(i,0)|^2+\sum_{j=0}^{\infty}|\bar{\boldsymbol{x}}^h(1,j)|^2)$. It follows that $\sum_{i=1}^{T}\sum_{j=0}^{\infty}|\bar{\boldsymbol{z}}(i,j)|^2\le$

$\lambda_{\max}(P)(\sum_{i=1}^{T}|\bar{\boldsymbol{x}}^v(i,0)|^2 + \sum_{j=0}^{\infty}|\bar{\boldsymbol{x}}^h(1,j)|^2) + (\lambda_{\max}(Q) - \lambda_{\min}(\Pi))\sum_{i=1}^{T}\sum_{j=0}^{\infty}|\bar{\boldsymbol{\varpi}}(i,j)|^2$ which yields the assertion (10) for all non-zero $\bar{\boldsymbol{\varpi}}$. Therefore, the proof is completed. □

Appendix D: *Proof of Theorem 4.1*

Note that $\boldsymbol{\psi}_k^T(t)\boldsymbol{\psi}_k(t) \le \begin{bmatrix} \delta\boldsymbol{x}_k(t) \\ \hat{\boldsymbol{e}}_k(t) \end{bmatrix}^T \tilde{\Theta} \begin{bmatrix} \delta\boldsymbol{x}_k(t) \\ \hat{\boldsymbol{e}}_k(t) \end{bmatrix}$, where $\tilde{\Theta} = \begin{bmatrix} [I_n\ 0] \\ \mathcal{K} \end{bmatrix}^T \Theta \begin{bmatrix} [I_n\ 0] \\ \mathcal{K} \end{bmatrix}$. Then, from Theorem 3.2, the 2-D H_∞ performance of the 2-D ILC system (7) with controller (13) can be guaranteed by

$$\begin{bmatrix} \Omega_{11} & (\mathcal{A}+\mathcal{B}_1\mathcal{K})^T P\mathcal{B}_2 & (\mathcal{A}+\mathcal{B}_1\mathcal{K})^T P\mathcal{D} \\ * & \mathcal{B}_2^T P\mathcal{B}_2 - Q & \mathcal{B}_2^T P\mathcal{D} \\ * & * & \mathcal{D}^T P\mathcal{D} - \mu I_s \end{bmatrix} < 0 \quad (19)$$

which is equivalent to

$$\begin{bmatrix} -P & P(\mathcal{A}+\mathcal{B}_1\mathcal{K}) & P\mathcal{B}_2 & P\mathcal{D} \\ * & -P+\mathcal{L}^T\mathcal{L}+\mu\tilde{\Theta} & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -\mu I_s \end{bmatrix} < 0, \quad (20)$$

where $\Omega_{11} = (\mathcal{A}+\mathcal{B}_1\mathcal{K})^T P(\mathcal{A}+\mathcal{B}_1\mathcal{K}) - P + \mathcal{L}^T\mathcal{L} + \mu\tilde{\Theta}$. Now, pre- and post-multiplying above inequality by $diag\{P^{-1}, P^{-1}, I, I\}$ and $diag\{P^{-1}, P^{-1}, I, I\}$, respectively, then the equivalence between (20) and $\Sigma < 0$ in (14) with $W = P^{-1}$ can be shown easily by using the Schur complements. Thus, we obtain the 2-D H_∞ performance with $\gamma_1 = \lambda_{\max}(W^{-1}), \gamma_2 = \lambda_{\max}(Q)$.

Furthermore, we can obtain the optimal H_∞ performance by minimizing $\lambda_{\max}(W^{-1}) + \lambda_{\max}(Q)$. This can be achieved by minimizing γ subject to $W^{-1} < \alpha_1\gamma I_{n+\varrho q}, Q < \alpha_2\gamma I_{n+q}$, where $\alpha_1 > 0, \alpha_2 > 0$ with $\alpha_1 + \alpha_2 = 1$ are the weights of initial state and disturbance, respectively. Then, combined with the inequality $\Sigma < 0$, we obtain the generalised eigenvalue problem (14). Furthermore, from Theorem 3.2, we can easily get the inequality (15). □

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